

Soret-Dufour Effects on Unsteady MHD flow of Dusty Fluid over Inclined Porous Plate Embedded in Porous Medium

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Abstract

An unsteady two dimensional MHD (Magneto Hydro Dynamics) flow of an incompressible viscous and electrically conducting dusty fluid past a continuously moving inclined plate with Soret-Dufour effects is analyzed. The governing equations are solved numerically using Crank-Nicolson finite difference method for different values of flow parameters. The velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number are explained through graphs and tables.

Keywords: MHD, Thermophoresis effect, Dufour effect, porous medium, Hall effect, radiation effect, Heat and Mass transfer, Crank-Nicolson method.

1. Introduction

During the last many decades, large number of mathematician has been attracted toward study of unsteady MHD flow of dusty fluid with effects of Thermophoresis, Dufour, radiation because of a large number of avenues for research work. The investigation of behavior of such flow have application in chemical engineering, MHD generators, in designing of underground water energy storage system, soil sciences, nuclear power reactors, aeronautics and so on. MHD convection flow past a semi-infinite vertical plate with thermal radiation was discussed by Venkataramana [1]. Sparrow and Cess [2] have discussed the effect of magnetic field on free convection heat transfer. Effect of magnetic field on heat and mass transfer by natural convection from vertical surface in porous medium with Soret and Dufour effects was discussed by Postelnicu [3].

Rajesh et al. [4] studied effects of radiation and mass transfer on MHD flow with exponentially accelerated vertical plate. Interaction of mixed convection with thermal radiation in laminar flow had been analyzed by Bestman et al. [5]. Olanrewaju [8] discussed Soret-Dufour and radiation effects on transient free convection flow past a moving plate. N. Pandya and A. K. Shukla [9] analyzed Soret-Dufour and Radiation Effects on Unsteady MHD Flow past an Impulsively Started Inclined Porous Plate with Variable Temperature and Mass Diffusion. N. Pandya and A. K. Shukla [10] Effects of Thermophoresis, Dufour, Hall and Radiation on an Unsteady MHD flow past an

Inclined Plate with Viscous Dissipation. N. Pandya and A. K. Shukla [11] has been discussed Soret-Dufour and Radiation effect on unsteady MHD flow over an inclined porous plate embedded in porous medium with viscous dissipation. Sudhakarraiah et al. [12] examined Soret effect on unsteady MHD flow past a vertical plate with heat source. Dubey et. al. [6] have studied on Effect of the dusty viscous fluid on unsteady free convective flow along a porous hot vertical plate with thermal diffusion and mass transfer solved by perturbation techniques. Recently, Dubey et. al. [7] have analyzed Effect of Dusty Viscous Fluid on Unsteady Laminar Free Convective Flow through Porous Medium along a Moving Porous Hot Vertical Plate with Thermal Diffusion.

The aim of this research work is to analyze combined effects of Soret or Thermophoresis and Dufour on unsteady MHD flow of dusty fluid past an inclined porous plate embedded in porous medium variable temperature and variable mass diffusion. Partial differential equations of dimensionless governing equations of flow have been solved by Crank-Nicolson implicit finite difference method. Effects of different physical parameters of flow field on velocity, temperature, concentration, coefficients of skin-friction, Nusselt number and Sherwood number have been discussed through graphs and tables.

2. Mathematical Analysis

An unsteady MHD flow of a viscous incompressible dusty fluid past an infinite inclined plate with variable temperature and mass diffusion has been analyzed. The plate is inclined at angle λ to vertical and embedded in porous medium. x' -axis has been considered along plate, y' -axis normal to it. A uniform magnetic field B_0 is taken along y' -axis and plate is electrically non-conducting. Magnetic Reynolds number and transversely applied magnetic field are very small so induced magnetic field is negligible in comparison to applied magnetic field, Cowling [14].

