IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 2 Issue 10, October 2015.
www.ijiset.com
ISSN 2348 – 7968

ON PAIRWISE FUZZY STRONGLY IRRESOLVABLE SPACES

G. Thangaraj, P.Vivakanandan

Department of Mathematics, Thiruvalluvar University, Vellore- 632 115, Tamilnadu, India.
Department of Mathematics, Jawahar Science College, Neyveli – 607 803, Tamilnadu, India.

ABSTRACT

In this paper, several characterizations of pairwise fuzzy strongly resolvable spaces are studied. The conditions under which pairwise fuzzy strongly resolvable spaces become pairwise fuzzy first category spaces, are also investigated.

KEY WORDS:

Pairwise fuzzy dense set , pairwise fuzzy nowhere dense set , pairwise fuzzy G_δ -set, pairwise fuzzy first category space, pairwise fuzzy Baire space, pairwise fuzzy almost resolvable space

2000 AMS Classification : 54 A 40, 03 E 72.

INTRODUCTION

The concept of fuzzy sets was introduced by **L.A. ZADEH** [20] in 1965 as a new approach for modeling uncertainities. Among the first field of Mathematics to be considered in the context of fuzzy sets, was general topology. In 1968, the concepts of fuzzy topology was defined by **C. L. CHANG** [2]. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then, much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. Today fuzzy topology has been firmly established as one of the basic disciplines of fuzzy mathematics.

The systematic study of resolvability began with the works of **E.Hewit** [6] and **Katetov** [7]. Resolvable and irresolvable spaces were studied extensively by Hewitt. In 1963, J. **G. Ceder** [3] introduced maximally resolvable spaces. **A. G. El'kin** [5] introduced open hereditarily irresolvable spaces in classical topology. The concept of almost resolvable spaces was introduced and studied by **Richard Bolstein** [9] as a generalization of resolvable spaces. The concept of strongly irresolvable spaces was introduced by Hewitt and it was also studied extensively by **David Rose , Kari Sizemore and Ben Thurston** [4]. In 1987, **A. Kandil** [8] introduced the concept of fuzzy bitopological spaces as a generalization of fuzzy topological spaces. The concept of pairwise fuzzy strongly irresolvable spaces was introduced by **G.Thangaraj** and **P.Sethuraman** [13] in fuzzy bitopological spaces. The aim of this paper is to study several characterizations of pairwise fuzzy strongly irresolvable spaces. The conditions under which pairwise fuzzy strongly resolvable spaces become pairwise fuzzy first category spaces, are also investigated.

ISSN 2348 – 7968

2 . PRELIMINARIES

Now we introduce some basic notions and results used in the sequel. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to CHANG (1968). By a fuzzy bitopological space (Kandil, 1989) we mean an ordered triple (X, T_1, T_2), where T_1 and T_2 are fuzzy topologies on the non-empty set X.

Definition 2.1 [2] : Let λ and μ be fuzzy sets in X. Then, for all $x \in X$,

(i) $\lambda = \mu \Leftrightarrow \lambda(x) = \mu(x)$

(ii) $\lambda \leq \mu \Leftrightarrow \lambda(x) \leq \mu(x)$

(iii) $\psi = \lambda \vee \mu \Leftrightarrow \psi(x) = \max \{ \lambda(x), \mu(x) \}$

(iv) $\delta = \lambda \wedge \mu \Leftrightarrow \psi(x) = \min \{\lambda(x), \mu(x) \}$

(v) $\eta = \lambda^c \Leftrightarrow \eta(x) = 1 - \lambda(x)$

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X,T), the union $\psi = \vee_i \lambda_i$ and intersection $\delta = \wedge_i \lambda_i$, are defined respectively as

(vi) $\psi(x) = sup_i \{\lambda_i(x) \,/\, x \in X \}$

(vii) $\delta(x) = inf_i \{\lambda_i(x) \,/ x \in X \}$

Definition 2.2 : Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). We define the interior and the closure of λ respectively as follows :

(i). $\text{int}(\lambda) = \vee \{ \mu / \mu \leq \lambda, \mu \in T \}$ and (ii) $\text{cl}(\lambda) = \wedge \{ \mu / \lambda \leq \mu, 1- \mu \in T \}$.

Lemma 2.1 [1] : For a fuzzy set λ of a fuzzy topological space X,

(i). $1 - \text{Int}(\lambda) = \text{Cl}(1 - \lambda)$,

(ii). $1 - \text{Cl}(\lambda) = \text{Int}(1 - \lambda)$.

