

# Simulation of Fuel Tank Model in A Hybrid Framework

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## Abstract

This paper presents the modeling and simulation of fuel tank model in a hybrid framework. Hybrid Dynamical Systems (HDS) show the characteristics of both continuous dynamical system and discrete dynamical system. The modeling of such Hybrid Systems is difficult because not only both continuous and discrete behaviors are present but also interact with each other. In this paper fuel tank model is modeled using the hybrid framework and to simulate the continuous as well as discrete behavior of the fuel tank model, SIMULINK and STATEFLOW packages of MATLAB are used.

**Keywords:**Hybrid Dynamical Systems, HDS, Fuel Tank Model, STATEFLOW, SIMULINK, MATLAB.

## 1. Introduction

Dynamical Systems can be classified into two categories namely continuous dynamical systems and discrete dynamical systems. Continuous dynamical systems can be represented using differential equations while discrete dynamical systems can be represented using difference equations. But in the real world, many systems are present which shows the characteristics of both continuous dynamical systems as well as discrete dynamical systems. These types of systems are defined as Hybrid dynamical Systems (HDS). Bouncing ball is the best example of Hybrid Dynamical System. Consider a ball is dropped from a certain height, and then it will fall freely satisfying the differential equations of motion under the gravity until it reaches to the ground. But at the impact with the surface its velocity gets reversed after losing some energy. In this manner it loses some energy at every impact with the surface followed by a free fall. A number of biological systems, such as groups of fireflies or crickets are able to produce synchronized behavior, flashing or chirping, respectively, through a dynamical mechanism that can be viewed as Hybrid.

Hybrid Systems were first introduced in 1966 by Witsenhausen. Hybrid Systems can be studied in different domains such as Control Systems, Computer Science, and Mathematics etc. It can be referred as a bridge between Control Theory and Communication Theory. There are two different techniques are used to analyze and develop

controllers for Hybrid Systems. First one is to apply discrete analysis and control design tools which is used in Computer Science domain to model the systems. And the other one is ignoring the discrete part and analyzing the specific dynamics at each location which is mostly used in the Control Systems domain. There are numerous applications of Hybrid Systems in embedded systems, chemical processes, robotics, manufacturing systems, traffic management, bio-molecular networks, automobiles, and power systems etc.

In this paper fuel tank model is designed using the Hybrid framework and simulated using SIMULINK and STATEFLOW in MATLAB.

## 2. Modeling Framework

The model of a hybrid system can be represented in following form

$$\begin{matrix} x \in C & \dot{x} \in F(x) \\ x \in D & x^+ \in G(x) \end{matrix} \quad (1)$$

Also hybrid systems can be represented using differential or difference inclusions in the following manner

$$\begin{matrix} x \in C & \dot{x} = f(x) \\ x \in D & x^+ = g(x) \end{matrix} \quad (2)$$

This representation suggests that, the state of the hybrid system represented by  $x$ , can change according to differential inclusion  $\dot{x} \in F(x)$  or differential equation  $\dot{x} = f(x)$  while in set  $C$ , and it can change according to a difference inclusion  $x^+ \in G(x)$  or difference equation  $x^+ = g(x)$  while in the set  $D$ .

This leads to the following names for the four objects involved in (1) or (2) :

- $C$  is the flow set
- $F$  (or  $f$ ) is the flow map
- $D$  is the jump set
- $G$  (or  $g$ ) is the jump map

As the model in (1) or (2) suggests, the flow set, the flow map, the jump set, and the jump map can be specialized to capture the dynamics of purely continuous time or discrete time systems on  $\mathbb{R}^n$ .

### 3. A Fuel Tank Example

The fuel is stored between a production unit and customer unit. The function of the fuel tank is to control the system in order to facilitate the balance between production and customer demand as shown in figure below.



Fig. 1 Fuel Tank Model

According to the production flow and customer demand, this model can be configured in four different configurations.

