

# Finding an Optimal Sequence in the Flowshop Scheduling Using Johnson's Algorithm

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## Abstract

This work is concerned about the minimization of the make span and also to establish the idle time for the machines in a flow shop scheduling environment by using Johnson's algorithm for two machine problems and Extension of Johnson's algorithm for three machine problems. The article also focuses on time-in and time-out strategy for two machines as well as for three machine problem. The work proposes to find the optimal sequence in which the job will be scheduled to flow in the machines using linear programming model which dominates the heuristic models used for NP hard problems.

**Keywords:** makespan, idle time, flow shop, scheduling, Johnson's algorithm, linear programming, heuristic models, NP hard.

## 1. Introduction

The production scheduling in general are three forms, Job shop production scheduling, Flow shop production scheduling and project production scheduling.

In the traditional flow shop scheduling problem, it is assumed that there is only one machine at each stage to execute passing jobs. With the development of hardware, software, and theory in parallel computing, the traditional model of flow shop scheduling is becoming somewhat unrealistic. Defined to capture the essence of parallel computing is the so-called hybrid flow shop model, in which each job has to go through multiple stages with parallel machines instead of a single machine (Havill & Mao, 2002).

Scheduling is one of the most important decisions in production control systems. Every production system should have a kind of production scheduling, no matter whether it is managed and organized traditionally or have a systematic and scientific approach to the planning in the production system. If a scientific approach to production planning is organized, we can be sure that a better usage of the resources especially the machinery and the manpower are considered and a better situation for competition are formed in the market. In this we try to use a mathematical optimization model for doing this job.

The goal is to minimize the total completion time of all the activities and the approach which are used is a linear programming which dominates the heuristic models which mostly used for the NP-Hard problems. To do this, first of all we convert the production system into the network form, then we find the critical activities which affects the total completion time (makespan), then we assign some budget to the activities to crash them, by assigning some budget to some of the operations, (Hojjati & Sahraeyan, 2009) the operation time of these activities reduces and affects the total completion time of all the operations and because of the shortage of the budget, the problem is solved and determines which activities are better to absorb the limited budget to minimize the makespan.

## 2. Job Shop Production System

A job shop is a type of manufacturing process structure where small batches of a variety of custom products are made. In the job shop process flow, most of the products produced require a unique set-up and sequencing of processing steps. Similar equipment or functions are grouped together, such as all drill presses in one area and grinding machines in another in a process layout. The layout is designed to minimize material handling, cost, and work in process inventories.

Job shops use general purpose equipment rather than specialty, dedicated product-specific equipment. Digital numerically controlled equipment is often used to give job shops the flexibility to change set-ups on the various machines very quickly. Job shops compete on quality, speed of product delivery, customization, and new product introduction, but are unlikely to compete on price as few scale economies exist. When an order arrives in the job shop, the part being worked on travels throughout the various areas according to a sequence of operations. Not all jobs will use every machine in the plant. Jobs often travel in a jumbled routing and may return to the same machine for processing several times. This type of layout is also seen in services like department stores or hospitals, where areas are dedicated to one particular product or one type of service.

A job is characterized by its route, its processing requirements, and its priority. In a job shop the mix of products is a key issue in deciding how and when to schedule jobs. Jobs may not be completed based on their arrival pattern in order to minimize costly machine set-ups and change-overs. Work may also be scheduled based on the shortest processing time.

Capacity is difficult to measure in the job shop and depends on lot sizes, the complexity of jobs, the mix of jobs already scheduled, the ability to schedule work well, the number of machines and their condition, the quantity and quality of labor input, and any process improvements.

### 2.1 Job shop characteristics

- Low volume, high variety customized products
- Flexible resources
- Skilled human resources
- Jumbled work flows
- High material handling
- Large of inventories
- Long flow time
- Highly structured information system
- High cost per unit of production but low investment

### 2.2 Scheduling

Scheduling is the allocation of start and finish time to each particular order. Therefore scheduling can bring productivity in shop floor by providing a calendar for processing a set of jobs. It is nothing but scheduling various jobs on a set of resources (machines) such that certain performance measures are optimized.

### 2.3 Shop scheduling

In flow shop scheduling problem, there are 'n' jobs; each requires processing on 'm' different machines. The order in which the machines are required to process a job is called process sequence of that job. The process sequences of all the jobs are the same. But the processing times for various jobs on a machine may differ. If an operation is absent in a job, and then the processing time of the operation of the job is assumed to be zero.