DEFINITION 2.3 [15] : A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy open set if $\lambda \in T_i$ ($i = 1,2$). The complement of pairwise fuzzy open set in (X, T_1, T_2) is called a pairwise fuzzy closed set.

Definition 2.4 [11] : A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy dense set if $\text{cl}_{T_1}\text{cl}_{T_2}(\lambda) = \text{cl}_{T_2}\text{cl}_{T_1}(\lambda) = 1$.

Definition 2.5 [12] : A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy nowhere dense set if $\text{int}_{T_1}\text{cl}_{T_2}(\lambda) = 0 = \text{int}_{T_2}\text{cl}_{T_1}(\lambda)$.

Definition 2.6 [12]: Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a pairwise fuzzy first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Any other fuzzy set in (X, T_1, T_2) is said to be a pairwise fuzzy second category set in (X, T_1, T_2).

ISSN 2348 – 7968

Definition 2.7 [12] : If λ is a pairwise fuzzy first category set in a fuzzy bitopological space (X, T_1, T_2), then the fuzzy set $(1 - \lambda)$ is called a pairwise fuzzy residual set in (X, T_1, T_2).

Definition 2.8 [16] : A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy σ -nowhere dense set if λ is a pairwise fuzzy F_σ-set in (X, T_1, T_2) such that $\text{int}_{T_1}\, \text{int}_{T_2}(\lambda) = \text{int}_{T_2}\, \text{int}_{T_1}(\lambda) = 0$.

Definition 2.9 [15] : A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy G_δ -set if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are pairwise fuzzy open sets in (X, T_1, T_2).

Definition 2.10 [15] : A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy F_σ -set if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are pairwise fuzzy closed sets in (X, T_1, T_2).

Theorem 2.1 [14] : Let (X, T_1, T_2) be a pairwise fuzzy strongly irresolvable space. Then, λ is a pairwise fuzzy dense set in (X, T_1, T_2), if and only if $(1 - \lambda)$ is a pairwise fuzzy nowhere dens set in (X, T_1, T_2).

Theorem 2.2 [19] : If λ is a pairwise fuzzy dense and pairwise fuzzy G_δ -set in a fuzzy bitopological space (X, T_1, T_2), then $(1 - \lambda)$ is a pairwise fuzzy σ-nowhere dense set in (X, T_1, T_2).

Definition 2.11 [10] : Let μ be a fuzzy set on a fuzzy bitopological space X. Then we call μ ,

(i) a (T_i, T_j)-fuzzy semi-open set on X if $\mu \leq T_j - cl\,(T_i\text{- Int}(\mu))$,

(ii) a (T_i, T_j)-fuzzy semi-closed set on X if T_j - int(T_i - cl (μ)) ≤ μ .

Definition 2.12 [12]: A fuzzy bitopological space (X,T_1,T_2) is called pairwise fuzzy first category space if the fuzzy set $\mathbf{1_X}$ is a pairwise fuzzy first category set in (X,T_1,T_2). That is, $1_X = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy nowhere dense sets in (X,T_1,T_2). Otherwise, (X,T_1,T_2) will be called a pairwise fuzzy second category space.

3. PAIRWISE FUZZY STRONGLY IRRESOLVABLE SPACES

Definition 3.1 [13] : A fuzzy bitopological space (X, T_1, T_2) is said to be a pairwise fuzzy strongly irresolvable space if $\mathrm{cl}_{T_i}\mathrm{int}_{T_j}(\lambda) = 1$ (i, j = 1,2), for each pairwise fuzzy dense set λ in (X, T_1, T_2). That is, (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space if $\mathrm{cl}_{T_1}\mathrm{cl}_{T_2}(\lambda) = 1 = \mathrm{cl}_{T_2}\mathrm{cl}_{T_1}(\lambda)$ for a fuzzy set λ in (X, T_1, T_2), then $\mathrm{cl}_{T_1}\mathrm{int}_{T_2}(\lambda) = 1 = \mathrm{cl}_{T_2}\mathrm{int}_{T_1}(\lambda)$ in (X, T_1, T_2).

Proposition 3.1 : If $\mathrm{int}_{T_i}(\mu) = 0$ (i = 1,2) for a fuzzy set μ in a pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then μ is a pairwise fuzzy nowhere dense set in (X,T_1,T_2).

