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Arithmetic Operation of Fuzzy Numbers Using A-Cut Method

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ABSTRACT

α-cut method is a standard method for performing different arithmetic operations like addition, multiplication, division, subtraction. Here to finding membership function for square root of X where X is a fuzzy number, is not possible by the standard alpha-cut method.

They have proposed a method of finding membership function from the simple assumption that the Dubois-Prade left reference function is a distribution function and similarly the Dubois-Prade right reference function is a complementary distribution function.

In this section we consider arithmetic operation on fuzzy numbers using α-cut cut method.

Addition of fuzzy Numbers:

Let $X = [a,b,c]$ and $Y = [p,q,r]$ be two fuzzy numbers whose membership functions are

$$\mu_X(X) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \end{cases}$$

$$\mu_Y(X) = \begin{cases} \dfrac{x-p}{p-q}, & p \le x \le q \\ \dfrac{r-x}{r-q}, & q \le x \le r \end{cases}$$

Then $\alpha_X = [(b-a)\alpha + a, c-(c-b)\alpha]$ and $\alpha_Y = [(q-p)\alpha + p, r-(r-q)\alpha]$ are the α –cuts of fuzzy numbers X and Y respectively. To calculate addition of fuzzy numbers X and Y we first add the α –cuts of X and Y using interval arithmetic

$$\alpha_X + \alpha_Y$$

$$= [(b-a)\alpha + a, c-(c-b)\alpha] + [(q-p)\alpha + p, r-(r-q)\alpha]$$

$$= [a + p + (b - a + q - p)\alpha, c + r - (c - b + r - q)\alpha] \ldots\ldots\ldots(4.1)$$

To find the membership function $\mu_{X+Y(x)}$ we equate to x both the first and second component in (4.1) which gives

$$x = [a + p + (b - a + q - p)\alpha] \text{ and}$$

$$x = [c + r - (c - b + r - q)\alpha]$$

Now, expressing α in terms of x and setting α = 0 and α =1 in (4.1) we get α together with the domain of x,

$\alpha = \frac{x-(a+p)}{(b+q)-(a+p)}, (a + p) \leq x \leq (b + q)$ and

$\alpha = \frac{(c+r)-x}{(c+r)-(b+q)}, (b + q) \leq x \leq (c + r)$

which gives

$$\mu_{X+Y}(x) = \begin{cases} \frac{x - (a+p)}{(b+q) - (a+p)}, & (a+p) \leq x \leq (b+q) \\ \frac{(c+r) - x}{(c+r) - (b+q)}, & (b+q) \leq x \leq (c+r) \end{cases}$$

Subtraction of Fuzzy Numbers:

Let $X = [a,b,c]$ and $Y = [p,q,r]$ be two fuzzy numbers. Then $\alpha_X = [(b - a)\alpha + a, c - (c - b)\alpha]$ and $\alpha_Y = [(q - p)\alpha + p, r - (r - q)\alpha]$ are the α –cuts of fuzzy numbers X and Y respectively. To calculate subtraction of fuzzy numbers X and Y we first subtract the α –cuts of X and Y using interval arithmetic.

$$\alpha_X - \alpha_Y$$

$$= [(b - a)\alpha + a, c - (c - b)\alpha] - [(q - p)\alpha + p, r - (r - q)\alpha]$$

$$= [(b - a)\alpha + a - (r - (r - q)\alpha), c - (c - b)\alpha - ((q - p)\alpha + p)]$$

$$= [(a - r) + (b - a + r - q)\alpha, (c - p) - (c - b + q - p)\alpha] \ldots\ldots(4.2)$$

To find the membership function $\mu_{X-Y}(x)$ we equate to x both the first and second component in (4.2) which gives

$x = (a - r) + (b - a + r - q)\alpha$ and

$x = (c - p) - (c - b + q - p)\alpha$

Now, expressing α in terms of x and setting α = 0 and α =1 in (4.2) we get α together with the domain of x,

