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Results on Cycle related product cordial graph

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Abstract

Let G = (V,E) be a graph with p vertices and q edges. A Product cordial labeling of a Graph G with vertex set V is a bijection from V to {0,1} such that if each edge uv is assigned the label (f(u) . f(v)) with the condition the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

The graph that admits a product cordial labeling is called a product cordial graph (PCG). In this paper,

we proved that Cycle C_n ,Double triangular snake $C_2(P_n)$: (n-odd) ,

($C_m \Theta K_{1,n}$) : (m-odd) , (n-odd) , ($C_3 @ S_n$) : (n-even) , ($C_n + v_1 v_3$) : (n-odd) are product cordial graphs.

Key words : *Product cordial labeling, Product cordial graph.*

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1. Introduction :

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e = {uv} of vertices in E is called an edge or a line of G. In this paper , we proved that Cycle C_n ,Double triangular snake $C_2(P_n)$: (n-odd) , ($C_m \Theta K_{1,n}$) : (m-odd) , (n-odd) , ($C_3 @ S_n$) : (n-even) , ($C_n + v_1 v_3$) : (n-odd) are product cordial graphs.

2. Preliminaries :

Let G = (V,E) be a graph with p vertices and q edges. A Product cordial labeling of a Graph G with vertex set V is a bijection from V to {0,1} such that if each edge uv is assigned the label (f(u) . f(v)) with the condition the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

The graph that admits a product cordial labeling is called a product cordial graph (PCG). we proved that Cycle C_n ,Double triangular snake $C_2(P_n)$: (n-odd) , ($C_m \Theta K_{1,n}$) : (m-odd) , (n-odd) , ($C_3 @ S_n$) : (n-even) , ($C_n + v_1 v_3$) : (n-odd) are product cordial graphs.

Definition 2.1

$C_m \boldsymbol{\Theta}$ $K_{1,n}$ is a graph obtained by joining one end vertex of a star having n edges, to one of the vertex of a cycle of length m.

Definition 2.2

A closed path is called a ***cycle*** and a cycle of length n is denoted by C_n.

Definition 2.3

Roots of a star is attached at any one of the vertex of C_n.It is denoted by $C_n @ S_4$.

Definition 2.4

In any cycle there is a chord between any two non-adjacent vertices.

Definition 2.5

Graph obtained from a path P_n , by joining each end vertices of an edge with two isolated vertex. It is denoted by $C_2(P_n)$.

3. Main Results

Theorem: 3.1

Cycle C_n : (n−odd) is product cordial.

Proof:

Let G be C_n.

Let V (G) = { [u_i : $1 \leq i \leq n$] } and

E(G) = { [$(u_i u_{i+1})$: $1 \leq i \leq n-1$] U [$(u_1 u_n)$] }.

Define $f : V(G) \rightarrow \{0,1\}$.

Case: 1

When n =3,

The labeling is ,

Case: 2

When n>3,

The vertex labelings are,

$f(u_1) = 0$.

$f(u_i) = 1 \quad 2 \leq i \leq (n+3)/2$

$f(u_i) = 0 \quad (n+5)/2 \leq i \leq n$.

The induced edge labelings are,

$f^*(u_1 u_2) = 0$.

$f^*(u_i u_{i+1}) = 1 \quad 2 \leq i \leq (n+1)/2$

$f^*(u_i u_{i+1}) = 0 \quad (n+3)/2 \leq i \leq n-1$.

$f^*(u_i u_n) = 0 \quad i = n$.

Here $v_f(1) + 1 = v_f(0)$ for all n and

$e_f(0) + 1 = e_f(1)$ for all n.

Therefore, the graph G satisfies the conditions $| v_f(0) - v_f(1) | \leq 1$ and $|e_f(0) - e_f(1) | \leq 1$.

Hence, C_n : (n-odd) is product cordial.

For example, the product cordial labeling of C_5 is shown in figure 3.2.

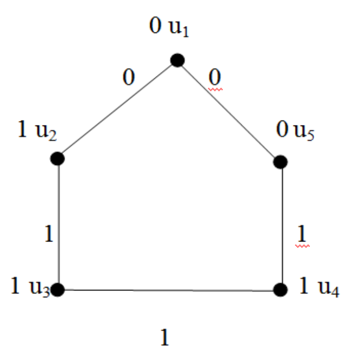


Figure 3.2

Theorem: 3.3

Cycle C_n : (n-even) is not product cordial.