Proof : Let μ be a fuzzy set in (X, T_1, T_2) such that $\mathrm{int}_{T_i}(\mu) = 0$ (i = 1,2). Then, $\mathrm{int}_{T_1}\mathrm{int}_{T_2}(\mu) = \mathrm{int}_{T_1}(0) = 0$ and $\mathrm{int}_{T_2}\mathrm{int}_{T_1}(\mu) = \mathrm{int}_{T_2}(0) = 0$ in (X,T_1,T_2). Then, we have $1 - \mathrm{int}_{T_1}\mathrm{int}_{T_2}(\mu) = 1$ and $1 - \mathrm{int}_{T_2}\mathrm{int}_{T_1}(\mu) = 1$

and hence $cl_{T_1} cl_{T_2}(1-\mu) = 1$ and $cl_{T_2} cl_{T_1}(1-\mu) = 1$. That is, $(1-\mu)$ is a pairwise fuzzy dense set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space, $cl_{T_1} int_{T_2}(1-\mu) = 1 = cl_{T_2} int_{T_1}(1-\mu)$, for the pairwise fuzzy dense set $(1-\mu)$ in (X, T_1, T_2). Then, we have $1 - cl_{T_1} int_{T_2}(1-\mu) = 0$ and $1 - cl_{T_2} int_{T_1}(1-\mu) = 0$ and hence $int_{T_1} cl_{T_2}(\mu) = 0$ and $int_{T_2} cl_{T_1}(\mu) = 0$. Therefore μ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).

Proposition 3.2 : If (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space and if $cl_{T_1} int_{T_2}(\lambda) \neq 1$ and $cl_{T_2} int_{T_1}(\lambda) \neq 1$ for a fuzzy set λ in (X, T_1, T_2), then $cl_{T_1} cl_{T_2}(\lambda) \neq 1$ and $cl_{T_2} cl_{T_1}(\lambda) \neq 1$ in (X, T_1, T_2).

Proof : Let $cl_{T_1} int_{T_2}(\lambda) \neq 1$ and $cl_{T_2} int_{T_1}(\lambda) \neq 1$, for a fuzzy set λ in the fuzzy bitopological space (X, T_1, T_2). Suppose that $cl_{T_1} cl_{T_2}(\lambda) = 1$ and $cl_{T_2} cl_{T_1}(\lambda) = 1$ in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space, $cl_{T_1} cl_{T_2}(\lambda) = 1$ and $cl_{T_2} cl_{T_1}(\lambda) = 1$ in (X, T_1, T_2), will imply that $cl_{T_1} int_{T_2}(\lambda) = 1$ and $cl_{T_2} int_{T_1}(\lambda) = 1$, a contradiction to the hypothesis. Hence we must have $cl_{T_1} cl_{T_2}(\lambda) \neq 1$ and $cl_{T_2} cl_{T_1}(\lambda) \neq 1$ in (X, T_1, T_2).

Proposition 3.3 : If $int_{T_1} cl_{T_2}(\lambda) \neq 0$ and $int_{T_2} cl_{T_1}(\lambda) \neq 0$, for a fuzzy set λ in a pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then $int_{T_1}(\lambda) \neq 0$ and $int_{T_2}(\lambda) \neq 0$ in (X, T_1, T_2).

Proof : Suppose that $\text{int}_{T_1}(\lambda) = 0$ and $\text{int}_{T_2}(\lambda) = 0$ for a fuzzy set λ in a pairwise fuzzy strongly irresolvable space (X, T_1, T_2). Then, $1 - \text{int}_{T_1}\text{int}_{T_2}(\lambda) = 1 - \text{int}_{T_1}(0) = 1$ and $1 - \text{int}_{T_2}\text{int}_{T_1}(\lambda) = 1 - \text{int}_{T_2}(0) = 1$. Then $\text{cl}_{T_1}\text{cl}_{T_2}(1-\lambda) = 1$ and $\text{cl}_{T_2}\text{cl}_{T_1}(1-\lambda) = 1$. Hence $(1-\lambda)$ is a pairwise fuzzy dense set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space, $\text{cl}_{T_1}\text{int}_{T_2}(1-\lambda) = 1$ and $\text{cl}_{T_2}\text{int}_{T_1}(1-\lambda) = 1$. Then, we will have $1 - \text{int}_{T_1}\text{cl}_{T_2}(\lambda) = 1$ and $1 - \text{int}_{T_2}\text{cl}_{T_1}(\lambda) = 1$ and hence $\text{int}_{T_1}\text{cl}_{T_2}(\lambda) = 0$ and $\text{int}_{T_2}\text{cl}_{T_1}(\lambda) = 0$, a contradiction. Hence we must have $\text{int}_{T_1}(\lambda) \neq 0$ and $\text{int}_{T_2}(\lambda) \neq 0$ in (X, T_1, T_2).

