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Cycle and Armed Cup cordial graphs

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Abstract

Let G = (V,E) be a graph with p vertices and q edges. A Cup (V) cordial labeling of a Graph G with vertex set V is a bijection from V to {0,1} such that if each edge uv is assigned the label $f(uv) = \begin{cases} 0 & if \ f(u) = f(v) = 0 \\ 1 & otherwise \end{cases}$ with the condition the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1.

The graph that admits a V cordial labeling is called a V cordial graph (CCG). In this paper, we proved that Cycle C_n (n : even), Bistar $B_{m,n}$, $P_m \Theta P_n$ and Helm are V cordial graphs.

Key words : *V cordial labeling, V cordial graph.*
2000 Mathematics Subject Classification 05C78.

1. Introduction :

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e = {uv} of vertices in E is called an edge or a line of G. In this paper , we proved that Cycle C_n (n : even), Bistar $B_{m,n}$, $P_m \Theta P_n$ and Helm are V cordial graphs.

2. Preliminaries :

Let G = (V,E) be a graph with p vertices and q edges. A V cordial labeling of a Graph G with vertex set V is a bijection from V to {0,1} such that if each edge uv is assigned the label $f(uv) = \begin{cases} 0 & if \ f(u) = f(v) = 0 \\ 1 & otherwise \end{cases}$ with the condition the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1.

The graph that admits a V cordial labeling is called a V cordial graph (CCG). We proved that Cycle C_n (n : even), Bistar $B_{m,n}$, $P_m \Theta P_n$ and Helm are V cordial graphs.

Definition 2.1 - Cycle

A graph with sequence of vertices u_1, u_2 …. u_n such that successive vertices are joined with an edge. P_n is a path of length n-1.

The closed path of length n is Cycle C_n.

Definition 2.2 - $P_m \Theta P_n$

It is a graph obtained from a path P_m by joining a path of length P_n at each vertex of P_m, it is denoted by $P_m \Theta P_n$.

Definition 2.3 – Bi-star

It is a graph obtained from a path P_2 by joining the root of stars S_m and S_n at the terminal vertices of P_2. It is denoted by $B_{m,n}$.

Definition 2.4 – Helm

It is a graph obtained from a Cycle C_n by joining a pendent vertex at each vertex of on C_n. It is denoted by $C_n \odot K_1$.

3. Main results :

Theorem 3.1

Cycle C_n (n : odd) is a V cordial graph

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Proof

Let $V(C_n)=\{u_i:1\le i\le n\}$ and

$$E(C_n)=\{[(u_iu_{i+1}):1\le i\le n-1]\cup(u_1u_n)\}$$

Define $f:V(C_n)\to\{0,1\}$

The vertex labeling are

$$f(u_i)=\begin{cases}0 & 1\le i\le\frac{n+1}{2}\\ 1 & \frac{n+3}{2}\le i\le n\end{cases}$$

The induced edge labeling are

$$f^*(u_iu_{i+1})=\begin{cases}0 & 1\le i\le\frac{n-1}{2}\\ 0 & \frac{n+1}{2}\le i\le n-1\end{cases}$$

$$f^*(u_1u_n)=1$$

Here $V_0(f)=V_1(f)+1$ and

$$e_0(f)+1=e_1(f)$$

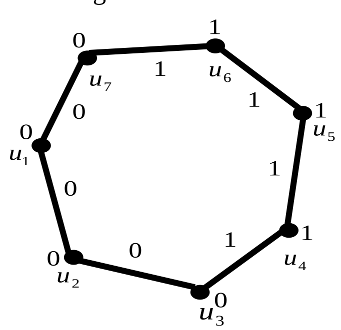
It satisfies the condition

$$|V_0(f)-V_1(f)|\le 1 \text{ and}$$

$$|e_0(f)-e_1(f)|\le 1$$

Hence, C_n is V cordial graph.

For example, C_5 and C_7 are V cordial graphs as shown in the figure 3.2.