Due to infinite length in x' direction, flow variables are function of t' and y' only. Consider usual Boussinesq approximation, governing equations of flow field are:

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v_0 \quad (1)$$

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) \cos(\lambda) + g\beta'(C' - C'_\infty) \cos(\lambda) + \frac{KN_0(V - u')}{\rho} \quad (2)$$

$$-\frac{\sigma B_0^2 u'}{\rho} - \frac{\nu u'}{K'} \quad m_1 \frac{\partial V}{\partial t'} = K(u' - V) \quad (3)$$

$$\rho C_p \left(\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} \right) = k \frac{\partial^2 T'}{\partial y'^2} + \frac{\rho D_m K_T}{c_s} \frac{\partial^2 C'}{\partial y'^2} \quad (4)$$

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} + \frac{D_m K_T}{T_m} \frac{\partial^2 T'}{\partial y'^2} \quad (5)$$

where γ^* is coefficient of volume expansion for mass transfer, γ is volumetric coefficient of thermal expansion, v' is velocity along y' -axis, K' is permeability of porous medium, σ is electrical conductivity, D_m is molecular diffusivity, g is acceleration due to gravity, K_T is thermal diffusion ratio, V is the velocity of dust particles, K is the stokes resistance coefficient, N_0 is the number density of the dust particles which is taken to be constant, m_1 is the mass of dust particles, ρ is fluid density, k is thermal conductivity of fluid, C' and T' are dimensional concentration and temperature, C'_∞ and T'_∞ are concentration and temperature of free stream, q_r is radiative heat along y' -axis, ν is kinematic viscosity and T_m is mean fluid temperature.

Boundary and initial conditional for this model are given as:

$$\begin{aligned} t' \leq 0 \quad u' = 0 \quad T' = T'_\infty \quad C' = C'_\infty \quad \forall y' \\ t' \geq 0 \quad u' = u_0 \quad v' = -v_0 \quad T' = T'_\infty + (T'_w - T'_\infty) e^{-At'} \\ C' = C'_\infty + (C'_w - C'_\infty) e^{-At'} \quad \text{at } y' = 0 \\ u' = 0 \quad T' \rightarrow T'_\infty \quad C' \rightarrow C'_\infty \quad y' \rightarrow \infty \end{aligned} \quad (6)$$

Where T'_w and C'_w are concentration and

temperature respectively of plate and $A = \frac{v_0^2}{\nu}$.

In order to obtain dimensionless partial differential equations, we introduce following quantities:

$$\begin{aligned} u = \frac{u'}{u_0}, t = \frac{t' v_0^2}{\nu}, y = \frac{y' v_0}{\nu}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Gm = \frac{\nu g \beta' (C'_w - C'_\infty)}{u_0 v_0^2}, \\ Gr = \frac{\nu g \beta (T'_w - T'_\infty)}{u_0 v_0^2}, K = \frac{v_0^2 K'}{\nu^2}, Pr = \frac{\mu c_p}{k}, \\ M = \frac{\sigma B_0^2 \nu}{\rho v_0^2}, Du = \frac{D_m K_T (C'_w - C'_\infty)}{c_s c_p \nu (T'_w - T'_\infty)}, Sc = \frac{\nu}{D_m}, \\ Sr = \frac{D_m K_T (T'_w - T'_\infty)}{T_m \nu (C'_w - C'_\infty)}, v = \frac{V}{u_0}, \\ B_1 = \frac{\nu K N_0}{\rho u_0^2}, B = \frac{m_1 u_0^2}{VK} \end{aligned} \quad (7)$$

Using quantities of equation 7, we get non-dimensional form of equations 2, 3, 4 and 5 respectively:

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr \cos(\lambda) \theta + Gm \cos(\lambda) C + B_1(v - u) - (M + \frac{1}{K})u \quad (8)$$

$$B \frac{\partial V}{\partial t} = u - V \quad (9)$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + Du \frac{\partial^2 C}{\partial y^2} \quad (10)$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + Sr \frac{\partial^2 \theta}{\partial y^2} \quad (11)$$

dimensionless boundary and initial conditions are:

$$\begin{aligned} t \leq 0 \quad u = 0 \quad \theta = 0 \quad C = 0 \quad \forall y \\ t \geq 0 \quad u = 1 \quad \theta = e^{-t} \quad C = e^{-t} \quad \text{at } y = 0 \\ u = 0 \quad \theta \rightarrow 0 \quad C \rightarrow 0 \quad y \rightarrow \infty \end{aligned} \quad (12)$$