Theorem 3.1 [19]: If λ is a pairwise fuzzy σ-nowhere dense set in a pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then λ is a pairwise fuzzy nowhere dense set and pairwise fuzzy F_σ-set in (X, T_1, T_2).

Proposition 3.4 : If λ is a pairwise fuzzy σ-nowhere dense set in a pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then λ is a pairwise fuzzy semi- closed set in (X, T_1, T_2).

Proof : Let λ be a pairwise fuzzy σ-nowhere dense set in a pairwise fuzzy strongly irresolvable space (X, T_1, T_2). Then, by theorem 3.1, λ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2) and hence $\text{int}_{T_1}\text{cl}_{T_2}(\lambda) = 0$ and $\text{int}_{T_2}\text{cl}_{T_1}(\lambda) = 0$. Then, we have $\text{int}_{T_1}\text{cl}_{T_2}(\lambda) \leq \lambda$ and $\text{int}_{T_2}\text{cl}_{T_1}(\lambda) \leq \lambda$. Therefore λ is a pairwise fuzzy semi- closed set in (X, T_1, T_2).

Proposition 3.5 : If λ is a pairwise fuzzy dense and pairwise fuzzy G_δ-set in a pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then λ is a pairwise fuzzy semi-open set in (X,T_1,T_2).

Proof : Let λ be a pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X,T_1,T_2). Then, by theorem 2.13, $(1-\lambda)$ is a pairwise fuzzy σ-nowhere dense set in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy strongly irresolvable space, by proposition 3.4, $(1-\lambda)$ is a pairwise fuzzy semi- closed set in (X, T_1, T_2). Hence λ is a pairwise fuzzy semi-open set in (X,T_1,T_2).

Proposition 3.6 : If $\lambda_i \leq (1-\lambda_j)$ ($i \neq j$), where λ_i is a pairwise fuzzy dense set in a pairwise fuzzy strongly irresolvable space (X,T_1,T_2), then λ_j is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).

Proof : Let λ_i be a pairwise fuzzy dense set in (X, T_1, T_2). Since (X, T_1,T_2) is a pairwise fuzzy strongly irresolvable space, for the pairwise fuzzy dense set λ_i, we have $cl_{T_1}\, int_{T_2}(\lambda_i) = 1$ and $cl_{T_2} int_{T_1}(\lambda_i) = 1$. Now $\lambda_i \leq (1-\lambda_j)(i \neq j)$ implies that $cl_{T_1}\, int_{T_2}(\lambda_i) \leq cl_{T_1}\, int_{T_2}(1-\lambda_j)$ and $cl_{T_2} int_{T_1}(\lambda_i) \leq cl_{T_2} int_{T_1}(1-\lambda_j)$. Then, we have $1 \leq cl_{T_1}\, int_{T_2}(1-\lambda_j)$ and $1 \leq cl_{T_2} int_{T_1}(1-\lambda_j)$. That is, $cl_{T_1}\, int_{T_2}(1-\lambda_j) = 1$ and $cl_{T_2} int_{T_1}(1-\lambda_j) = 1$. Hence $int_{T_1}\, cl_{T_2}(\lambda_j) = 0$ and $int_{T_2}\, cl_{T_1}(\lambda_j) = 0$. Therfore λ_j is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).

4. PAIRWISE FUZZY STRONGLY IRRESOLVABLE SPACES and OTHER FUZZY TOPOLOGICAL SPACES

Definition 4.1 [19] : A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy σ-Baire space if $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$, $(i = 1, 2)$ where $(\lambda_k)'s$ are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2).

Proposition 4.1 : If (X,T_1,T_2) is a pairwise fuzzy strongly irresolvable space and pairwise fuzzy σ-Baire space, then $cl_{T_i}(\wedge_{k=1}^{\infty}(\mu_k)) = 1 (i = 1, 2)$, where $(\mu_k)'s$ are pairwise fuzzy semi-open sets in (X,T_1,T_2).