## 3. Methodology Selection

Makespan is one of the most important criteria in every production systems; it is equal to the total completion time of all the activities. Minimizing this criterion caused better usage of the resources specially machinery and manpower. In both simple and hybrid flow shop, the methodology is to convert the flow shop into a network form, then a linear programming model with the objective of minimizing the total completion time of all the activities are constructed. Minimizing total completion time of all the activities is equivalent to minimizing makespan in the production system. The result is that the sequencing and scheduling of all the activities are determined. The data is analyzed and the understood there is enormous opportunities for improvement. The methodologies are considered for the implementation Flow shop Scheduling using Johnson's Algorithm and Extension of Johnson's rule

### 3.1 Johnson's algorithm

Johnson's algorithm is a way to find the shortest paths between all pairs of vertices in a sparse directed graph. In operations research Johnson's rule is a method of scheduling jobs in two work centers. Its primary objective is to find an optimal sequence of jobs to reduce makespan (the total amount of time it takes to complete all jobs). It also reduces the number of idle time between the two work centers. Results are not always optimal, especially for a small group of jobs.

### 3.2 Techniques

- The time for each job must be constant.
- Job times must be mutually exclusive of the job sequence.
- All jobs must go through first work center before going through the second work center.
- There must be no job priorities.

### 3.3 Johnson's rules

- List the jobs and their times at each work center.
- Select the job with the shortest activity time. If that activity time is for the first work center, then schedule the job first. If that activity time is for the second work center then schedule the job last. Break ties arbitrarily.
- Eliminate the shortest job from further consideration.

- Repeat steps 2 and 3, working towards the center of the job schedule until all jobs have been scheduled.

### 3.4 Procedure

**STEP 1:** Find the minimum among various  $t_{i1}$  and  $t_{i2}$ .

**STEP 2a:** If the minimum processing time requires machine 1, place the associated job in the first available position in sequence. Go to STEP 3

**STEP 2b:** If the minimum processing time requires machine 2, place the associated job in the last available position in the sequence. Go to STEP 3

**STEP 3:** Remove the assigned job from consideration and return to Step 1 until all positions in sequence are filled. (Ties may be broken randomly)

### 3.5 Extension Of Johnson’s Rule

Extension of Johnson’s algorithm is applicable only if the machine exceeds more than 2 and it also should satisfy the condition ( $\min t_{i1} \geq \max t_{i2}$ ,  $\min t_{i3} \geq \max t_{i2}$ ). The sequence can be found out by the same procedure instead we have to make the three machine problem to a two machine by combining (Machine1 and Machine 2) (Machine 2 and Machine 3).the combination of Machine 1 and Machine 2 will be considered as machine 1 and then the combination of Machine 2 and Machine 3 as Machine 2.then it is proceeded in the usual as Johnson’s algorithm. In this algorithm machine 3 will also be included and idle time will be calculated for Machine 3 also. The jobs can be many in number but if the machine exceeds more than three then we should use another heuristics to solve the conditions.

### 3.6 Heuristic Algorithms

Choi and Kim developed two simple heuristic algorithms, two hybrid algorithms, and three constructive algorithms for m-machine re-entrant flow shop, where the key idea of a hybrid algorithm is a combination of simple heuristic algorithms and the essence of a constructive algorithm is an improvement of an initial sequence which can be obtained randomly or by simple heuristic algorithms.

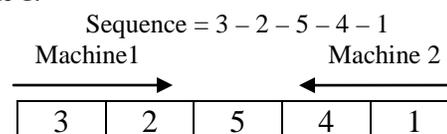
### 3.6 Heuristic solution

Given a sequence of jobs on both machines, the model transforms to a linear programming problem. When the sequence is known, the binary variables are fixed. The model can determine a schedule given a sequence. Furthermore, given a sequence, the problem can be reformulated as a single machine problem with release times. The formulation for the single machine would have

the objective of minimizing the sum of absolute deviation using the processing times on the second machine where the ready times are actually the completion times of the jobs on the first machine. The completion times on the first machine (or the release times for the second machine) are computed such that the jobs are scheduled with no idle time on the first machine since the insertion of idle time on the first machine does not improve the objective function value.

### 3.7 Sequencing

The most important thing in finding sequence is to find out the flow of the jobs in which sequence it has to load to the machines. The sequence can be found out by their processing times of the jobs in machine 1 and machine 2.it also allows finding out the idle time of the machines. The jobs will be in a flow for the first machine when it comes to the second machine it will moving according to the jobs completed on the machine 1.if the job is completed before the first machine then the next job should continue with the end time of the first machine, if the job exceeds the time on the first machine then the next job should continue the proceedings. Sequence can be found out by minimum processing time, it should be calculated according to the ascending order. If the minimum processing time is on machine 2 means it should start from right to left and if it is on machine 1 means it will be from right to left. In the table 1.1 the minimum processing time is machine 2 and job is 1, so it is started from the right. And the next minimum processing time is 4 and job is 4, so that is also started from right. If the processing times on both the machines are same means we can select randomly the machines. An example for the sequencing is shown from the table 1.



