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Cube Difference Labeling Of Some Cyclerelated Graphs

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Abstract

A graph G =(V,E) with p vertices and q edges is said to admit cube difference labeling,if there exists a bijectionf:v(G)→{0,1,2,…,p} such that the induced function f* : E(G) → N given by f*(uv) =| [f(u)]3 - [f(v)]3 | for everyuv∈ E(G) are all distinct. A graph which admits cube difference labeling is called cube difference graph. In this paper we prove that some classes ofgraphs like Double Triangular Snake, Umbrella Graph, Pn(Qsn) graph, Cn(Qsn) graphs arecube difference graphs.

Keywords:Cube difference labeling,Cube difference graph.

INTRODUCTION

All graphs in this paper are simple finite undirected and nontrivial graph G = (V, E) with vertex set V and the edge set E. For graph theoretic terminology, we refer to Harary [2]. A dynamic survey on graph labeling is regularly updated by Gallian [3] and it is published by Electronic Journal of Combinatorics. Vast amount of literature is available on different types of graph labeling and more than 1000 research papers have been published so far in past three decades. The squaredifference labeling is previously defined by V. Ajitha, S. Arumugam and K. A. Germina [1]. The concept of cube difference labeling was first introduced by J. Shiama and it was proved in [7] that many standard graphs like Pn, Cn, complete graphs, ladder, lattice grids, wheels, comb, star graphs,crown,dragon, coconut trees and shell graphs are cube difference Labeling. Andalso somereferences [4]-[6].Some graphs like cycle cactus graph,special tree and a New Key graph[8] can also be investigated for the cube difference.

Definition:1.1: Let G = (V (G), E (G)) be a graph. G is said to be cube difference labeling if there exist a bijectionf: V(G)→{0,1,2,….p-1} such that the induced function f*: E(G) → N given by f*(u v) = $[f(u)]^3$ - $[f(v)]^3$| is injective.

Definition:1.2:An umbrella graph U(m,n) to be a graph obtained by joining a path $p_{n \text{ with}}$ the central vertex of a fan fn.

Definition:1.3: Let G = Pn (Qs_n) is a graph. Let V(G) = {u_1,u_2, . .,u_n,v_1,v_2,…,v_{mn},w_1,w_2,…,w_{mn}}be the vertices of the graph and E(G) ={u_iu_{i+1}/1≤i≤n-1}U{u_iv_{2i-1},w_iv_{2i-1}/1≤i≤n}U{u_iv_{2i},w_iv_{2i}/1≤i≤n}. Let G_1,G_2,…G_n be m copies of C_4 and P_n : u_1,u_2,…u_n be a path. The P_n (Qs_n) is 4mn + (n-1) copies of P_2.

Definition:1.4: Let G = Cn (Qs_n) is a graph. let V(G) = {u_1,u_2, . .,u_n,v_1,v_2,…,v_{mn},w_1,w_2,…,w_{mn}}be the vertices of the graph and E(G) ={u_iu_{i+1}/1≤i≤n-1}U{u_iv_{2i-1},w_iv_{2i-1}/1≤i≤n}U{u_iv_{2i},w_iv_{2i}/1≤i≤n}. Let G_1,G_2,…G_n be m copies of C_4 and Let C_n : u_1,u_2,…u_n be a cycle.

MAIN RESULT

Theorem:1

The Double Triangular Snake G_n is a Cube difference graph.

Proof

Let G be the Double Triangular Snake and Let V(G) ={$u_1,u_2, . . .u_n,v_1,v_2,\ldots,v_{n-1},w_1,w_2,\ldots,w_{n-1}$}be the vertices of the graph and E(G) = {u_iu_{i+1}/1≤i≤n}U{v_iu_i,w_iu_i/1≤i≤n-1} U{v_iu_{i+1},w_iu_{i+1}/1≤i≤n-1} be the edges of the graph.