$\alpha = \frac{x-(a-r)}{(b-q)-(a-r)}$, $(a - r) \leq x \leq (b - q)$ and

$\alpha = \frac{(c-p)-x}{(c-p)-(b-q)}$, $(b - q) \leq x \leq (c - p)$

which gives

$$\mu_{X-Y}(x) = \begin{cases} \frac{x-(a-r)}{(b-q)-(a-r)}, & (a-r) \leq x \leq (b-q) \\ \frac{(c-p)-x}{(c-p)-(b-q)}, & (b-q) \leq x \leq (c-p) \end{cases}$$

Multiplication of Fuzzy Numbers:

Let $X = [a, b, c]$ and $Y = [p, q, r]$ be two positive fuzzy numbers. Then $\alpha_X = [(b-a)\alpha + a, c - (c-b)\alpha]$ and $\alpha_Y = [(q-p)\alpha + p, r - (r-q)\alpha]$ are the α cuts of fuzzy numbers X and Y respectively. To calculate multiplication of fuzzy numbers X and Y we first multiply the α –cuts of X and Y using interval arithmetic

$$\alpha_X * \alpha_Y$$
$$= [(b-a)\alpha + a, c - (c-b)\alpha] * [(q-p)\alpha + p, r - (r-q)\alpha]$$
$$= [((b-a)\alpha + a) * ((q-p)\alpha + p), (c - (c-b)\alpha) * (r - (r-q)\alpha)] \quad ...(4.3)$$

To find the membership function $\mu_{XY}(x)$ we equate to x both the first and second component in (4.3) which gives

$x = (b-a)(q-p)\alpha^2 + ((b-a)p + (q-p)a)\alpha + ap$ and

$x = (c-b)(r-q)\alpha^2 - ((r-q)c + (c-b)r)\alpha + cr$

Now, expressing α in terms of x and setting α = 0 and α =1 in (4.3) we get α together with the domain of x,

$$\alpha = \frac{-((b-a)p+(q-p)a)+\sqrt{((b-a)p+q-p)a)^2-4(b-a)(q-p)(ap-x)}}{2(b-a)(q-p)} \quad , ap \le x \le bq$$

Which gives,

$$\alpha = \frac{-((r-q)c+(c-b)r)-\sqrt{((r-q)c+(c-b)r)^2-4(c-b)(r-q)(cr-x)}}{2(c-b)(r-q)} \quad , bq \le x \le cr$$

$$\mu_{XY}(x) = \begin{cases} \frac{-((b-a)p+(q-p)a)+\sqrt{((b-a)p+q-p)a)^2-4(b-a)(q-p)(ap-x)}}{2(b-a)(q-p)}, & ap \le x \le bq \\ \frac{-((r-q)c+(c-b)r)-\sqrt{((r-q)c+(c-b)r)^2-4(c-b)(r-q)(cr-x)}}{2(c-b)(r-q)}, & bq \le x \le cr \end{cases}$$

Division of Fuzzy Numbers:

Let $X = [a,b,c]$ and $Y = [p,q,r]$ be two positive fuzzy numbers. Then $\alpha_X = [(b-a)\alpha + a, c-(c-b)\alpha]$ and

$\alpha_Y = [(q-p)\alpha + p, r-(r-q)\alpha]$ are the α –cuts of fuzzy numbers X and Y respectively. To calculate division of fuzzy numbers X and Y we first divide the α –cuts of X and Y using interval arithmetic.

$$\frac{\alpha_X}{\alpha_Y} = \frac{[(b-a)\alpha+a,c-(c-b)\alpha]}{[(q-p)\alpha+p,r-(r-q)\alpha}$$

$$= \left[\frac{(b-a)\alpha+a}{r-(r-q)\alpha}, \frac{c-(c-b)\alpha}{(q-p)\alpha+p}\right] \ldots\ldots\ldots\ldots(4.4)$$

To find the membership function $\mu_{x/y}(x)$ we equate to x both the first and second component in (4.4) which gives,

$$x = \frac{(b-a)\alpha + a}{r-(r-q)\alpha}$$

and

$$x = \frac{c-(c-b)\alpha}{(q-p)\alpha + p}$$

Now, expressing α in terms of x and setting α = 0 and α =1 in (4.4) we get α together with the domain of x,