#### 3.1 Configuration 1

In the first configuration, the production flow rate and the customer flow rate are identical. Because of that the storage can be by-passed. Configuration 1 is presented in fig. 2 below.

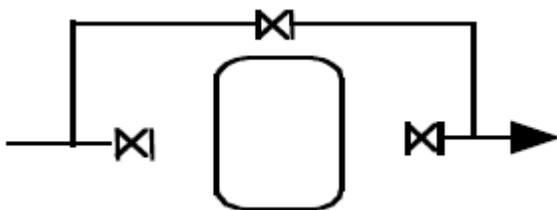


Fig. 2 Configuration 1

The differential equation associated with configuration 1 is given as

$$\frac{dx}{dt} = 0 \quad (3)$$

Where, x is the stored fuel.

#### 3.2 Configuration 2

In this configuration production is greater than the demand so the surplus is transferred to the storage as shown in figure below.

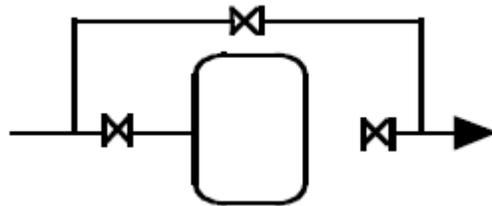


Fig. 3 Configuration 2

Configuration 2 can be described by the differential equation as

$$\frac{dx}{dt} = Dp - Dc \quad (4)$$

Where, x is the stored fuel, Dp is production flow, and Dc is the customer demand.

#### 3.3 Configuration 3

In configuration 3 the production flow is lower than the customer demand. So, the lack of fuel is drawn from the storage. Following Fig. 4 shows the representation of configuration 3.

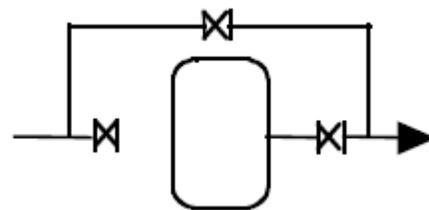


Fig. 4 Configuration 3

The evolution of the configuration 3 can be explained by the differential equation represented by (5)

$$\frac{dx}{dt} = Dp - Dc \quad (5)$$

### 3.4 Configuration 4

If the production flow is stopped, then this can be represented by configuration 4. Sometimes production unit can be stopped manually or automatically due to technical problems etc. This configuration is as shown in figure below.

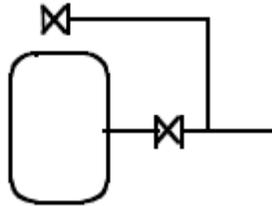


Fig. 5 Configuration 4

Here, production flow is not present, so the configuration 4 can be modeled using the following differential equation.

$$\frac{dx}{dt} = -Dc \quad (6)$$

In this way, a fuel tank storage model follows any of the above differential equations at a particular instant depending upon the specific production flow and customer demand.

## 4. Simulation and Results

A fuel tank model described above is simulated using SIMULINK and STATEFLOW in MATLAB. As this is a hybrid dynamical system comprises of both continuous and discrete part, simulation with ode45 is not possible due to presence of discrete part. Therefore, STATEFLOW is used and interfaced with the SIMULIK. STATEFLOW is a graphical design and development tool for control and supervisory logic used in conjunction with SIMULINK. It provides clear, concise descriptions of complex system behavior using finite machine theory, flow diagram notations, and state-transition diagrams all in the same stateflow diagram.

SIMULINK model of a fuel tank is as shown in figure below.