Proof:

Let G be C_n.

Let V(G) = { [u_i : $1 \leq i \leq n$] } and

E(G) = { [$(u_i u_{i+1})$: $1 \leq i \leq n-1$] }

If a vertex u_i is assigned label 0 then the labels of adjacent edge $(u_i u_{i+1})$ and $(u_i u_{i-1})$ are Zero.

Moreover, vertex assigning labels 0 and 1 are equal i.e $v_f(0) = v_f(1) = n/2$ it does not satisfy the condition $|e_f(0) - e_f(1) | \leq 1$.

If $v_f(0) \neq v_f(1)$ it does not satisfy the condition $|v_f(0) - v_f(1)| \leq 1$.

Hence C_n : (n even) is not product cordial.

Theorem: 3.4

$C_2(P_n)$: (n–odd) is product cordial.

Proof:

Let G be $C_2(P_n)$.

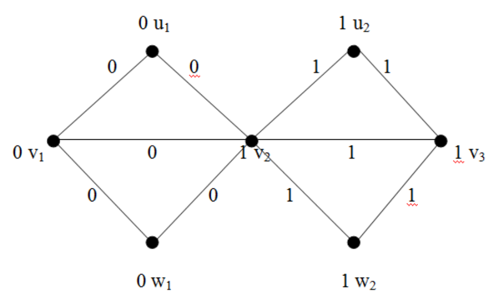
Let V (G) = { [u_i ,w_i : $1 \leq i \leq n-1$] ,[v_i : $1 \leq i \leq n$] } and E(G) = { [$(v_i v_{i+1})$: $1 \leq i \leq n-1$] U [$(u_i v_i)$ U $(w_i v_i)$: $1 \leq i \leq n-1$] U [$(u_i v_{i+1})$U $(w_i v_{i+1})$: $1 \leq i \leq n-1$}.

Define $f : V(G) \rightarrow \{0,1\}$.

Case: 1

When n =3,

The labeling is ,



Case: 2

When n>3,

The vertex labelings are,

$f(u_1) = 0.$

$f(u_i) = 1 \quad 2 \le i \le (n+1)/2$

$f(u_i) = 0 \quad (n+3)/2 \le i \le n-1.$

$f(w_1) = 0.$

$f(w_i) = 1 \quad 2 \le i \le (n+1)/2$

$f(w_i) = 0 \quad (n+3)/2 \le i \le n-1.$

$f(v_1) = 0.$

$f(v_i) = 1 \quad 2 \le i \le (n+3)/2$

$f(v_i) = 0 \quad (n+5)/2 \le i \le n.$

The induced edge labelings are,

$f^*(v_1v_2) = 0.$

$f^*(v_iv_{i+1}) = 1 \quad 2 \le i \le (n+1)/2$

$f^*(v_iv_{i+1}) = 0 \quad (n+3)/2 \le i \le n-1.$

$f^*(u_1v_1) = 0.$

$f^*(u_iv_i) = 1 \quad 2 \le i \le (n+1)/2$

$f^*(u_iv_i) = 0 \quad (n+3)/2 \le i \le n-1.$

$f^*(w_1v_1) = 0.$

$f^*(w_iv_i) = 1 \quad 2 \le i \le (n+1)/2$

$f^*(w_iv_i) = 0 \quad (n+3)/2 \le i \le n-1.$

$f^*(u_1v_2) = 0.$

$f^*(u_iv_{i+1}) = 1 \quad 2 \le i \le (n+1)/2$

$f^*(u_iv_{i+1}) = 0 \quad (n+3)/2 \le i \le n-1.$

$f^*(w_1v_2) = 0.$

$f^*(w_iv_{i+1}) = 1 \quad 2 \le i \le (n+1)/2$

$f^*(w_iv_{i+1}) = 0 \quad (n+3)/2 \le i \le n-1$

Here $v_f(1) = v_f(0) + 1$ for all n and

$e_f(0) = e_f(1)$ for all n.