$\alpha = \frac{xr-a}{(b-a)+(q-r)x}$,$a/r \leq x \leq b/q$ and

$\alpha = \frac{c-px}{(c-b)+(q-p)x}$,$b/q \leq x \leq c/p$

which gives,

$$\mu_{X/Y}(x) = \begin{cases} \frac{xr-a}{(b-a)+(q-r)}, & a/r \leq x \leq b/q \\ \frac{c-px}{(c-p)+(b-q)}, & b/q \leq x \leq c/r \end{cases}$$

Inverse of fuzzy number:

Let $X = [a,b,c]$ be a positive fuzzy number.

Then $\alpha_X = [(b-a)\alpha + a, c-(c-b)\alpha]$ is the α-cut of the fuzzy numbers X. To calculate inverse of the fuzzy number X we first take the inverse of the α-cut of X using interval arithmetic.

$$\frac{1}{\alpha_X} = \frac{1}{[(b-a)\alpha+a, c-(c-b)\alpha]}$$

$$= \left[\frac{1}{c-(c-b)\alpha}, \frac{1}{(b-a)\alpha+a}\right] \ldots\ldots\ldots(4.5)$$

To find the membership function $\mu_{1/X}(x)$ we equate to x both the first and second component in (4.5),

which gives,

$x = \frac{1}{c-(c-b)\alpha}$ and

$x = \frac{1}{(b-a)\alpha+a}$

Now, expressing α in terms of x and setting α = 0 and α =1 in (4.5) we get α together with the domain of x,

$\alpha = \frac{cx-1}{x(c-b)}$,$1/c \leq x \leq 1/b$ and

$\alpha = \frac{1-ax}{x(b-a)}$,$1/b \leq x \leq 1/a$

which gives,

$$\mu_{1/x}(x) = \begin{cases} \dfrac{cx-1}{x(c-b)}, & 1/c \le x \le 1/b \\ \dfrac{1-ax}{x(b-a)}, & 1/b \le x \le 1/a \end{cases}$$

Exponential of a Fuzzy number:

Let $X = [a,b,c] > 0$ be a fuzzy number.

Then $\alpha_A = [(b-a)\alpha + a, c-(c-b)\alpha]$ is the α -cut of the fuzzy numbers A. To calculate exponential of the fuzzy number *A* we first take the exponential of the α cut of A using interval arithmetic.

$$exp(\alpha_A) = exp([(b-a)\alpha + a, c-(c-b)\alpha])$$
$$= [exp((b-a)\alpha + a), exp(c-(c-b)\alpha)] \ldots\ldots\ldots\ldots\ldots(4.6)$$

To find the membership function $\mu_{exp(x)}(x)$ we equate to x both the first and second component in (4.6).

which gives,

$$x = exp\big((b-a)\alpha + a\big) and$$
$$x = exp(c-(c-b)\alpha)$$

Now, expressing α in terms of x and setting α = 0 and α =1 in (4.6) we get α together with the domain of x,

$\alpha = \frac{In(x)-a}{b-a}$, exp $(a) \le x \le exp$(b) and

$\alpha = \frac{c-In(x)}{c-b}$,exp $(b) \le x \le$ exp (c)

which gives,

$$\mu_{exp\,(x)}(x) = \begin{cases} \dfrac{In(x)-a}{b-a}, & exp\,(a) \le x \le exp\,(b) \\ \dfrac{c-In(x)}{c-b}, & exp\,(b) \le x \le exp\,(c) \end{cases}$$

Logarithm of a fuzzy number:

Let $X = [a,b,c] > 0$ be a fuzzy number.