Further, there is primary interest for research workers to calculate physical quantities skin-friction coefficients t along wall x -axis, Nusselt number Nu and Sherwood number Sh . Non-dimensional form of these physical quantities are:

$$\begin{aligned}
 \tau &= -\left(\frac{\partial u}{\partial y}\right)_{y=0} \\
 Nu &= -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \\
 Sh &= -\left(\frac{\partial C}{\partial y}\right)_{y=0}
 \end{aligned}
 \tag{13}$$

$$\begin{aligned}
 &= \frac{1}{2Sc} \left(\frac{C_{i-1,j} - 2C_{i,j} + C_{i+1,j} + C_{i-1,j+1} - 2C_{i,j+1} + C_{i+1,j+1}}{2(\Delta y)^2} \right) \\
 &+ \frac{Sr}{2} \left(\frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j} + \theta_{i-1,j+1} - 2\theta_{i,j+1} + \theta_{i+1,j+1}}{2(\Delta y)^2} \right)
 \end{aligned}
 \tag{17}$$

initial and boundary conditions are also expressed as:

$$\begin{aligned}
 u_{i,0} &= 0, \quad \theta_{i,0} = 0, \quad C_{i,0} = 0 \quad \forall i \\
 u_{0,j} &= 1, \quad \theta_{0,j} = e^{-j\Delta t}, \quad C_{0,j} = e^{-j\Delta t} \\
 u_{X,j} &= 0, \quad \theta_{X,j} \rightarrow 0, \quad C_{X,j} \rightarrow 0
 \end{aligned}
 \tag{18}$$

3. Mathematical Analysis

Partial differential equations 8 to 11 are solved with initial and boundary conditions 12. To find exact solution of these partial differential equations are impossible. So, these are solved numerically using Crank-Nicolson implicit finite difference method. First, equations 8, 9, 10 and 11 are expressed as:

$$\begin{aligned}
 &\frac{u_{i,j+1} - u_{i,j}}{\Delta t} - \frac{u_{i+1,j} - u_{i,j}}{\Delta y} \\
 &= \frac{1}{2} \left(\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j} + u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{2(\Delta y)^2} \right) \\
 &+ Gr \cos(\lambda) \left(\frac{\theta_{i,j+1} + \theta_{i,j}}{2} \right) + Gm \cos(\lambda) \left(\frac{C_{i,j+1} + C_{i,j}}{2} \right) \\
 &+ B_1 \left(\left(\frac{v_{i,j+1} + v_{i,j}}{2} \right) - \left(\frac{u_{i,j+1} + u_{i,j}}{2} \right) \right) \\
 &- \left(M + \frac{1}{K} \right) \left(\frac{u_{i,j+1} + u_{i,j}}{2} \right)
 \end{aligned}
 \tag{14}$$

$$B \left(\frac{v_{i,j+1} - v_{i,j}}{\Delta t} \right) = \left(\frac{u_{i,j+1} + u_{i,j}}{2} \right) - \left(\frac{v_{i,j+1} + v_{i,j}}{2} \right)
 \tag{15}$$

$$\begin{aligned}
 &\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} - \frac{\theta_{i+1,j} - \theta_{i,j}}{\Delta y} \\
 &= \frac{1}{2Pr} \left(\frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j} + \theta_{i-1,j+1} - 2\theta_{i,j+1} + \theta_{i+1,j+1}}{2(\Delta y)^2} \right) \\
 &+ \frac{Du}{2} \left(\frac{C_{i-1,j} - 2C_{i,j} + C_{i+1,j} + C_{i-1,j+1} - 2C_{i,j+1} + C_{i+1,j+1}}{2(\Delta y)^2} \right) \\
 &+ \frac{C_{i,j+1} - C_{i,j}}{\Delta t} - \frac{C_{i+1,j} - C_{i,j}}{\Delta y}
 \end{aligned}
 \tag{16}$$

where index i refers to y , j refers to time t , $\Delta t = t_{j+1} - t_j$ and $\Delta y = y_{i+1} - y_i$. Known values of u, θ and C at t , we solved above equations for values $t + \Delta t$ as follows: We obtain these values to substitute $i = 1, 2, \dots, X - 1$, where X pertains to ∞ then equations 14 to 17 give tridiagonal system of equations with initial and boundary conditions in equation 18 are solved using Thomos algorithm as discussed in Carnahan et al. [15], we have found values of θ and C for all values of y at $t + \Delta t$. Equation 16 and 17 are solved by same to substitute these values of θ and C , we get solution for u till desired time t . calculation were execute for $\Delta y = 0.1$, $\Delta t = 0.001$ and repeated till $y = 4$.