Let . Let │V(G)│= 2n+2 and │E(G)│= 4n-1

Define the vertex labeling f:V(G)→ {0,1,2,…p-1}

f(ui) = i-1 ,1≤ i ≤ n

f(vi) = i+n-1, 1≤ i ≤ n-3

= i+k , 1≤ i ≤ k

f(wi) = 2k+i , 1≤ i ≤ k

and the induced edge labeling function

f:E(G) →N defined by

f(uv) = │$[f(u)]^3$-$[f(v)]^3$ │ for every uv∈ E(G)

are all distinct .such that f(ei)≠f(ej) for every ei ≠ej

The edge sets are

E1= {u_iu_{i+1}/ 1≤ i ≤ n-1}

E2 = {v_iu_i/ 1≤ i ≤ n-1}

E3 = {v_iu_{i+1}/ 1≤ i ≤ n-1}

E4 = {w_iu_i/1≤ i ≤ n-1}

E5 = {w_iu_{i+1}/ 1≤ i ≤ n-1}

and the edge labeling are

In E1

$f^*(u_iu_{i+1}) = \cup_{i=1}^{n} \left| f(u_i)^3 - f(u_{i+1})^3 \right|$

$= \cup_{i=1}^{n} \left| (i-1)^3 - i^3 \right|$

$= \cup_{i=1}^{n} |3i2 - 3i + 1|$

= {1,7,19,…,3n2-3n+1}

In E_2

f*(v_i u_i) = $\cup_{i=1}^{n-1} |f(v_i)^3 - f(u_i)^3|$

$= \cup_{i=1}^{n-1} |k3 + 3i(k+1) + 3i(k2-1) + 1|$

={64,124,208,…,}

In E_3

f*(v_i u_{i+1}) = $\cup_{i=1}^{n-1} |f(v_i)^{23} - f(u_{i+1})^3|$

$= \cup_{i=1}^{n-1} |k3 + 3ki(k+i)|$

={63,117,189,…,}

In E_4

f*($w_i u_i$) = $\cup_{i=1}^{n-1} |f(w_i)^3 - f(u_i)^3|$

$= \cup_{i=1}^{n-1} |(i+2k)3 - (i-1)3|$

$= \cup_{i=1}^{n-1} |8k3 + 3i(2k+1) + 3i(4k2-1) + 1|$

={343,511,721,…,}

In E_5

f*($w_i u_{i+1}$) = $\cup_{i=1}^{n-1} |f(w_i)^3 - f(u_{i+1})^3|$

$= \cup_{i=1}^{n-1} |8k2 + 6ki(2k+i)|$

={342,504,702,…,}

Here the edges are distinct.

Hence the Double triangular snake graph admits a cube difference labeling.

Theorem:2

The Umbrella graph is a cube difference labeling.

Proof:

Let U(G) be the umbrella graph and the vertex is : u_1, u_2 ,…, u_n . Let v_1, v_2 ,…, $v_n$be the another vertex.

Let $|V(G)|$ = n and

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$|E(G)| = 3n$

Define the vertex labeling $f:E\rightarrow \{0,1,2,\ldots p-1\}$

$f(u_i) = i-1 \quad ,1\leq i \leq n$

$f(v_i) = i+2,\ 1\leq i \leq n+1$

and the induced edge labeling function $f:E \rightarrow N$ defined by

$f(uv) = |[f(u)]^3-[f(v)]^3|$ for every $uv \in E(G)$

are all distinct .such that $f(e_i)\neq f(e_j)$ for every $e_i \neq e_j$

The edge sets are

$E_1= \{u_iu_{i+1}/ 1\leq i \leq n-1\}$

$E_2 = \{v_iv_{i+1}/ 1\leq i \leq n\}$

$E_3 = \{v_iu_n/ 1\leq i \leq n+1\}$

And the edge labeling are

$f^*(E_1) = \cup_{i=1}^{n-1} |f(u_i)^3 - f(u_{i+1})^3|$

$=\cup_{i=1}^{n-1} |2 - 3i2|$

$=\{1,7,\ldots,3n2-9n+7\}$

$f^*(E_2) = \cup_{i=1}^{n} |f(v_i)^3 - f(v_{i+1})^3|$

$=\cup_{i=1}^{n} |(i+2)3 - (i+3)3|$

$=\cup_{i=1}^{n} |-3i^2 - 15i - 19|$

$=\{37,61,91,\ldots,-(3n^2+15n+19)\}$

$f^*(E_3) = \cup_{i=1}^{n+1} |f(v_i)^3 - f(u_n)^3|$

$= \cup_{i=1}^{n+1} |i^3 + 6i(i+2) - n^3 + 3n(n-1+9)|$

$=\{19,56,117,\ldots,12n^2+24n+28\}$

Here the edges are distinct.

Hence the umbrella graph admits a cube difference labeling.