Proof : Let (X, T_1, T_2) be a pairwise fuzzy strongly irresolvable space and pairwise fuzzy σ-Baire space. Let $(\lambda_k)'s$ be pairwise fuzzy σ-nowhere dense sets in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy σ-Baire space,

$int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$, $(i = 1, 2)$ where $(\lambda_k)'s$ are pairwise fuzzy σ-nowhere dense sets in (X,T_1,T_2). Then, $1 - int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 1$. This implies that $cl_{T_i}(\wedge_{k=1}^{\infty}(1-\lambda_k)) = 1$. By proposition 3.4, the pairwise fuzzy σ-nowhere dense sets $(\lambda_k)'s$ in the pairwise fuzzy strongly irresolvable space (X, T_1, T_2), are pairwise fuzzy semi- closed sets in (X, T_1, T_2) and hence $(1-\lambda_k)'s$ are pairwise fuzzy semi-open sets in (X, T_1, T_2). Let $\mu_k = 1-\lambda_k$. Hence we have $cl_{T_i}(\wedge_{k=1}^{\infty}(\mu_k)) = 1$ $(i = 1, 2)$, where $(\mu_k)'s$ are pairwise fuzzy semi-open sets in (X,T_1,T_2).

Definition 4.2 [12]: A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy Baire space if $\mathrm{int}_{T_i} (\vee_{k=1}^{\infty} (\lambda_k)) = 0$, $(i = 1, 2)$ where $(\lambda_k)'s$ are pairwise fuzzy nowhere dense sets in (X, T_1, T_2).

Proposition 4.2 : If $\lambda_i \leq (1-\lambda_j)$ $(i \neq j)$, where λ_i is a pairwise fuzzy dense set in a pairwise fuzzy strongly irresolvable space (X,T_1,T_2) and if $\mathrm{cl}_{T_k} (\wedge_{i=1}^{\infty} (\lambda_i)) = 1, (k = 1,2)$, then (X,T_1,T_2) is a pairwise fuzzy Baire space.

Proof : Let $\lambda_i \leq (1-\lambda_j)$ $(i \neq j)$, where λ_i is a pairwise fuzzy dense set in a pairwise fuzzy strongly irresolvable space (X,T_1,T_2). Then , by proposition 3.6, (λ_j)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Now $\lambda_i \leq (1-\lambda_j)$, implies that $\mathrm{cl}_{T_k} (\wedge_{i=1}^{\infty} \lambda_i) \leq \mathrm{cl}_{T_k}(\wedge_{j=1}^{\infty} (1-\lambda_j)$.Then we have $1 \leq \mathrm{cl}_{T_k}(\wedge_{j=1}^{\infty} (1-\lambda_j))$and hence $1 \leq 1-\mathrm{int}_{T_k} (\vee_{j=1}^{\infty} (\lambda_j))$. This implies that $\mathrm{int}_{T_k} (\vee_{j=1}^{\infty} (\lambda_j)) \leq 1-1 = 0$.That is, $\mathrm{int}_{T_k} (\vee_{j=1}^{\infty} (\lambda_j)) = 0$. Hence $\mathrm{int}_{T_k} (\vee_{j=1}^{\infty} (\lambda_j)) = 0$, where (λ_j)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2), implies that (X,T_1,T_2) is a pairwise fuzzy Baire space.

Proposition 4.3 : If $\mathrm{cl}_{T_i} (\wedge_{k=1}^{\infty} (\lambda_k) = 1$ $(i =1,2)$ where $(\lambda_k)'s$ are pairwise fuzzy dense and pairwise fuzzy G_δ -sets in a pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy Baire space.

Proof : Now $\mathrm{cl}_{T_i} (\wedge_{k=1}^{\infty} (\lambda_k)) = 1(i =1,2)$, implies that $\mathrm{int}_{T_i} (\vee_{k=1}^{\infty} (1-\lambda_k)) = 0$. Since $(\lambda_k)'s$ are pairwise fuzzy dense and pairwise fuzzy G_δ -sets in a pairwise fuzzy strongly irresolvable space (X, T_1, T_2), by theorem 2.13, $(1-$

ISSN 2348 – 7968

$\lambda_k)'s$ are pairwise fuzzy σ-nowhere dense sets in (X,T_1,T_2). Also, by theorem 3.1, $(1-\lambda_k)'s$ are pairwise fuzzy nowhere dense sets in (X,T_1,T_2). Hence $\text{int}_{T_i}\,(\vee_{k=1}^{\infty}(1-\lambda_k)) = 0$, where $(1-\lambda_k)'s$ are pairwise fuzzy nowhere dense sets in (X,T_1,T_2), implies that (X, T_1, T_2) is a pairwise fuzzy Baire space.