. *Figure 3.2*

Theorem 3.3

Star S_n is a V cordial graph

Proof

Let $V(S_n)=\{u,u_i:1\le i\le n\}$ and

$$E(S_n)=\{(uu_i):1\le i\le n\}$$

Define $f:V(S_n)\to\{0,1\}$

The vertex labeling are

Case 1: When n is even

$$f(u)=0$$

$$f(u_i)=\begin{cases}0 & 1\le i\le\frac{n}{2}\\ 1 & \frac{n}{2}+1\le i\le n\end{cases}$$

The induced edge labeling are

$$f^*(uu_i)=\begin{cases}0 & 1\le i\le\frac{n}{2}\\ 1 & \frac{n}{2}+1\le i\le n\end{cases}$$

Here $V_0(f)=V_1(f)+1$ and

$$e_0(f)=e_1(f)$$

It satisfies the condition

$$|V_0(f)-V_1(f)|\le 1 \text{ and}$$

$$|e_0(f)-e_1(f)|\le 1$$

Case 2: When n is odd

$$f(u)=0$$

$$f(u_i)=\begin{cases}0 & 1\le i\le\frac{n-1}{2}\\ 1 & \frac{n+1}{2}\le i\le n\end{cases}$$

The induced edge labeling are

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$$f^*(uu_i)=\begin{cases}0 & 1\le i\le \dfrac{n-1}{2}\\ 1 & \dfrac{n+1}{2}\le i\le n\end{cases}$$

Here $V_0(f)=V_1(f)$ and

$$e_0(f)+1=e_1(f)$$

It satisfies the condition

$$|V_0(f)-V_1(f)|\le 1 \text{ and}$$

$$|e_0(f)-e_1(f)|\le 1$$

Hence, S_n is V cordial graph.

For example, S_5 and S_6 are V cordial graphs as shown in the figure 3.4 and 3.5

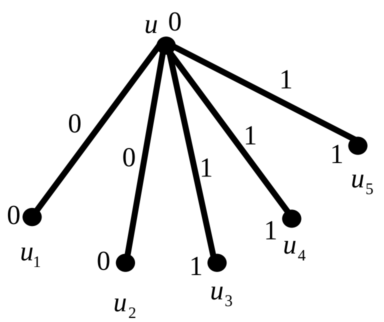

Figure 3.4

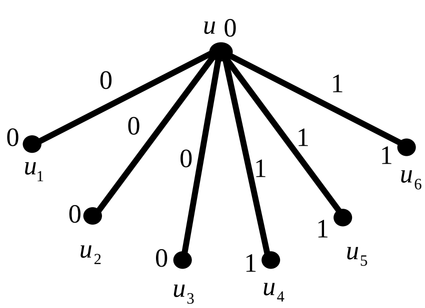

Figure 3.5

Theorem 3.6

Bistar $B_{m,n}$ is a V cordial graph.

Proof

Let $V(B_{m,n})=\{u,v,(u_i:1\le i\le m),(v_j:1\le j\le n)\}$

and

$$E(B_{m,n})=\begin{Bmatrix}[(uu_i):1\le i\le m]\cup[(vv_i):1\le i\le m]\\ \cup[(uv)]\end{Bmatrix}$$

Define $f:V(B_{m,n})\to\{0,1\}$

Case 1:

When m = n

The vertex labeling are

$$f(u)=\{0\}$$
$$f(v)=\{1\}$$
$$f(u_i)=\{0 \quad 1\le i\le m\}$$
$$f(v_i)=\{1 \quad 1\le i\le m\}$$

The induced edge labeling are

$$f^*(uu_i)=\{0 \quad 1\le i\le m\}$$
$$f^*(vv_i)=\{1 \quad 1\le i\le m\}$$
$$f^*(uv)=1$$

Here $V_0(f)=V_1(f)$ and

$$e_0(f)=e_1(f)+1$$

It satisfies the condition

$$|V_0(f)-V_1(f)|\le 1 \text{ and}$$

$$|e_0(f)-e_1(f)|\le 1$$

Case 2:

When m < n

The vertex labeling are

$$f(u)=\{0\}$$
$$f(v)=\{1\}$$
$$f(u_i)=\{0 \quad 1\le i\le m\}$$
$$f(v_i)=\{1 \quad 1\le i\le m\}$$

$$f(v_{m+i})=\begin{Bmatrix}0 & i\equiv 1 \bmod 2\\ 1 & i\equiv 0 \bmod 2\end{Bmatrix} 1\le i\le n-m$$

The induced edge labeling are

$$f^*(uu_i)=\{0 \quad 1\le i\le m\}$$

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$f^*(uv) = 1$

$f^*(vv_j) = \{1 \; i \le j \le m\}$

$$f^*(vv_{m+i}) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} 1 \le i \le n-m$$

Here, when n-m is odd

$V_0(f)+1 = V_1(f)$ and

$e_0(f) = e_1(f)$

When n-m is even

$V_0(f) = V_1(f)$ and

$e_0(f) = e_1(f)+1$

It satisfies the condition

$|V_0(f) - V_1(f)| \le 1$ and

$|e_0(f) - e_1(f)| \le 1$

Case 3:

When n < m

By substituting m by n and n by m in case 2 the result follows.