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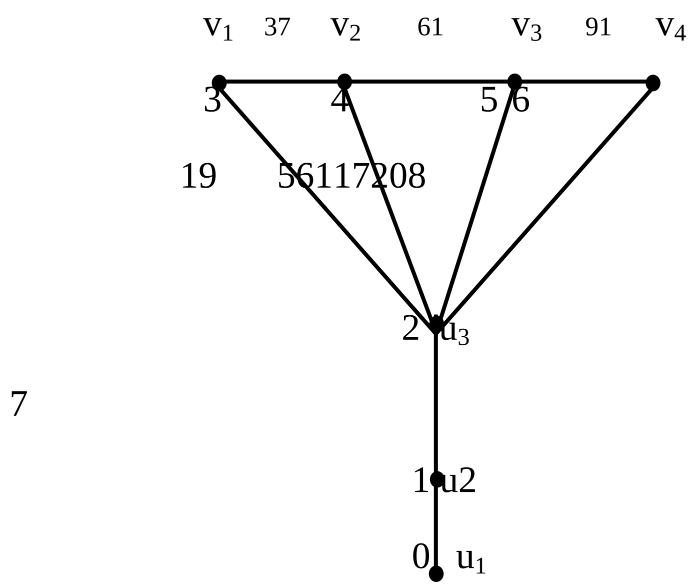
For example;



Theorem:3

The graph $G = P_n (QS_n)$ is a cube difference labeling ($n \geq 1$, $m \geq 1$).

Proof:

Let $G = P_n (QS_n)$ is a graph. let $V(G) = \{u_1,u_2, . . .,u_n,v_1,v_2,\dots,v_{mn},w_1,w_2,\dots,w_{mn}\}$be the vertices of the graph and $E(G) =\{u_iu_{i+1}/1\leq i\leq n-1\}U\{u_iv_{2i-1},w_iv_{2i-1}/1\leq i\leq n\}U\{u_iv_{2i},w_iv_{2i}/1\leq i\leq n\}$.Let $G_1,G_2,\dots G_n$be m copies of C_4 and $P_n : u_1,u_2,\dots u_n$ be a path . The $P_n (QS_n)$ is $4mn + (n-1)$ copies of P_2.