Then $\alpha_X = [(b-a)\alpha + a, c - (c-b)\alpha]$ is the α -cut of the fuzzy number X. To calculate logarithm of the fuzzy number X we first take the logarithm of the α -cut of X using interval arithmetic.

$$In(\alpha_X) = In([(b-a)\alpha + a, c-(c-b)\alpha])$$
$$= [In((b-a)\alpha + a), In(c-(c-b)\alpha)] \ldots\ldots\ldots\ldots\ldots (4.7)$$

To find the membership function $\mu_{In(x)}(x)$ we equate to x both the first and second component in (4.7).

which gives,

$$x = In\big((b-a)\alpha + a\big) \quad and$$
$$x = In(c-(c-b)\alpha)$$

Now, expressing α in terms of x and setting α = 0 and α =1 in (4.7) we get α together with the domain of x,

$$\alpha = \frac{exp(x)-a}{b-a}, \ In(a) \le x \le In(b) \ \ and$$
$$\alpha = \frac{c-exp(x)}{c-b}, In(b) \le x \le In(c)$$

which gives,

$$\mu_{In(x)}(x) = \begin{cases} \dfrac{exp(x)-a}{b-a}, & In(a) \le x \le In(b) \\ \dfrac{c-exp(x)}{c-b}, & In(b) \le x \le In(c) \end{cases}$$

Square root of fuzzy number by α-cut method

Here we determine square root of a fuzzy number by α-cut cut method. Let $X = [a,b,c] > 0$ be a fuzzy number. Then $\alpha_X = [(b-a)\alpha + a, c-(c-b)\alpha]$ is the α -cut of the fuzzy numbers X. To calculate square root of the fuzzy number X we first take the square root of the α -cut of X using interval arithmetic.

$$\sqrt{\alpha_X} = \sqrt{[(b-a)\alpha + a, c-(c-b)\alpha]}$$
$$= \sqrt{(b-a)\alpha + a}, \sqrt{c-(c-b)\alpha} \ \ldots\ldots\ldots (4.8)$$

To find the membership function $\mu_{\sqrt{x}}(x)$ we equate to x both the first and second component in (4.8),

which gives,

$$x = \sqrt{(b-a)\alpha + a} \quad \text{and}$$

$x = \sqrt{c - (c-b)\alpha}$

Now, expressing α in terms of x and setting α = 0 and α =1 in (4.8) we get α together with the domain of x,

$\alpha = \frac{x^2 - a}{b-a}$, $\sqrt{a} \le x \le \sqrt{b}$ and

$\alpha = \frac{c - x^2}{c-b}$, $\sqrt{b} \le x \le \sqrt{c}$

which gives,

$$\mu_{\sqrt{x}}(x) = \begin{cases} \dfrac{x^2 - a}{b-a}, & \sqrt{a} \le x \le \sqrt{b} \\ \dfrac{c - x^2}{c-b}, & \sqrt{b} \le x \le \sqrt{c} \end{cases}$$

n^th root of a Fuzzy number:

Let $X = [a, b, c] > 0$ be a fuzzy number.

Then $\alpha_X = [(b-a)\alpha + a, c - (c-b)\alpha]$ is the α -cut of the fuzzy numbers X. To calculate nth root of the fuzzy number X we first take the nth root of the α -cut of X using interval arithmetic.

$$\alpha_X = ([(b-a)\alpha + a, c - (c-b)\alpha])^{1/n}$$

$$= [((b-a)\alpha + a)1/n, (c - (c-b)\alpha)]^{1/n} \quad \text{………(4.9)}$$

To find the membership function $\mu_{\sqrt[n]{X}}(x)$ we equate to x both the first and second component in (4.9),

which gives,

$x = ((b-a)\alpha + a)^{1/n}$ and

$x = (c - (c-b)\alpha)^{1/n}$

Now, expressing α in terms of x and setting α = 0 and α =1 in (4.9) we get α together with the domain of x,

$\alpha = \frac{x^n - a}{b-a}$, $\sqrt[n]{a} \le x \le \sqrt[n]{b}$ and

$\alpha = \frac{c - x^n}{c-b}$, $\sqrt[n]{b} \le x \le \sqrt[n]{c}$

which gives,

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$$\mu_{\sqrt[n]{x}}(x) = \begin{cases} \dfrac{x^n - a}{b - a}, & \sqrt[n]{a} \leq x \leq \sqrt[n]{b} \\ \dfrac{c - x^n}{c - b}, & \sqrt[n]{b} \leq x \leq \sqrt[n]{c} \end{cases}$$

A comparison with the proposed method:

In this section we solve the problems as by α -cut method.