### 3.8 Optimal Sequences:

Optimal sequencing is the term where there will be a sequence of flow in the jobs in which the make span and the idle time will be less so that in that sequence of flow of jobs can be allocated to the machines.in the flow shop problem the sequences will be in factorial depending upon the machine in which the jobs will be allotted. For the problem in table in there are five jobs so there will be 5 factorial so many no of sequences are possible. The calculation of all the sequences should be necessary, so that we can find out the optimal sequences.an example for the sequence 3-2-5-4-1 is shown in figure 1,where the

make span or completion time of the jobs is 30 and the idle time is 3. like this the make span and the idle time should be calculated for all the sequences, so that we can find out the optimal sequence and then that sequence is allotted to the machine. the main disadvantage in this is it will take sometime to find out the optimal sequence.

### 4. Results and Discussion

#### 4.1 Flowshop Scheduling For Two Machines

Condition: This problem has two machines and five jobs with processing times. Machine 1 drilling operation and machine 2 with boring operation. The processing times will vary with jobs. Their make span and idle time are calculated below using the Gantt chart and time-in and time-out strategy

Table 1 Flow shop scheduling for two machines

Jobs	Drilling (mins)	Boring (mins)
J1	4	1
J2	5	8
J3	3	9
J4	6	4
J5	7	5

Table 3 Flow shop scheduling for combination of two machines

Jobs	Lathe + Drilling (mins)	Drilling + Boring (mins)
J1	13	9
J2	16	15
J3	8	10
J4	10	9
J5	15	9

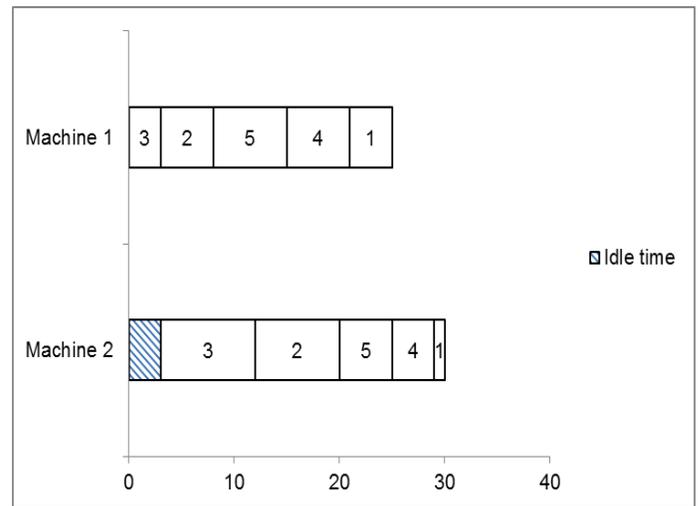


Figure 1 Gantt Chart, Tables And Graphs

#### 4.2 Extension Of Johnson’s Algorithm For Three Machines

Condition: this problem has three machines and five jobs with processing times. Machine 1 turning operation, machine 2 with drilling operation, machine 3 with boring operation. The processing times will vary with jobs. Their make span and idle time are calculated below using the Gantt chart and time-in and time-out strategy

Table 2 Flow shop scheduling for three machines

Jobs	Lathe (mins)	Drilling (mins)	Boring (mins)
J1	8	5	4
J2	10	6	9
J3	6	2	8
J4	7	3	6
J5	11	4	5

Table 4 Time-In And Time-Out Strategy For Two Machines

Jobs	Process to Drilling Machine		Process to Boring Machine		Idle time in Boring Machine (mins)
	Time In (mins)	Time Out (mins)	Time In (mins)	Time Out (mins)	
3	0	3	3	12	3
2	3	8	12	20	0
5	8	15	20	25	0
4	15	21	25	29	0
1	21	25	29	30	0

Table 5 Sequences

Sequence	Make Span (mins)	Idle Time (mins)	Sequence	Make Span (mins)	Idle Time (mins)
1-2-3-4-5	35	8	2-1-5-4-3	35	8
1-2-3-5-4	35	8	2-1-5-3-4	34	7
1-2-5-4-3	35	8	3-1-2-4-5	30	3
1-2-4-3-5	35	8	3-1-2-5-4	30	3
1-2-5-3-4	35	8	3-1-4-5-2	33	6
1-2-4-5-3	36	9	3-1-4-2-5	31	4
2-1-3-4-5	32	5	3-1-5-2-4	31	4
2-1-3-5-4	32	5	3-1-5-4-2	33	6
2-1-4-3-5	33	6	4-1-3-5-2	35	8
2-1-4-5-3	36	9	4-1-3-2-5	35	8