Let $|V(G)| = 3mn + n$ and $|E(G)| = 4mn + (n-1)$

Define the vertex labeling $f:E \rightarrow \{0,1,\dots p-1\}$

$f(u_i) = i-1$, $1\leq i \leq n$

$f(v_{2nk+i}) = (3k+1)n+(i-1)$, $1\leq i \leq n-1$, $k=0,1,2,3,\dots,n$

$f(w_{nk+1})=3n(k+1)+(i-1)$, $1\leq i \leq n-1$, $k=0,1,2,3,\dots,n$

$f(v_i) =n+i-1$ for $k=0$

$f(w_i)=3n+i-1$ for $k=0$

and the induced edge labeling function

$f:E \rightarrow N$ defined by

$f(uv) = |[f(u)]^3-[f(v)]^3|$ for every $uv \in E(G)$

are all distinct .such that $f(e_i) \neq f(e_j)$ for every $e_i \neq e_j$

The edge sets are

$E_1= \{u_iu_{i+1}/ 1\leq i \leq n\text{-}1\}$

$E_2 = \{u_iv_{2i-1}/ 1\leq i \leq n\}$

$E_3 = \{u_iv_{2i}/ 1\leq i \leq n\}$

$E_4 = \{w_iv_{2i-1}/ 1\leq i \leq n\}$

$E_5 = \{w_iv_{2i}/ 1\leq i \leq n\}$

And the edge labeling are

In E_1

$f^*(u_iu_{i+1}) = \bigcup_{i=1}^{n-1} |(1-i)^3 - i^3|$

$= \bigcup_{i=1}^{n-1} |(1-3i(1-i)|$

$=\{1,7,\ldots,1\text{-}3n(1\text{-}n)\}$

In E_2

$f^*(u_iv_{2i-1}) = \bigcup_{i=1}^{n} |f(u_i)^3 - f(v_{2i-1})^3|$

$= \bigcup_{i=1}^{n} |(i-1)^3 - (n+2i-2)^3|$

$=\bigcup_{i=1}^{n} |i2(-7i+21-12n) + i(-21-6n2+24n) + (-n3+6n2-12n+7)|$

$= \{8,63,\ldots,(-26n3+51n^2-33n+7)$

In E_3

$f^*(u_iv_{2i}) = \bigcup_{i=1}^{n} |f(u_i)^3 - f(v_{2i})^3|$

$= \bigcup_{i=1}^{n} |(i-1)^3 - (n+2i-1)^3|$

$= \bigcup_{i=1}^{n} |-7i3 + +9i2 - 3i - n3 - 6n2i - 12ni2 + 3n2 + 12ni - 3n|$

$=\bigcup_{i=1}^{n} |i2(-7i+9-12n) + i(-3-6n2+12n) + (-n2+3n2- 3n)|$

$=\{27,124,\ldots,(-26n^3+24n^2-6n)\}$

In E_4

$f^*(w_iv_{2i-1}) = \bigcup_{i=1}^{n} |f(w_i)^3 - f(v_{2i})^3|$

$= \bigcup_{i=1}^{n} |(3n+i-1)^3 - (n+2i-2)^3|$

$= \cup_{i=1}^{n} \left| 26n3 - 7i3 - 21n2 + 21n2i - 3ni2 + 6ni + 21i2 - 3n - 21i + 7 \right|$

$= \cup_{i=1}^{n} \left| i2(-7i + 21 - 3n) + i(21n2 + 6n - 21) + (26n3 - 21n2 - 3n + 7) \right|$

$= \{208, 279, \ldots, (37n^3+6n^2-24n+7)\}$

In E_5

$f^*(w_i v_{2i}) = \cup_{i=1}^{n} \left| f(w_i)^3 - f(v_{2i})^3 \right|$

$= \cup_{i=1}^{n} \left| (3n + i - 1)3 - (n + 2i - 1)3 \right|$

$= \cup_{i=1}^{n} \left| 26n3 + 21n2i - 3ni2 - 7i3 - 24n2 - 6ni + 9i2 + 6n - 3i \right|$

$= \cup_{i=1}^{n} \left| i2(-7i + 9 - 3n) + i(21n2 - 6n - 3) + (26n3 - 24n2 + 6n) \right|$

$= \{189, 218, \ldots, (37n^3-21n^2+3n)\}$

Here the edges are distinct.

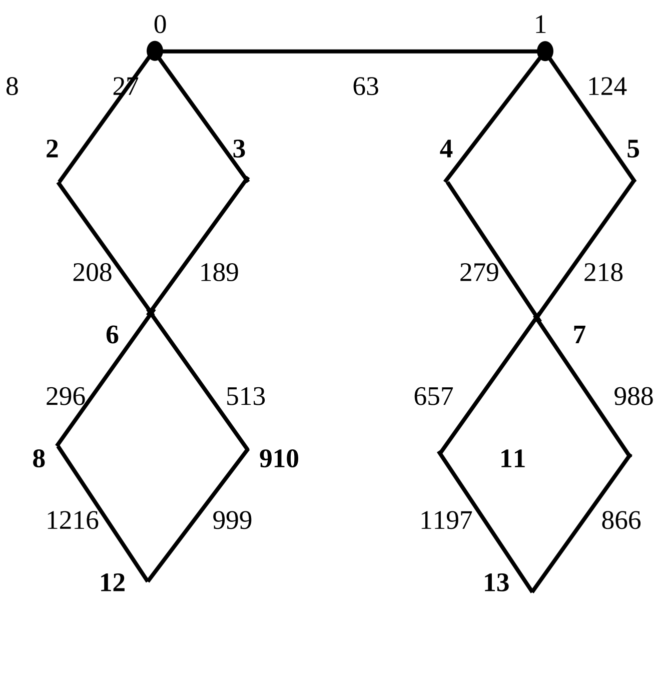
Hence the $P_n(QS_n)$ graph admits a cube difference labeling.

For example;

The graph P_2 (QS_2) is a cube difference labeling.

Solution:

If $n \geq 1$ and $m \geq 1$



Theorem:4

The graph G = C_n (QS_n) is a cube difference labeling.

Proof:

Let G = C_n (QS_n) is a graph . let V(G) = {$u_1,u_2, . . .,u_n,v_1,v_2,\ldots,v_{mn},w_1,w_2,\ldots,w_{mn}$}be the vertices of the graph and E(G) ={$u_iu_{i+1}/1\le i\le n-1$}U{$u_iv_{2i-1},w_iv_{2i-1}/1\le i\le n$}U{$u_iv_{2i},w_iv_{2i}/1\le i\le n$}.