Addition of fuzzy Numbers:

Let $X = [1,2,4]$ and $Y = [3,5,6]$ be two fuzzy numbers whose membership functions are

$$\mu_X(x) = \begin{cases} x - 1, & 1 \leq x \leq 2 \\ \dfrac{4 - x}{2}, & 2 \leq x \leq 4 \end{cases}$$

$$\mu_Y(x) = \begin{cases} \dfrac{x - 3}{2}, & 3 \leq x \leq 5 \\ 6 - x, & 5 \leq x \leq 6 \end{cases}$$

Then $\alpha_X = [1 + \alpha, 4 - 2\alpha]$ and $\alpha_Y = [2\alpha + 3, 6 - \alpha]$ are the α –cuts of fuzzy numbers X and Y respectively.

$\alpha_X + \alpha_Y$

$= [1 + \alpha, 4 - 2\alpha] + [2\alpha + 3, 6 - \alpha]$

$= [3\alpha + 4, 10 - 3\alpha]$..................(5.1)

We take,

$x = 3\alpha + 4, and$

$x = 10 - 3\alpha,$

Now, expressing α in terms of x and setting $\alpha = 0$ and $\alpha = 1$ in (5.1) we get α together with the domain of x,

$\alpha = \frac{x-4}{3}$ $, 4 \leq x \leq 7$ and

$\alpha = \frac{10-x}{3}$ $, 7 \leq x \leq 10$

which gives

$$\mu_{X+Y}(x) = \begin{cases} \dfrac{x-4}{3}, & 4 \le x \le 7 \\ \dfrac{10-x}{3}, & 7 \le x \le 10 \end{cases}$$

Subtraction of fuzzy Numbers:

Let $X = [1,2,4]$ and $Y = [3,5,6]$ be two fuzzy numbers.

Then $\alpha_X = [1+\alpha, 4-2\alpha]$ and $\alpha_Y = [2\alpha+3, 6-\alpha]$ are the α –cuts of fuzzy numbers X and Y respectively.

$\alpha_X - \alpha_Y$

$= [1+\alpha, 4-2\alpha] + [2\alpha+3, 6-\alpha]$

$= [2\alpha-5, 1-4\alpha]$…………………(5.2)

We take,

$x = 2\alpha - 5$, and

$x = 1 - 4\alpha,$

Now, expressing α in terms of x and setting α = 0 and α =1 in (5.2) we get α together with the domain of x,

$\alpha = \frac{x+5}{2}$ $, -5 \le x \le -3$ and

$\alpha = \frac{1-x}{4}$ $, -3 \le x \le 1$

which gives,

$$\mu_{X-Y}(x) = \begin{cases} \dfrac{x+5}{4}, -5 \le x \le -3 \\ \dfrac{1-x}{4}, -3 \le x \le 1 \end{cases}$$

Multiplication of fuzzy Numbers:

Let $X = [1,2,4]$ and $Y = [3,5,6]$ be two fuzzy numbers.

Then $\alpha_X = [1+\alpha, 4-2\alpha]$ and $\alpha_Y = [2\alpha+3, 6-\alpha]$ are the α –cuts of fuzzy numbers X and Y respectively.
Therefore,

$\alpha_X * \alpha_Y$

$= [1+\alpha, 4-2\alpha] + [2\alpha+3, 6-\alpha]$

$= [2\alpha^2 + 5\alpha + 3, 2\alpha^2 - 16\alpha + 24]$...................(5.3)

We take,

$x = 2\alpha^2 + 5\alpha + 3$ and

$x = 2\alpha^2 - 16\alpha + 24$

Now, expressing α in terms of x and setting α = 0 and α =1 in (5.3) we get α together with the domain of x,