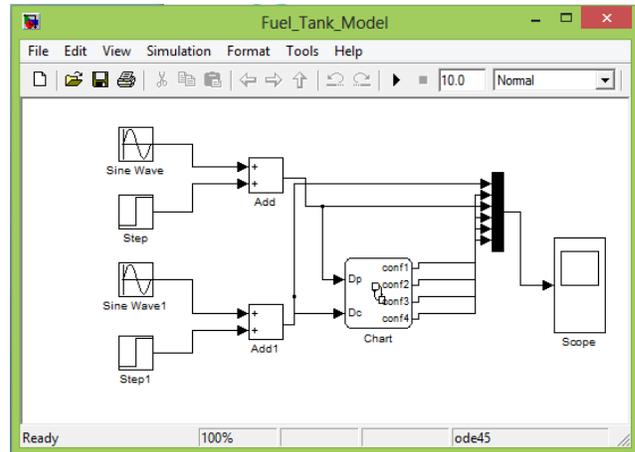


Fig. 6 SIMULINK model

The discrete part of a fuel tank model is developed using STATEFLOW, as shown in figure below.

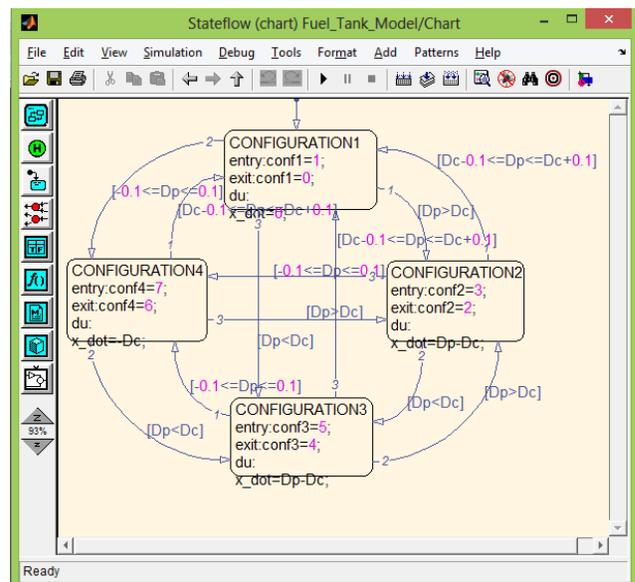


Fig. 7 STATEFLOW model

Fig. 8 shows the evolution of two continuous variables, in these case customer demand and production flows. And also the particular configuration at a particular instant of time.

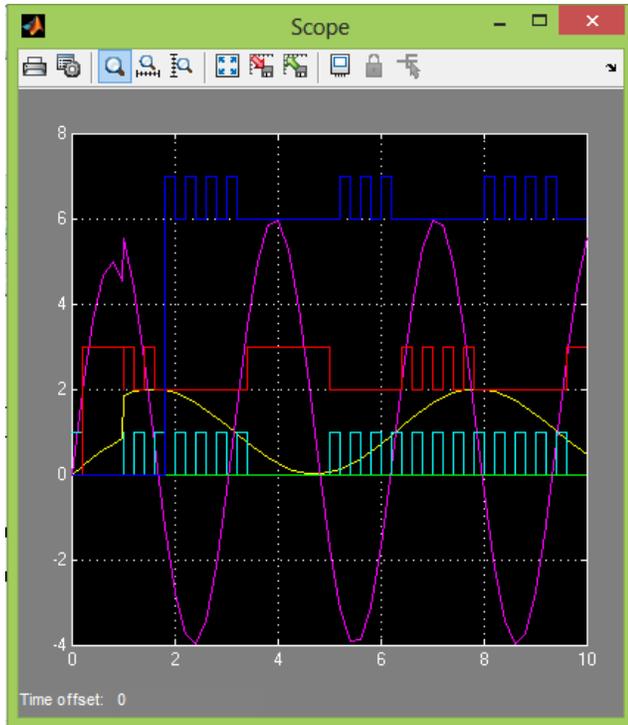


Fig. 8 Evolution of States

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## 5. Conclusion

In this paper , a fuel tank is modeled in hybrid framework and simulated using SIMULINK and STATEFLOW in MATLAB. Hybrid dynamical system framework can be used to model complex real world problems. Stability analysis tools can be used to predict the behavior of hybrid systems and to design hybrid control algorithms.

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