4. Result and Discussion

In order to analysis, we would like to see numerical results for velocity profile u , temperature profile θ and concentration profile C by giving numerical values of thermal Grashof number Gr , Thermophoresis or Soret number Sr , Dufour number Du , solutal Grashof number Gm , Schmidt number, Sc magnetic parameter M , dusty fluid parameter B_1 , dusty particle parameter B , permeability of porous medium K , prandtl number Pr , and inclination angle λ with help of graphs. As well as coefficients of skin-friction t along wall x -axis, Nusselt number Nu and Sherwood number Sh are discussed through tables.

Figures 1, 2 and 3 depict that on increasing Sr , velocity profile u , temperature profile θ first decreases after then increases and concentration profile C increase rapidly when Sr increases respectively. velocity profiles u in figure 4 decreases and concentration profile in figure 6 first increases after then decreases when Schmidt number

Sc increases. Figure 8 show that temperature profile θ increases when Dufour number Du increases while concentration profile C first increases, at some distance from plate, it decreases and at last it again increases in figure 9.

Velocity profile u decreases as inclination angle λ increases in figure 10. Figures 11, 12 and 13 explain that velocity profile u , temperature profile θ and concentration profile C increase as time t increases. on increasing dusty particle parameter B , in figure 5 velocity profile u decreases, in figure 7 velocity profile u decreases rapidly when dusty fluids parameter B_1 increases.

From table 1, it is clear that on increasing Dusty fluids parameter, Dusty particle parameter, inclination angle and Schmidt number, skin-friction coefficient τ decreases. Further skin-friction coefficient τ increases as Dufour number, Soret number and time increase. In table 2, we have seen that Nusselt number Nu decreases as Schmidt number increases and Sherwood number Sh increases as Dufour number, Soret number and time increase. On other hand, Nusselt number increases as Dufour number, Soret number and time increase and Sherwood number decreases as Schmidt number increases.