Definition 4.3[13] : A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy almost resolvable space if $\vee_{k=1}^{\infty}(\lambda_k) = 1$, where the fuzzy sets $(\lambda_k)'s$ in (X, T_1, T_2) are such that $\text{int}_{T_1}\text{int}_{T_2}((\lambda_k) = \text{int}_{T_2}\text{int}_{T_1}((\lambda_k) = 0$. Otherwise (X, T_1, T_2) is called a pairwise fuzzy almost irresolvable space.

Theorem 4.1 [18] : If a fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy second category space, then (X, T_1, T_2) is a pairwise fuzzy almost irresolvable space .

Theorem 4.2 [17]: If a fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy first category space, then (X, T_1, T_2) is a pairwise fuzzy almost resolvable space.

Proposition 4.4 : Let the fuzzy bitopological space (X, T_1, T_2) be a pairwise fuzzy strongly irresolvable space. Then, we have the following :

(i). (X, T_1, T_2) is a pairwise fuzzy almost irresolvable space , then (X, T_1, T_2) is a pairwise fuzzy second category space.

(ii). (X,T_1,T_2) is a pairwise fuzzy almost resolvable space, then (X, T_1, T_2) is a pairwise fuzzy first category space.

ISSN 2348 – 7968

Proof : (i). Let (X, T_1, T_2) be a pairwise fuzzy almost irresolvable space. Then, $\vee_{k=1}^{\infty}(\lambda_k) \neq 1$, where the fuzzy sets $(\lambda_k)'s$ in (X, T_1, T_2) are such that $int_{T_1} int_{T_2}(\lambda_k) = int_{T_2} int_{T_1}(\lambda_k) = 0$. Now, $1 - int_{T_1} int_{T_2}(\lambda_k) = 1$ and $1 - int_{T_2} int_{T_1}(\lambda_k) = 1$. Then, $cl_{T_1} cl_{T_2}(1 - \lambda_k) = 1$ and $cl_{T_2} cl_{T_1}(1 - \lambda_k) = 1$ and hence $(1 - \lambda_k)'s$ are pairwise fuzzy dense sets in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space, for the pairwise fuzzy dense set λ_k, we have $cl_{T_1} int_{T_2}(1 - \lambda_k) = 1$ and $cl_{T_2} int_{T_1}(1 - \lambda_k) = 1$. This implies that $int_{T_1} cl_{T_2}(\lambda_k) = int_{T_2} cl_{T_1}(\lambda_k) = 0$ and hence $(\lambda_k)'s$ are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Thus, $\vee_{k=1}^{\infty}(\lambda_k) \neq 1$, where $(\lambda_k)'s$ are pairwise fuzzy nowhere dense sets in (X, T_1, T_2), implies that (X, T_1, T_2) is a pairwise fuzzy second category space.

(ii). Let (X, T_1, T_2) be a pairwise fuzzy almost resolvable space .Then, $\vee_{k=1}^{\infty}(\lambda_k) = 1$, where the fuzzy sets $(\lambda_k)'s$ in (X, T_1, T_2) are such that $int_{T_1} int_{T_2}(\lambda_k) = 0$ and $int_{T_2} int_{T_1}(\lambda_k) = 0$. Now, $1 - int_{T_1} int_{T_2}(\lambda_k) = 1$ and $1 - int_{T_2} int_{T_1}(\lambda_k) = 1$. Then, $cl_{T_1} cl_{T_2}(1 - \lambda_k) = 1$ and $cl_{T_2} cl_{T_1}(1 - \lambda_k) = 1$ and hence $(1 - \lambda_k)'s$ are pairwise fuzzy dense sets in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space, for the pairwise fuzzy dense sets λ_k, we have, $cl_{T_1} int_{T_2}(1 - \lambda_k) = 1$ and $cl_{T_2} int_{T_1}(1 - \lambda_k) = 1$. This implies that $int_{T_1} cl_{T_2}(\lambda_k) = int_{T_2} cl_{T_1}(\lambda_k) = 0$ and hence $(\lambda_k)'s$ are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Thus, $\vee_{k=1}^{\infty}(\lambda_k) = 1$, where $(\lambda_k)'s$ are pairwise fuzzy nowhere dense sets in (X, T_1, T_2), implies that (X, T_1, T_2) is a pairwise fuzzy first category space.