Hence, $B_{m,n}$ is a V cordial Graph

For example, $B_{3,3}$, $B_{2,6}$ and $B_{6,2}$ are V cordial graphs as shown in the figure 3.7, figure 3.8 and figure 3.9.

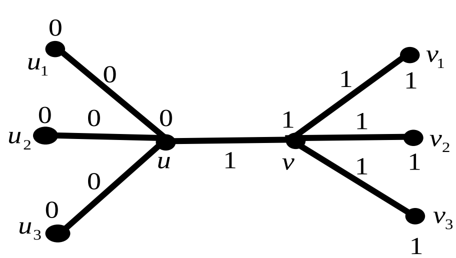

Figure 3.7

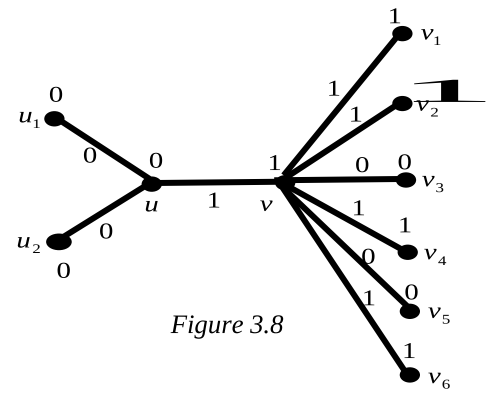

Figure 3.8

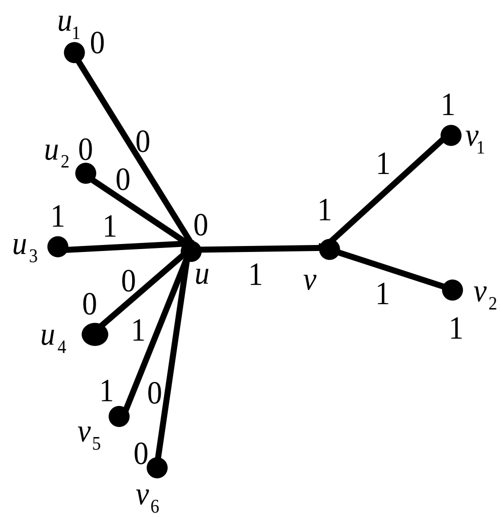

Figure 3.9

Theorem 3.10

The Graph $P_m \Theta P_n$ is V cordial.

Proof

Let G be [$P_m \Theta P_n$]

Let $V(G) = \begin{Bmatrix} [u_i : 1 \le i \le m], \\ [v_{ij} : 1 \le i \le m, 1 \le j \le n-1] \end{Bmatrix}$

and $E(G) = \begin{Bmatrix} [(u_i u_{i+1}) : 1 \le i \le m-1] \cup \\ [(u_i v_{i1}) : i \le i \le m] \cup \\ [(v_{ij} v_{ij+1}) : 1 \le i \le m, 1 \le j \le n-2 \end{Bmatrix}$

Define $f : V(G) \to \{0,1\}$

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Case 1:

When m is even

The vertex labeling are

$$f(u_i)=\begin{cases}0 & 1\le i\le \dfrac{m}{2}\\ 1 & \dfrac{m}{2}+1\le i\le m\end{cases}$$

$$f(v_{ij})=\begin{cases}0 & 1\le i\le \dfrac{m}{2}, 1\le j\le n-1\\ 1 & \dfrac{m}{2}+1\le i\le m, 1\le j\le n-1\end{cases}$$

The induced edge labeling are

$$f^*(u_iu_{i+1})=\begin{cases}0 & 1\le i\le \dfrac{m}{2}-1\\ 1 & \dfrac{m}{2}\le i\le m-1\end{cases}$$

$$f^*(u_iv_{i1})=\begin{cases}0 & 1\le i\le \dfrac{m}{2}\\ 1 & \dfrac{m}{2}+1\le i\le m\end{cases}$$

$$f^*(v_{ij}v_{ij+1})=\begin{cases}0 & 1\le i\le \dfrac{m}{2}, 1\le j\le n-2\\ 1 & \dfrac{m}{2}+1\le i\le m, 1\le j\le n-2\end{cases}$$

Here $V_0(f)=V_1(f)$ and

$e_0(f)+1=e_1(f)$

It satisfies the condition

$|V_0(f)-V_1(f)|\le 1$ and

$|e_0(f)-e_1(f)|\le 1$

Case 2:

When m is odd and n is odd

The vertex labeling are

$$f(u_i)=\begin{cases}0 & 1\le i\le \dfrac{m+1}{2}\\ 1 & \dfrac{m+3}{2}\le i\le m\end{cases}$$

$$f(v_{ij})=\begin{cases}0 & 1\le i\le \dfrac{m+1}{2}, 1\le j\le n-1\\ 1 & \dfrac{m+1}{2}\le i\le m, 1\le j\le n-1\end{cases}$$

$$f(v_{\frac{m+1}{2}j})=\begin{cases}0 & 1\le j\le \dfrac{n-1}{2}\\ 1 & \dfrac{n+1}{2}\le j\le n-1\end{cases}$$

The induced edge labeling are

$$f^*(u_iu_{i+1})=\begin{cases}0 & 1\le i\le \dfrac{m-1}{2}\\ 1 & \dfrac{m+1}{2}\le i\le m-1\end{cases}$$

$$f^*(u_iv_{i1})=\begin{cases}0 & 1\le i\le \dfrac{m+1}{2}\\ 1 & \dfrac{m+3}{2}\le i\le m\end{cases}$$

$$f^*(v_{ij}v_{ij+1})=\begin{cases}0 & 1\le i\le \dfrac{m-1}{2}, 1\le j\le n-2\\ 1 & \dfrac{m+3}{2}\le i\le m, 1\le j\le n-2\end{cases}$$

$$f^*(v_{\frac{m+1}{2}j}v_{\frac{m+1}{2}j+1})=\begin{cases}0 & 1\le j\le \dfrac{n-3}{2}\\ 1 & \dfrac{n-1}{2}\le j\le n-2\end{cases}$$

Here $V_0(f)+1=V_1(f)$ and

$e_0(f)=e_1(f)$

It satisfies the condition

$|V_0(f)-V_1(f)|\le 1$ and

$|e_0(f)-e_1(f)|\le 1$

Case 3:

When m is odd and n is even

The vertex labeling are

$$f(u_i)=\begin{cases}0 & 1\le i\le \frac{m+1}{2}\\ 1 & \frac{m+3}{2}\le i\le m\end{cases}$$

$$f(v_{ij})=\begin{cases}0 & 1\le i\le \frac{m+1}{2}, 1\le j\le n-1\\ 1 & \frac{m+1}{2}\le i\le m, 1\le j\le n-1\end{cases}$$

$$f(v_{\frac{m+1}{2}j})=\begin{cases}0 & 1\le j\le \frac{n-1}{2}\\ 1 & \frac{n}{2}\le j\le n-1\end{cases}$$

The induced edge labeling are

$$f^*(u_iu_{i+1})=\begin{cases}0 & 1\le i\le \frac{m-1}{2}\\ 1 & \frac{m+1}{2}\le i\le m-1\end{cases}$$

$$f^*(u_iv_{i1})=\begin{cases}0 & 1\le i\le \frac{m+1}{2}\\ 1 & \frac{m+3}{2}\le i\le m\end{cases}$$

$$f^*(v_{ij}v_{ij+1})=\begin{cases}0 & 1\le i\le \frac{m-1}{2}, 1\le j\le n-2\\ 1 & \frac{m+3}{2}\le i\le m, 1\le j\le n-2\end{cases}$$

$$f^*(v_{\frac{m+1}{2}j}v_{\frac{m+1}{2}j+1})=\begin{cases}0 & 1\le j\le \frac{n-4}{2}\\ 1 & \frac{n-2}{2}\le j\le n-2\end{cases}$$

Here $V_0(f)=V_1(f)$ and

$$e_0(f)+1=e_1(f)$$

It satisfies the condition

$$|V_0(f)-V_1(f)|\le 1 \text{ and}$$

$$|e_0(f)-e_1(f)|\le 1$$

Hence, the graph $P_m\Theta P_n$ is V cordial.

For example, $P_5\Theta P_4$, $P_4\Theta P_4$ and $P_5\Theta P_5$ are V cordial as shown in the figure 3.11, figure 3.12 and figure 3.13.