$\alpha = \frac{\sqrt{1+8x}-5}{4}$,$3 \le x \le 10$ and

$\alpha = \frac{8-\sqrt{16+2x}}{2}$,$10 \le x \le 24$

which gives

$$\mu_{XY}(x) = \begin{cases} \dfrac{\sqrt{1+8x}-5}{4}, 3 \le x \le 10 \\ \dfrac{8-\sqrt{16+2x}}{2}, 10 \le x \le 24 \end{cases}$$

Division of fuzzy Numbers:

Let $X = [1,2,4]$ and $Y = [3,5,6]$ be two fuzzy numbers.

Then $\alpha_X = [1+\alpha, 4-2\alpha]$ and $\alpha_Y = [2\alpha+3, 6-\alpha]$ are the α –cuts of fuzzy numbers X and Y respectively.
Therefore,

$$\frac{\alpha_X}{\alpha_Y} = \frac{[1+\alpha, 4-2\alpha]}{[2\alpha+3, 6-\alpha]}$$

$$= \left[\frac{1+\alpha}{6-\alpha}, \frac{4-2\alpha}{2\alpha+3}\right]$$

We take,

ISSN 2348 – 7968

$x = \frac{1+\alpha}{6-\alpha}$ and

$x = \frac{4-2\alpha}{2\alpha+3}$

Now, expressing α in terms of x and setting α = 0 and α =1 in (5.4) we get α together with the domain of x,

$\alpha = \frac{6x-1}{1+x}$,$1/6 \leq x \leq 2/5$ and

$\alpha = \frac{4-3x}{2(1+x)}$,$2/5 \leq x \leq 4/3$

which gives,

$$\mu_{X/Y}(x) = \begin{cases} \frac{6x-1}{1+x}, 1/6 \leq x \leq 2/5 \\ \frac{4-3x}{2(1+x)}, 2/5 \leq x \leq 4/3 \end{cases}$$

Inverse of fuzzy number:

Let *X*= [1, 2, 4] be a positive fuzzy number.

Then $\alpha_X = [1+\alpha, 4-2\alpha]$ is the α -cut of the fuzzy numbers *X*. To calculate inverse of the fuzzy number *X* we first take the inverse of the α -cut of *X* using interval arithmetic.

$$\frac{1}{\alpha_X} = \frac{1}{[1+\alpha, 4-2\alpha]}$$

$$= \left[\frac{1}{4-2\alpha}, \frac{1}{1+\alpha}\right]$$

To find the membership function $\mu_{1/X}(x)$ we equate to x both the first and second component in (5.5),

which gives,

$x = \frac{1}{4-2\alpha}$ and

$x = \frac{1}{1+\alpha}$

Now, expressing α in terms of x and setting α = 0 and α =1 in (5.5) we get α together with the domain of x,

$\alpha = \frac{4x-1}{2x}$,$1/4 \leq x \leq 1/2$ and

$\alpha = \frac{1-x}{x} \quad , 1/2 \leq x \leq 1$

which gives,

$$\mu_{1/X}(x) = \begin{cases} \frac{4x-1}{2x}, 1/4 \leq x \leq 1/2 \\ \frac{1-x}{x}, 1/2 \leq x \leq 1 \end{cases}$$

Exponential of a Fuzzy number:

Let $X = [1,2,4] > 0$ be a fuzzy number.

Then $\alpha_X = [1+\alpha, 4-2\alpha]$ is the α -cut of the fuzzy numbers A. To calculate exponential of the fuzzy number *A* we first take the exponential of the α -cut of A using interval arithmetic.

$$exp(\alpha_X) = exp([1+\alpha, 4-2\alpha])$$
$$= [exp((1+\alpha), exp(4-2\alpha)] \ldots\ldots\ldots\ldots\ldots (5.6)$$

To find the membership function $\mu_{exp(x)}(x)$ we equate to x both the first and second component in (5.6),
which gives,