Theorem 4.3 [18] : If $\vee_{k=1}^{\infty}(\lambda_k) = 1$, where the fuzzy sets (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in a fuzzy bitopological space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy almost resolvable space.

Definition 4.4 [19] : A fuzzy bitopological space (X,T_1,T_2) is called pairwise fuzzy σ- first category space if the fuzzy set $\mathbf{1_X}$ is a pairwise fuzzy σ-first category set in (X,T_1,T_2). That is, $1_X = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy σ-nowhere dense sets in (X,T_1,T_2). Otherwise, (X,T_1,T_2) will be called a pairwise fuzzy σ-second category space.

Proposition 4.5 : If (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable and pairwise fuzzy σ-first category space, then (X, T_1, T_2) is a pairwise fuzzy first category space.

Proof : Let (X,T_1,T_2) be a pairwise fuzzy σ-first category space. Then, $\vee_{k=1}^{\infty}(\lambda_k) = 1$, where the fuzzy sets (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space, by theorem 3.1, then (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Hence $\vee_{k=1}^{\infty}(\lambda_k) = 1$, where the fuzzy sets (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2), implies that (X, T_1, T_2) is a pairwise fuzzy first category space.

Definition 4.5 [16] : A fuzzy bitopological space (X, T_1,T_2) is called a pairwise fuzzy hyper connected space if each non- null pairwise fuzzy open set λ in (X,T_1,T_2), is a pairwise fuzzy dense set in (X,T_1,T_2). That is, if λ is a pairwise fuzzy open set in a fuzzy bitopological space (X,T_1,T_2), then $cl_{T_1} cl_{T_2} (\lambda) = cl_{T_2} cl_{T_1} (\lambda) = 1$.

Theorem 4.4 [18] : If $\vee_{k=1}^{\infty} (\lambda_k) = 1$, where the fuzzy sets $(\lambda_k)'s$ are pairwise fuzzy closed sets in a pairwise fuzzy hyper connected space (X,T_1,T_2), then (X,T_1,T_2) is a pairwise fuzzy almost resolvable space.

Proposition 4.6 : If $\vee_{k=1}^{\infty} (\lambda_k) = 1$, where the fuzzy sets $(\lambda_k)'s$ are pairwise fuzzy closed sets in a pairwise fuzzy hyper connected and pairwise fuzzy strongly irresolvable space (X,T_1,T_2), then (X, T_1, T_2) is a pairwise fuzzy first category space.

Proof : Let $\vee_{k=1}^{\infty} (\lambda_k) = 1$, where the fuzzy sets $(\lambda_k)'s$ are pairwise fuzzy closed sets in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy hyper connected space, by theorem 4.3, (X,T_1,T_2) is a pairwise fuzzy almost resolvable space. Then, (X,T_1,T_2) is a pairwise fuzzy almost resolvable and pairwise fuzzy strongly irresolvable space, by proposition 4.3, (X,T_1,T_2) is a pairwise fuzzy first category space.

Proposition 4.7 : If (X,T_1,T_2) is a pairwise fuzzy Baire and pairwise fuzzy strongly irresolvable space, then $cl_{T_i} (\wedge_{k=1}^{\infty} \mu_k) = 1$ $(i = 1,2)$ where $(\mu_k)'s$ are pairwise fuzzy dense in (X,T_1,T_2).

Proof: Let (X,T_1,T_2) be a pairwise fuzzy Baire space. Then, $int_{T_i} (\vee_{k=1}^{\infty} (\lambda_k)) = 0$, $(i = 1, 2)$, where $(\lambda_k)'s$ are pairwise fuzzy nowhere dense sets in (X,T_1,T_2).Since $(\lambda_k)'s$ are pairwise fuzzy nowhere dense sets in (X,T_1,T_2), by theorem 2.14, $(1-\lambda_k)'s$ are pairwise fuzzy dense sets in (X, T_1, T_2). Now $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$, implies that $1 - int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 1$. Then, we have $cl_{T_i}(\wedge_{k=1}^{\infty}(1-\lambda_k)) = 1$. Let $(1-\lambda_k) = \mu_k$. Then $cl_{T_i}(\wedge_{k=1}^{\infty}\mu_k) = 1$, where $(\mu_k)'s$ are pairwise fuzzy dense sets in (X, T_1, T_2).