Therefore, the graph G satisfies the conditions $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence, $C_2(P_n)$: (n-odd) is product cordial.

For example, the product cordial labeling of $C_2(P_5)$ is shown in figure 3.5.

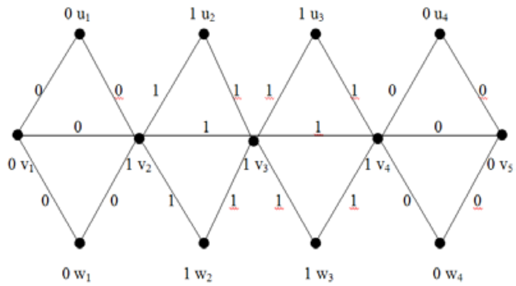


Figure : 3.5 : $C_2(P_5)$

Theorem: 3.6

$(C_m \Theta K_{1,n})$: (m−odd),(n−even) is product cordial.

Proof:

Let G be $(C_m \Theta K_{1,n})$.

Let V (G) = { $[u_i : 1 \le i \le m]$,$[u_{ij} : 1 \le i \le m, 1 \le j \le n]$ } and

E(G) = { $[(u_iu_{i+1}) : 1 \le i \le m-1]$ U $[(u_1u_m)]$ U $[(u_iu_{ij}) : 1 \le i \le m, 1 \le j \le n]$ }.

Define $f : V(G) \rightarrow \{0,1\}$.

The vertex labelings are,

$f(u_1) = 0.$

$f(u_i) = 1 \quad 2 \le i \le (m+3)/2$

$f(u_i) = 0 \quad (m+5)/2 \le i \le m.$

$f(u_{1j}) = 0 \quad 1 \le j \le n.$

$f(u_{ij}) = 1 \quad 2 \le i \le (m+1)/2, 1 \le j \le n.$

$$f(u_{ij}) = \begin{cases} 0 & j \equiv 1 \bmod 2 \\ 1 & j \equiv 0 \bmod 2 \end{cases}$$

$i = (m+3)/2, \ 1 \le j \le n.$

$f(u_{ij}) = 0 \quad (m+5)/2 \le i \le m, \ 1 \le j \le n.$

The induced edge labelings are,

$f^*(u_1u_2) = 0.$

$f^*(u_iu_{i+1}) = 1 \quad 2 \le i \le (m+1)/2.$

$f^*(u_iu_{i+1}) = 0 \quad (m+3)/2 \le i \le m-1.$

$f^*(u_1u_m) = 0.$

$f^*(u_1u_{1j}) = 0 \quad 1 \le j \le n.$

$f^*(u_iu_{ij}) = 1 \quad 2 \le i \le (m+1)/2, 1 \le j \le n.$

$$f^*(u_iu_{ij}) = \begin{cases} 0 & j \equiv 1 \bmod 2 \\ 1 & j \equiv 0 \bmod 2 \end{cases}$$

$i = (m+3)/2 , 1 \le j \le n.$

$f^*(u_iu_{ij}) = 0 \; (m+5)/2 \le i \le m , 1 \le j \le n.$

Here $v_f(1) = v_f(0) + 1$ for all n and

$e_f(0) = e_f(1) + 1$ for all n.

Therefore, the graph G satisfies the conditions $| v_f(0) - v_f(1) | \le 1$ and $|e_f(0) - e_f(1) | \le 1$.

Hence, $(C_m \Theta K_{1,n})$: (m-odd) , (n-even) is product cordial.

For example, the product cordial labeling of $(C_5 \Theta K_{1,2})$ is shown in figure 3.7.

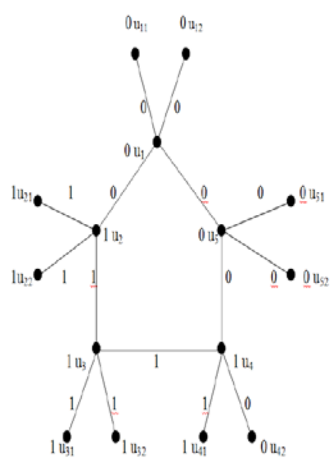


Figure 3.7 : $(C_5 \Theta K_{1,2})$

Theorem: 3.8

$(C_3@)S_n)$: (n−even) is product cordial.