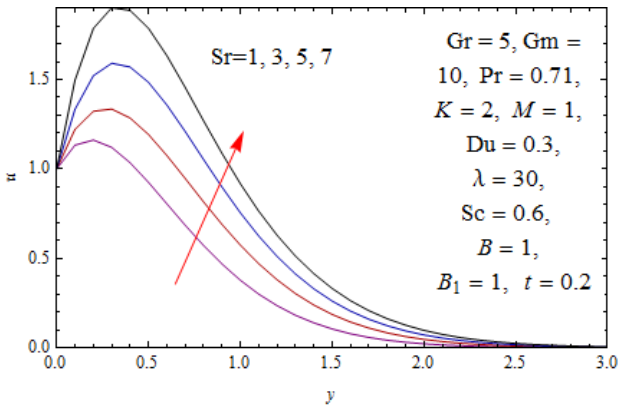


Fig. 1 Velocity Profile for Different Values of Sr

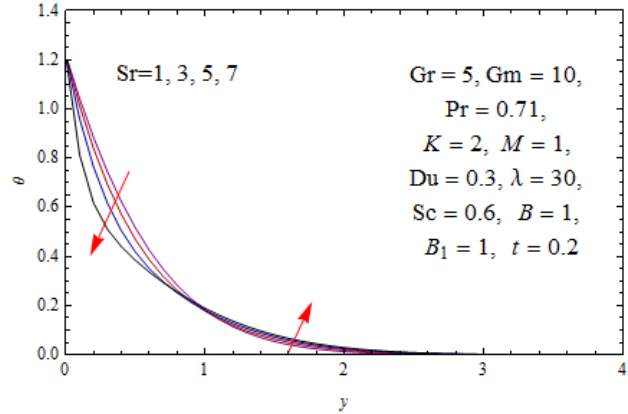


Fig. 2 Temperature Profile for Different Values of Sr

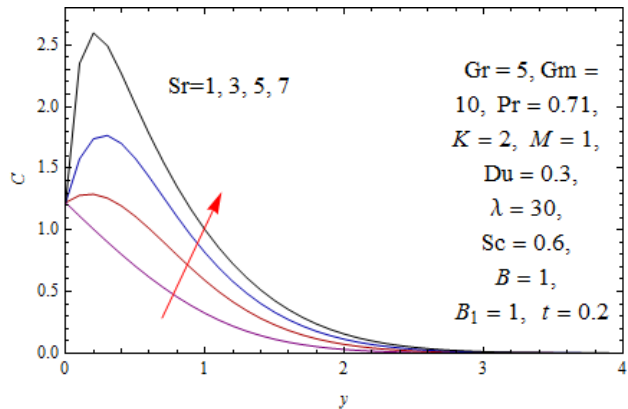


Fig. 3 Concentration Profile for Different Values of Sr

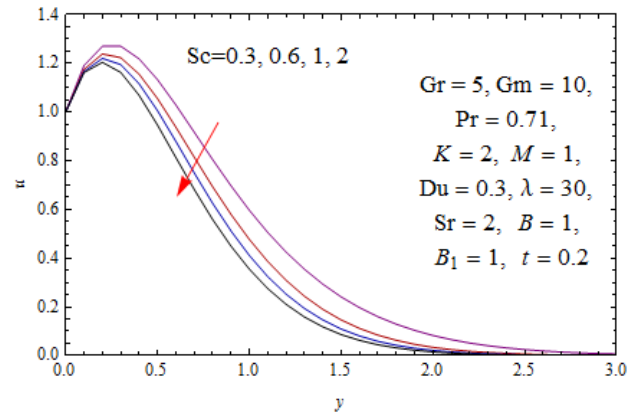


Fig. 4 Velocity Profile for Different Values of Sc

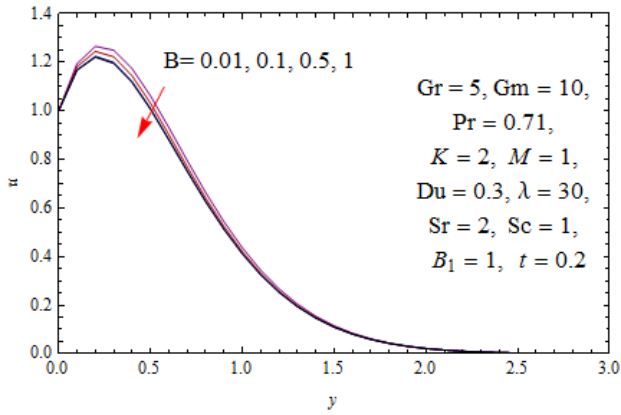


Fig. 5 Velocity Profile for Different Values of B

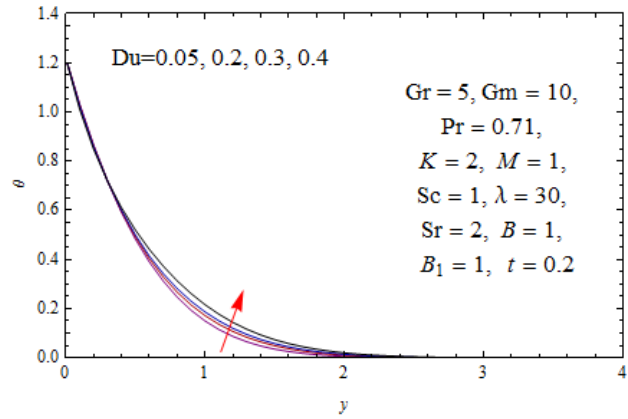


Fig. 8 Temperature Profile for Different Values of Du

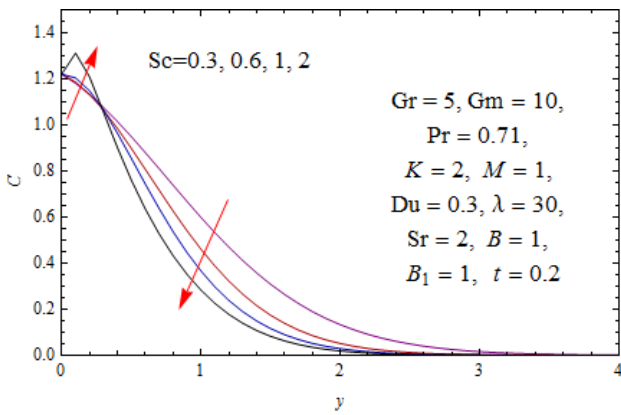


Fig. 6 Concentration Profile for Different Values of Sc

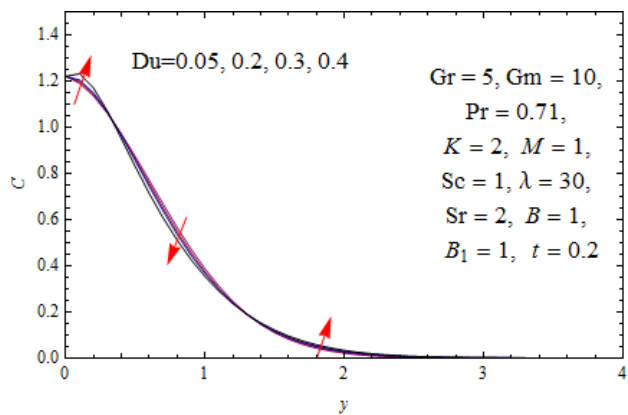


Fig. 9 Concentration Profile for Different Values of Du

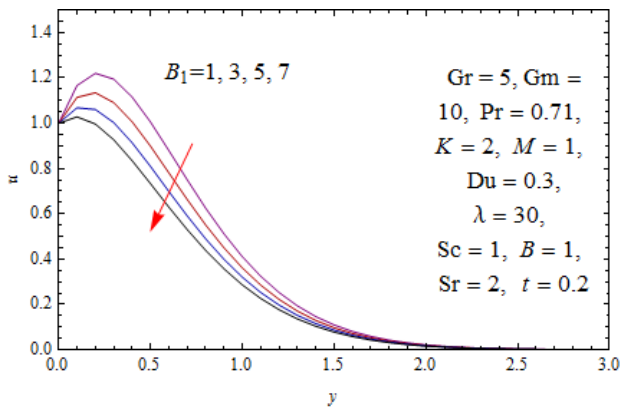