REFERENCES

[1] . **K.K.AZAD**, *On Fuzzy semi continuity*, *Fuzzy almost continuity and Fuzzy weakly continuity*, J. Math. Anal. Appl., 82 (1981), 14 – 32.

[2]. **C.L. CHANG**, *Fuzzy Topological Spaces*, J. Math. Anal. Appl., 24, (1968),
182 – 190.

[3]. **J.G.CEDER**, *On Maximally Resolvable Spaces* , Fund. Math., 55 (1964),
87 – 93.

[4]. **DAVID ROSE, KARI SIZEMORE and BEN THURSTON**, *Strongly Irresolvable Spaces*, J. Math. Math. Sci., (2006), 1 -12.

[5]. **A.G. EL'KIN**, *Ultrafilters and Undecomposable spaces*, Vestnik. Moskov. Univ. Ser. I Mat. Meh., 24 (5), (1969), 51– 56.

[6]. **E.HEWITT**, *A Problem In Set - Theoretic Topology*, Duke Math. Jour., 10,
[1943], 309 – 333.

[7]. **M. KATETOV**, *On Topological Spaces containing no disjoint dense subsets,*
Math. Sbornik . N.S., 21 (63) (1947), 3 – 12.

[8]. **A. KANDIL.,** *Biproximities and Fuzzy Bitopological Spaces,* Simon Stevin, 63 (1989), 45 – 66.

[9]. **RICHARD BOLSTEIN**, *Sets Of Points Of Discontinuity*., Proc. Amer.
Math. Soc., Vol. 38. No. 1, (1973), 193 – 197.

[10]. **S. S.Thakur and R. Malviya**., *Semi-Open Sets and Semi-Continuity in Fuzzy Bitopological Spaces,* Fuzzy Sets and Sys., 79 (1996) , 251–256.

[11]. **G. THANGARAJ** , *On Pairwise Fuzzy Resolvable And Fuzzy Irresolvable*
Spaces, Bull. Cal. Math. Soc., 102, (1) (2010) , 59 – 68.

[12]. **G.THANGARAJ** and **S. SETHURAMAN**, *On Pairwise Fuzzy Baire Bitopological*
Spaces, Gen. Math. Notes., Vol. 20, No.2, (2014), 12– 21.

[13]. **G.THANGARAJ** and **S.SETHURAMAN**, *A Note on Pairwise Fuzzy Baire*
Spaces, Ann. Fuzzy Math. Inform., Vol. 8, No. 5, (2014), 729 – 737.

[14]. **G.THANGARAJ** and **S.SETHURAMAN**, *Some Remarks on Pairwise Fuzzy Baire Spaces*, Ann. Fuzzy Math. Inform., Vol. 9, No. 5, (2014), 683 – 691.

[15]. **G.THANGARAJ** and **V.CHANDIRAN**, *On Pairwise Fuzzy Volterra Spaces*, Ann. Fuzzy Math. Inform., Vol. 7 No. 6 ,(2014), 1005 – 1012.

[16]. **G. THANGARAJ** and **V.CHANDIRAN,** *A Note On Pairwise Fuzzy Volterra Spaces* , Ann. Fuzzy Math. Inform., Vol. 9, No.3 (2015), 365 – 372.

[17]. **G. THANGARAJ** and **P.VIVAKANANDAN**, *Pairwise Fuzzy Almost Resolvable Spaces*, Inter. J. Fuzzy Math. Sys., Vol. 4, No. 2 (2014), 245 – 253.

[18]. **G. THANGARAJ** and **P.VIVAKANANDAN**, *A Note on Pairwise Fuzzy Almost Resolvable Spaces*, (To appear in Inter. Review Fuzzy Math .,(2015),

[19]. **G.THANGARAJ** and **A.VINOTHKUMAR**, *On Pairwise Fuzzy σ-Baire spaces, Spaces*, Ann. Fuzzy Math. Inform., Vol. 9, No. 4 (2015), 529 – 536.

[20]. **L.A. ZADEH**, *Fuzzy sets*, Inform. And Control, Vol.8, (1965), 338 – 358.