$x = exp((1+\alpha)\ and$
$x = exp(4-2\alpha)$

Now, expressing α in terms of x and setting $\alpha = 0$ and $\alpha = 1$ in (5.6) we get α together with the domain of x,

$\alpha = In(1+\alpha) - 1, exp(1) \leq x \leq exp(2) \quad and$

$\alpha = \frac{4-In(4-2\alpha)}{2}, exp(2) \leq x \leq exp(4$

which gives,

$$\mu_{exp(x)}(x) = \begin{cases} In(1+\alpha) - 1, & exp(1) \leq x \leq exp(2) \\ \frac{4-In(4-2\alpha)}{2}, & exp(2) \leq x \leq exp(4) \end{cases}$$

Logarithm of a fuzzy number:

Let $X = [1,2,4] > 0$ be a fuzzy number.

Then $\alpha_X = [1+\alpha, 4-2\alpha]$ is the α -cut of the fuzzy number X. To calculate logarithm of the fuzzy number X we first take the logarithm of the α -cut of X using interval arithmetic.

$In(\alpha_X) = In([1+\alpha, 4-2\alpha])$

$= [In(1+\alpha), In(4-2\alpha)]$..............(5.7)

To find the membership function $\mu_{In(x)}(x)$ we equate to x both the first and second component in (5.7),
which gives,

$x = In(1+\alpha) and$

$x = In(4-2\alpha)$

Now, expressing α in terms of x and setting α = 0 and α =1 in (5.7) we get α together with the domain of x,

$\alpha = exp(1+\alpha) - 1, In(1) \le x \le In(2)$ and

$\alpha = \frac{4-exp(4-2\alpha)}{2}, \mathrm{In}(2) \le x \le \mathrm{In}(4)$

which gives,

$$\mu_{\mathrm{In}\,(x)}(x) = \begin{cases} \exp(1+\alpha) - 1, & \mathrm{In}(1) \le x \le \mathrm{In}(2) \\ \dfrac{4 - \exp(4-2\alpha)}{2}, & \mathrm{In}(2) \le x \le \mathrm{In}(4) \end{cases}$$

Square root of fuzzy number by α-cut method:

Let $X = [1,2,4] > 0$ be a fuzzy number.

Then $\alpha_X = [1+\alpha, 4-2\alpha]$ is the α -cut of the fuzzy numbers X. To calculate square root of the fuzzy number X we first take the square root of the α -cut of X using interval arithmetic.

$\sqrt{\alpha_X} = \sqrt{[1+\alpha, 4-2\alpha]}$

$= [\sqrt{1+\alpha}, \sqrt{4-2\alpha}]$...........(5.8)

To find the membership function $\mu_{\sqrt{x}}(x)$ we equate to x both the first and second component in (5.8),

which gives,

$x = \sqrt{1+\alpha}$ and

$x = \sqrt{4-2\alpha}$

Now, expressing α in terms of x and setting α = 0 and α =1 in (5.8) we get α together with the domain of x,

$\alpha = x^2 - 1, \sqrt{1} \le x \le \sqrt{2}$

$\alpha = \frac{4-x^2}{2}, \sqrt{2} \le x \le \sqrt{4}$

Which gives,

$$\mu_{\sqrt{(x)}}(x) = \begin{cases} x^2 - 1, & \sqrt{1} \le x \le \sqrt{2} \\ \dfrac{4 - x^2}{2}, & \sqrt{2} \le x \le \sqrt{4} \end{cases}$$

n^th root of a Fuzzy number:

Let $X = [1,2,4] > 0$ be a fuzzy number.