Fig. 7 Velocity Profile for Different Values of B_1

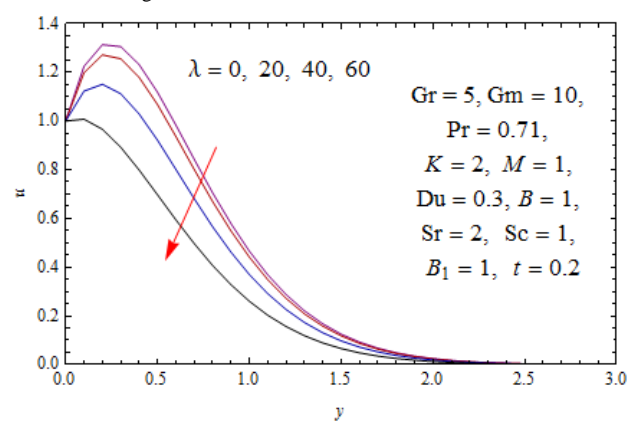


Fig. 10 Velocity Profile for Different Values of λ

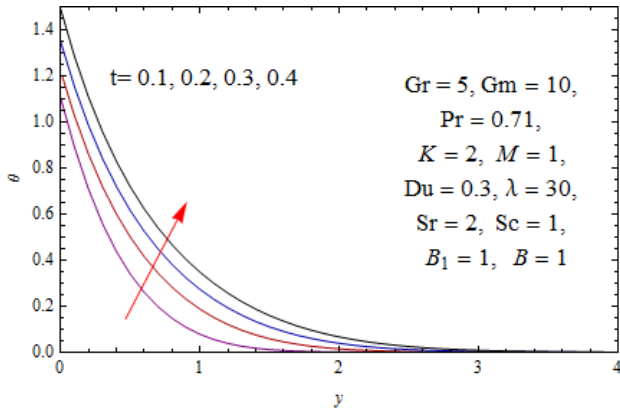


Fig. 11 Temperature Profile for Different Values of t

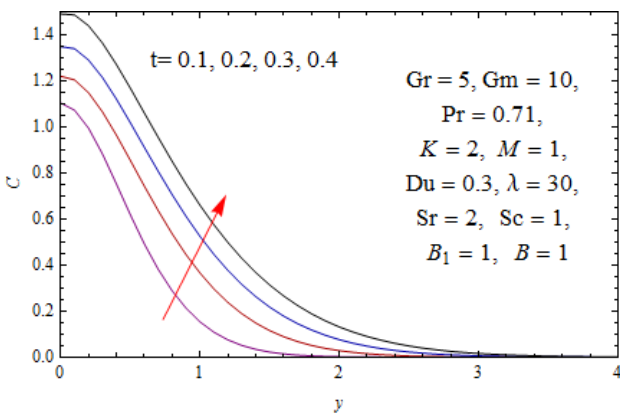


Fig. 12 Concentration Profile for Different Values of t

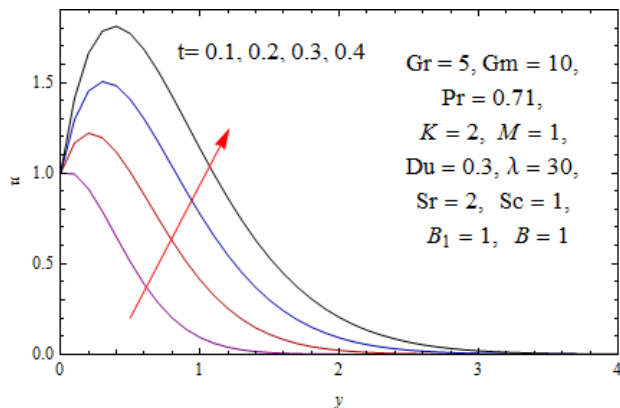


Fig. 13 Velocity Profile for Different Values of t

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Table-1: Skin friction coefficient τ for different values of parameter

B	B1	Du	λ	Sc	Sr	t	t
0	1	0.3	30	1	2	0.2	1.94092
0.1	1	0.3	30	1	2	0.2	1.81853
0.5	1	0.3	30	1	2	0.2	1.69244
1	1	0.3	30	1	2	0.2	1.14206
1	3	0.3	30	1	2	0.2	0.684971
1	5	0.3	30	1	2	0.2	0.282376
1	1	0.05	30	1	2	0.2	1.65696
1	1	0.2	30	1	2	0.2	1.66174
1	1	0.4	30	1	2	0.2	1.67891
1	1	0.3	0	1	2	0.2	2.24728