Proof:

Let G be $(C_3@S_n)$.

Let V (G) = { $[u_1 , u_2 , u_3] ,[u_{3j} : 1 \le j \le n]$ } and

E(G) = { $[(u_1u_2)] \cup [(u_2u_3)] \cup [(u_3u_1)]$

$\cup [(u_3u_{3j}) : 1 \le j \le n]$ }.

Define $f : V(G) \to \{0,1\}$.

The vertex labelings are,

$f(u_1) = 0.$

$f(u_2) = 1$

$f(u_3) = 1$

$$f(u_{3j}) = \begin{cases} 0 & j \equiv 0 \bmod 2 \\ 1 & j \equiv 1 \bmod 2 \end{cases} \quad 1 \le j \le n.$$

The induced edge labelings are,

$f^*(u_1u_2) = 0.$

$f^*(u_2u_3) = 1$

$f^*(u_3u_1) = 0$

$$f^*(u_3u_{3j}) = \begin{cases} 0 & j \equiv 0 \bmod 2 \\ 1 & j \equiv 1 \bmod 2 \end{cases}$$

$1 \le j \le n.$

$f^*(u_iu_{ij}) = 0 \quad (m+5)/2 \le i \le m , 1 \le j \le n.$

Here $v_f(1) = v_f(0) + 1$ for all n and

$e_f(0) = e_f(1) + 1$ for all n.

Therefore, the graph G satisfies the conditions $| v_f(0) - v_f(1) | \le 1$ and $|e_f(0) - e_f(1) | \le 1$.

Hence, $(C_3@S_n)$: (n-even) is product cordial.

For example, the product cordial labeling of $(C_3@S_4)$ is shown in figure 3.9.

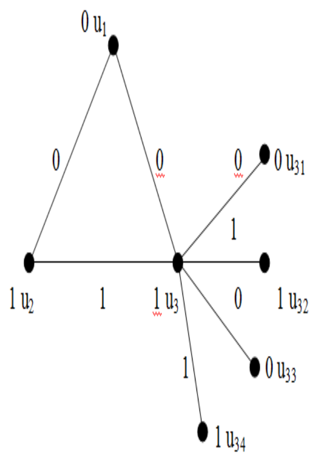


Figure 3.9 : ($C_3@S_4$)

Theorem: 3.10

Cycle $C_n + v_1v_3$: (n−odd) ,(n>3) is product cordial.

Proof:

Let G be $C_n + v_1v_3$.

Let V (G) = { [v_i : $1 \leq i \leq n$] } and

E(G) = { [(v_iv_{i+1}) : $1 \leq i \leq n-1$] U [(v_1v_n)]

U [(v_1v_3)] }.

Define $f : V(G) \rightarrow \{0,1\}$.

The vertex labelings are,

$f(v_i) = 1 \qquad 1 \leq i \leq (n+1)/2$

$f(v_i) = 0 \qquad (n+3)/2 \leq i \leq n.$

The induced edge labelings are,

$f^*(v_1v_3) = 1.$

$f^*(v_1v_n) = 0.$

$f^*(u_iu_{i+1}) = 1 \qquad 2 \leq i \leq (n+1)/2$

$f^*(v_iv_{i+1}) = 1 \qquad 1 \leq i \leq (n-1)/2.$

$f^*(v_iv_{i+1}) = 0 \qquad (n+1)/2 \leq i \leq n-1.$

Here $v_f(0) + 1 = v_f(1)$ for all n and

$e_f(0) = e_f(1)$ for all n.

Therefore, the graph G satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, $C_n+v_1v_3$: (n-odd) is product cordial.

For example the product cordial labeling of $C_5+v_1v_3$ is shown in figure 3.11.

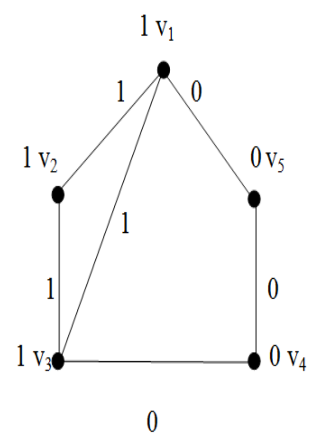


Figure 3.11 : $C_5+v_1v_3$

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