Then $\alpha_X = [1 + \alpha, 4 - 2\alpha]$ is the α -cut of the fuzzy numbers X. To calculate nth root of the fuzzy number *X* we first take the nth root of the α -cut of X using interval arithmetic.

$$\alpha_X = ([1 + \alpha, 4 - 2\alpha])^{1/n}$$

$$= (1 + \alpha)^{1/n}, (4 - 2\alpha)^{1/n} \quad \ldots\ldots\ldots\ldots\ldots(5.9)$$

To find the membership function $\mu_{\sqrt[n]{(x)}}(x)$ we equate to x both the first and second component in (5.9), which gives,

$x = (1 + \alpha)^{1/n}$ and

$x = (4 - 2\alpha)^{1/n}$

Now, expressing α in terms of x and setting α = 0 and α =1 in (5.9) we get α together with the domain of x,

$\alpha = x^n - 1$, $\sqrt[n]{1} \le x \le \sqrt[n]{2}$ and

$\alpha = \frac{4-x^n}{2}, \sqrt[n]{2} \le x \le \sqrt[n]{4}$

Which gives,

$$\mu_{\sqrt[n]{x}}(x) = \begin{cases} x^n - 1, & \sqrt[n]{1} \le x \le \sqrt[n]{2} \\ \dfrac{4 - x^n}{2}, & \sqrt[n]{2} \le x \le \sqrt[n]{4} \end{cases}$$

Addition of a triangular and a non-triangular fuzzy number:

Let $X = [2,3,4]$ and $Y = [4,16,25]$ be triangular and non triangular fuzzy numbers respectively whose membership functions are given as,

$$\mu_y(x) = \begin{cases} \frac{x-2}{2}, 2 \le x \le 4 \\ 5 - x, 4 \le x \le 25 \\ 0, otherwise \end{cases}$$

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Then $\alpha_X = [2+2\alpha, 5-\alpha]$ and $\alpha_Y = [(2+2\alpha)2, (5-\alpha)2]$ are the α –cuts of fuzzy numbers X and Y respectively. Therefore,

$\alpha_X + \alpha_Y$

$$= [2+2\alpha, 5-\alpha] + [(2+2\alpha)^2, (5-\alpha)^2]$$

$$= [4\alpha^2 + 10\alpha + 6, \alpha^2 - 11\alpha + 30] \text{...................}(5.10)$$

To find the membership function $\mu_{X+Y}(x)$ we equate to x both the first and second component in (5.10) which gives

$x = 4\alpha^2 + 10\alpha + 6$ and

$x = \alpha^2 - 11\alpha + 30$

Now, expressing α in terms of x and setting α = 0 and α =1 in (5.10) we get α together with the domain of x,

$\alpha = \frac{\sqrt{1+4x}-5}{4}$ $,6 \le x \le 20$ and

$\alpha = \frac{11-\sqrt{1+4x}}{2}$ $,6 \le x \le 20$

which gives,

$$\mu_{X+Y}(x) = \begin{cases} \frac{\sqrt{1+4x}-5}{4}, 6 \le x \le 20 \\ \frac{11-\sqrt{1+4x}}{2}, 6 \le x \le 20 \\ 0, otherwise \end{cases}$$

We have seen that the α –cuts method is simpler than the method proposed by them. Infact for performing division (and subtraction), their method is lengthy compared to α cuts method, because they have to first perform inverse (negative) of the divisor (fuzzy number to be subtracted).

$$\mu_{X+Y}(x) = \begin{cases} \frac{\sqrt{1+4x}-5}{4}, 6 \le x \le 20 \\ \frac{11-\sqrt{1+4x}}{2}, 6 \le x \le 20 \\ 0, otherwise \end{cases}$$

Instead of

$$\mu_{X+Y}(x) = \begin{cases} \dfrac{\sqrt{1+4x}-5}{4}, 6 \le x \le 20 \\ \dfrac{11-\sqrt{1+4x}}{2}, 6 \le x \le 20 \\ 0 , otherwise \end{cases}$$

Conclusion

In this project we have shown that α-cut method is a method general enough to deal with all kinds of fuzzy arithmetic including nth root, exponentiation and taking log. We have solved problems using this method and compared them with the method in proposed [2] and [6]. We have seen that the alpha-cut method is simpler than the proposed method. As a passing remark we would like to mention that example 3 in [2] is confusing. Because there they started to extract nth root of $X = [a, b, c], (a, b, c) > 0$ but ended up performing exponentiation of the fuzzy number X.