1	1	0.3	20	1	2	0.2	1.98556
1	1	0.3	40	1	2	0.2	1.23199
1	1	0.3	30	0.3	2	0.2	1.91098
1	1	0.3	30	0.6	2	0.2	1.75188
1	1	0.3	30	2	2	0.2	1.62117
1	1	0.3	30	1	1	0.2	1.33274
1	1	0.3	30	1	3	0.2	2.21532
1	1	0.3	30	1	5	0.2	3.33664
1	1	0.3	0	1	2	0.1	-0.04974
1	1	0.3	0	1	2	0.3	2.96368
1	1	0.3	0	1	2	0.4	4.14245

Table-2: Nusselt number Nu and Sherwood number Sh for different values of parameter

B	B1	Du	λ	Sc	Sr	t	Nu	Sh
1	1	0.05	30	1	2	0.2	1.88424	0.302048
1	1	0.2	30	1	2	0.2	1.91975	0.231161
1	1	0.4	30	1	2	0.2	2.08851	-0.110048
1	1	0.3	30	3	2	0.2	1.88499	0.401814
1	1	0.3	30	5	2	0.2	1.90713	0.34354
1	1	0.3	30	7	2	0.2	2.16847	-0.914806
1	1	0.3	30	1	1	0.2	1.78111	1.09094
1	1	0.3	30	1	3	0.2	2.06896	-0.593851
1	1	0.3	30	1	5	0.2	2.59959	-3.55071
1	1	0.3	30	1	2	0.1	2.15095	0.316292
1	1	0.3	30	1	2	0.3	1.97009	0.0896506
1	1	0.3	30	1	2	0.4	1.07012	0.0434141