Let $G_1,G_2,\ldots G_n$be m copies of C_4 and Let C_n : $u_1,u_2,\ldots u_n$ be a cycle . Let │V(G)│= 3mn + n and │E(G)│= 4mn + n

Define the vertex labeling f: E → {0,1,…p-1}

$f(u_i) = i-1$, $1\le i \le n$

$f(v_{2nk+i}) = (3k+1)n+(i-1)$, $1\le i \le n-1$, k=0,1,2,3,…,n

$f(w_{nk+1})=3n(k+1)+(i-1)$, $1\le i \le n-1$, k=0,1,2,3,…,n

f(vi) =n+i-1 for k=0f(wi)=3n+i-1 for k=0

and the induced edge labeling function

f:E → N defined by

$f(uv) = \left| [f(u)]^3 - [f(v)]^3 \right|$ for every uv ∈ E(G)

are all distinct .such that $f(e_i)\ne f(e_j)$ for every $e_i \ne e_j$

The edge sets are

E_1= {u_iu_{i+1}/ $1\le i \le n-1$}

E_2 = {u_iv_{2i-1}/ $1\le i \le n$}

E_3 = {u_iv_{2i}/ $1\le i \le n$}

E_4 = {w_iv_{2i-1}/ $1\le i \le n$}

E_5 = {w_iv_{2i}/ $1\le i \le n$}

and the edge labeling are

In E_1

$f^*(u_iu_{i+1}) = \bigcup_{i=1}^{n-1} \left| (1-i)^3 - i^3 \right|$

$= \bigcup_{i=1}^{n-1} \left| (1-3i(1-i) \right|$

={1,7,…,1-3n(1-n)}

In E_2

$f^*(u_iv_{2i-1}) = \bigcup_{i=1}^{n} \left| f(u_i)^3 - f(v_{2i-1})^3 \right|$

$= \bigcup_{i=1}^{n} \left| (i-1)^3 - (n+2i-2)^3 \right|$

$= \bigcup_{i=1}^{n} \left| -7i^3 + 21i^2 - 21i + 7 - n^3 - 6n^2i - 12ni^2 + 6n^2 + 24ni - 12n \right|$

$= \bigcup_{i=1}^{n} \left| i^2(-7i + 21 - 12n) + i(-21 - 6n^2 + 24n) + (-n^3 + 6n^2 - 12n + 7) \right|$

$= \{27, 124, 335, .., (-26n^3 + 56n^2 - 33n + 7)\}$

InE3

$f^*(u_iv_{2i}) = \bigcup_{i=1}^{n} \left| f(u_i)^3 - f(v_{2i})^3 \right|$

$= \bigcup_{i=1}^{n} \left| (i-1)^3 - (n+2i-1)^3 \right|$

$= \bigcup_{i=1}^{n} \left| -7i^3 + +9i^2 - 3i - n^3 - 6n^2i - 12ni^2 + 3n^2 + 12ni - 3n \right|$

$= \bigcup_{i=1}^{n} \left| i^2(-7i + 9 - 12n) + i(-3 - 6n^2 + 12n) + (-n^{3`} + 3n^2 - 3n) \right|$

$= \{64, 215, 504, \ldots, (-26n3 + 24n2 - 6n)\}$

In E_4

$f^*(w_iv_{2i-1}) = \bigcup_{i=1}^{n} \left| f(w_i)^3 - f(v_{2i})^3 \right|$

$= \bigcup_{i=1}^{n} \left| (3n + i - 1)^3 - (n + 2i - 2)^3 \right|$

$= \bigcup_{i=1}^{n} \left| 26n3 - 7i3 - 21n2 + 21n2i - 3ni2 + 6ni + 21i2 - 3n - 21i + 7 \right|$

$= \bigcup_{i=1}^{n} \left| i2(-7i + 9 - 3n) + i(21n + 6n - 21) + (26n3 - 21n2 - 3n + 7) \right|$

$= \{702, 875, 988, \ldots, 37n^3 + 6n^2 - 24n + 7\}$

In E_5

$f^*(w_iv_{2i}) = \bigcup_{i=1}^{n} \left| f(w_i)^3 - f(v_{2i})^3 \right|$

$= \bigcup_{i=1}^{n} \left| (3n + i - 1)3 - (n + 2i - 1)3 \right|$

$= \bigcup_{i=1}^{n} \left| 26n3 + 21n2i - 3ni2 - 7i3 - 24n2 - 6ni + 9i2 + 6n - 3i \right|$

$= \bigcup_{i=1}^{n} \left| i2(-7i + 9 - 3n) + i(21n2 - 6n - 3) + (26n3 - 4n2 + 6n) \right|$

$= \{665, 784, 819, \ldots, 37n3 - 21n2 + 3n\}$

Here the edges are distinct.