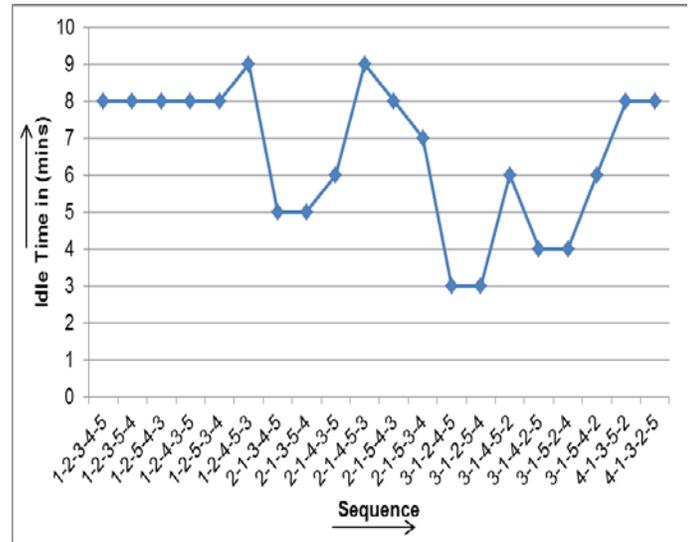


Figure 3 Sequences & Idle Time

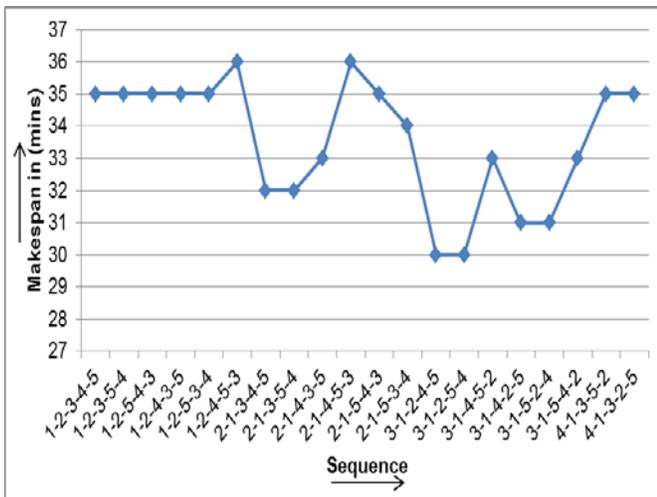


Figure 2 Sequences & Make Span

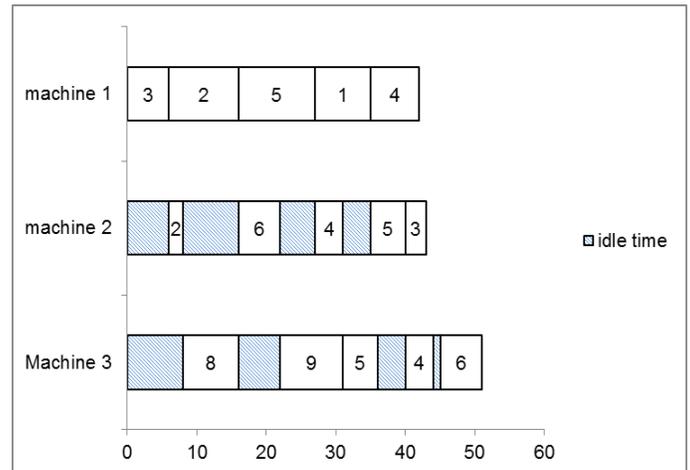


Figure 4 Gantt Chart For Three Machines

Table 6 Time-In And Time-Out Strategy For Three Machines

Jobs	Process to Lathe (mins)		Process to Drilling (mins)		Process to Boring (mins)		Idle time in drilling machine (mins)	Idle time in boring machine (mins)
	Time In	Time Out	Time In	Time Out	Time In	Time Out		
3	0	6	6	8	8	16	6	8
2	6	16	16	22	22	31	8	6
5	16	27	27	31	31	36	5	0
1	27	35	35	40	40	44	4	4
4	35	42	42	45	45	51	2	1

### 5. Conclusion & Future Guidelines

From this study it is clear that for two machine and more than two machine problems, Johnsons and extended Johnsons algorithm are used to find the makespan and idle time for a particular factorial series. The prime critic observed in this algorithm is, it will take some time to find the optimal sequences. On practicing this strategy to three components and results are really good and we suggested to follow the system to all the parts with continuous monitoring and improvement. This work can also be extended to design a algorithm to minimize the idle time.

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