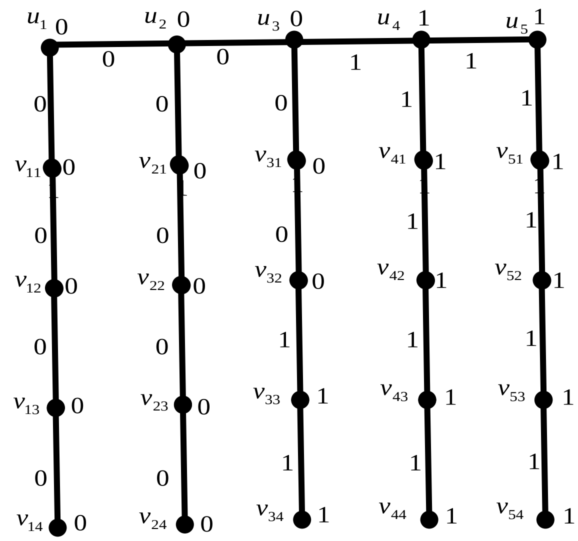

Figure 3.11

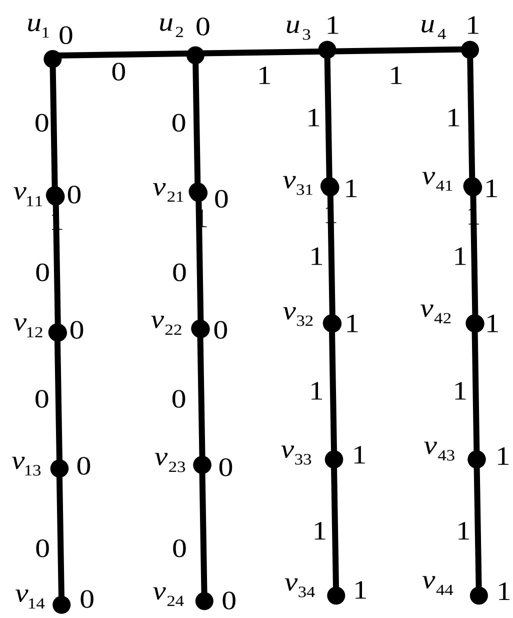

Figure 3.12

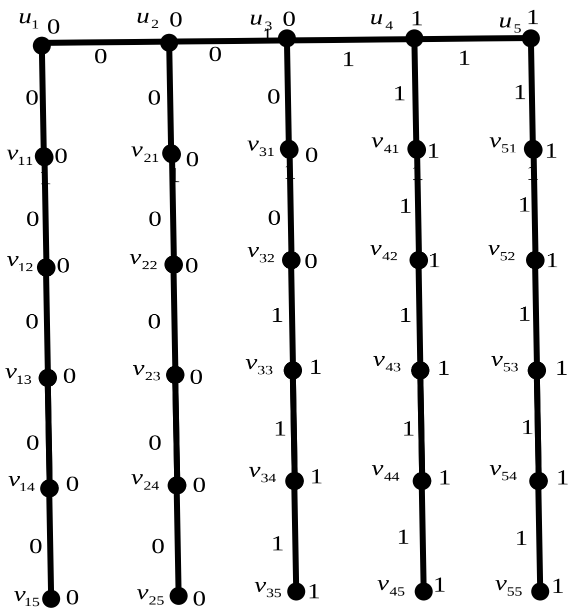

Figure 3.13

Theorem 3.14

Helm ($C_n \odot K_1$) is V cordial.

Proof

Let G be [$C_n \odot K_1$]

Let $V(G) = \{u_i, v_i : 1 \le i \le m\}$

and $E(G) = \{(u_i v_i) : 1 \le i \le m\}$

The vertex labeling are

$$f(u_i) = \{0 \quad 1 \le i \le m\}$$
$$f(v_i) = \{1 \quad 1 \le i \le m\}$$

The induced edge labeling are

$$f^*(u_i u_{i+1}) = \{0 \quad 1 \le i \le m-1\}$$
$$f^*(u_m u_1) = 0$$
$$f^*(u_i v_i) = \{1 \quad 1 \le i \le m\}$$

Here $V_0(f) = V_1(f)$ and

$$e_0(f) = e_1(f)$$

It satisfies the condition

$$|V_0(f) - V_1(f)| \le 1 \text{ and}$$

$$|e_0(f) - e_1(f)| \le 1$$

Hence, Helm is V cordial.

For example, Helm ($C_6 \odot K_1$) is V cordial as shown in the figure 3.15.

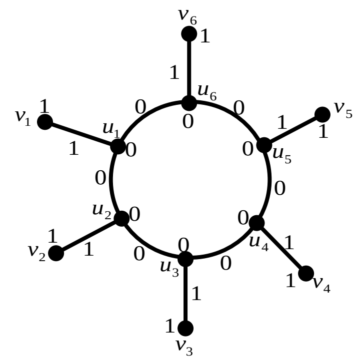

Figure 3.13

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