Hence the $C_n(QS_n)$ graph admits a square difference labeling.

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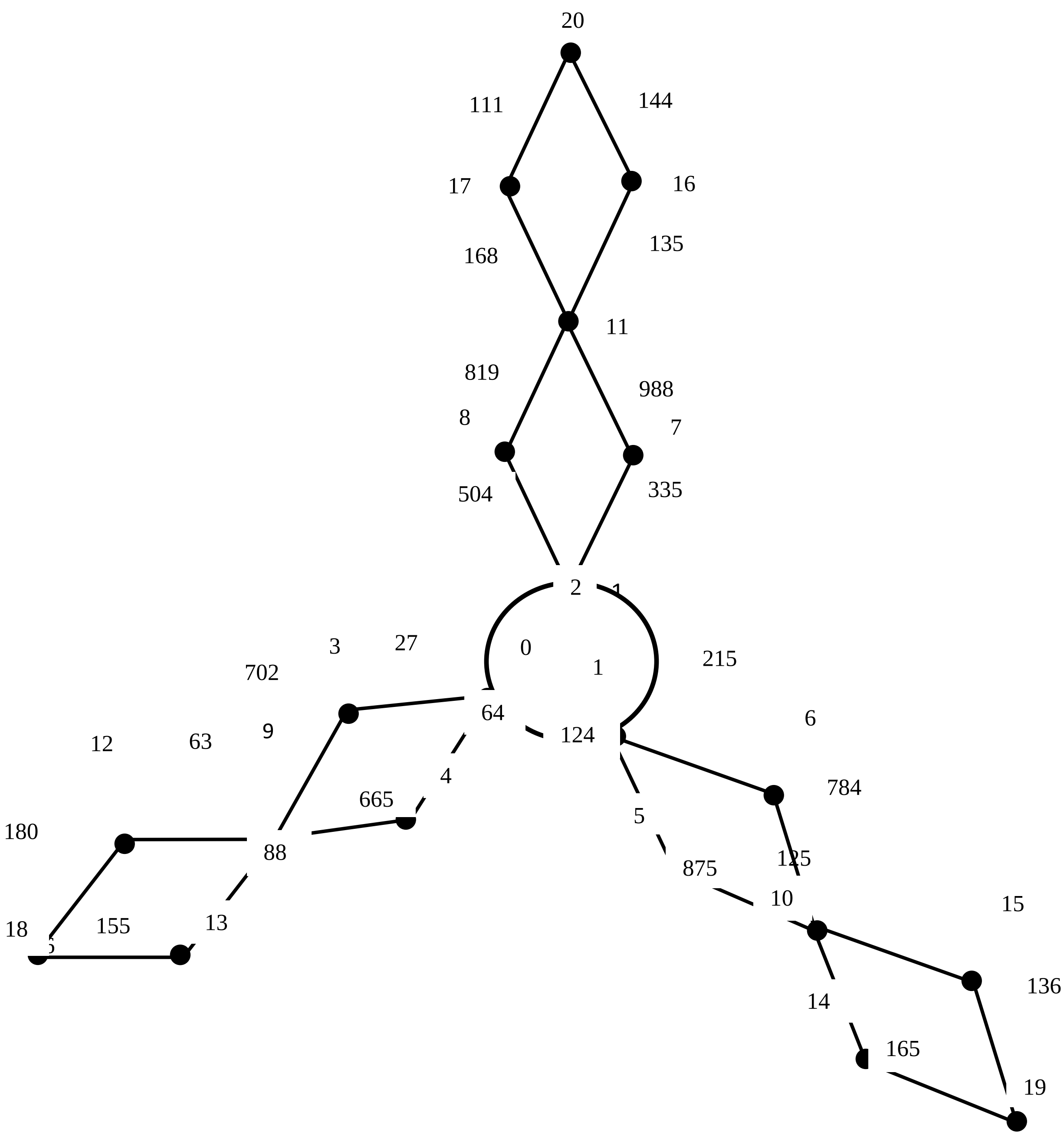
Eample;The graph $C_3(QS_2)$ is a cube difference labeling. ($n \geq 3$, $m \geq 1$).

Solution: If $n \geq 3$ and $m \geq 1$



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