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Vertex Odd Divisor Cordial Labeling of Graphs

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Abstract

In this paper, the vertex odd divisor cordial labeling of wheel graph W_n, switching of a pendent vertex in path P_n, switching of a vertex in cycle C_n, Bistar $B_{n,n}$, $S(K_{1,n})$, $B^2_{n,n}$, $DS(B_{n,n})$, $S'(B_{n,n})$ and $< K^{(1)}_{1,n}, K^{(2)}_{1,n}, K^{(3)}_{1,n} >$ are presented.

Keywords: *Divisor Cordial labeling, Divisor Cordial Graphs, Vertex Odd Divisor Cordial labeling, Vertex Odd Square Divisor Cordial Graphs.*

1. Introduction

By a graph, we mean a finite, undirected graph without loops and multiple edges, for terms not defined here, we refer to Harary [4]. For standard terminology and notations related to number theory we refer to Burton [2] and graph labeling, we refer to Gallian [3]. In [1], Cahit introduce the concept of cordial labeling of graph. In [12], Varatharajan et al. introduce the concept of divisor cordial labeling of graph. The divisor cordial labeling of various types of graph are presented in [5-8, 10-13]. In [9], Muthaiyan et al introduce vertex odd divisor cordial labeling of graph. Further they proved path, cycle, $K_{2,n}$, $K_{1,n}\cup K_{1,m}$, helm H_n, flower Fl_n, $< K^{(1)}_{1,n}, K^{(2)}_{1,n} >$, switching of the apex vertex in helm H_n and $S'(K_{1,n})$ are vertex odd divisor cordial graphs under some conditions. Every divisor cordial graphs need not be vertex odd divisor cordial graphs. Also, every vertex odd divisor cordial graphs need not be divisor cordial graphs. The brief summaries of definition which are necessary for the present investigation are provided below.

2. Definitions

Definition : 2.1

A graph labeling is the assignment of unique identifiers to the edges and vertices of a graph.

Definition : 2.2

A mapping $f : V(G) \rightarrow \{0,1\}$ is called binary vertex labeling of G and f(v) is called the label of the vertex v of G under f. If for an edge e = uv, the induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ is given by $f^*(e) = | f(u) - f(v) |$. Then $v_f(i)$ = number of vertices of having label i under f and $e_f(i)$ = number of edges of having label i under f*.

A binary vertex labeling f of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0)–e_f(1)| \leq 1$. A graph G is cordial if it admits cordial labeling.

Definition : 2.3

Let a and b be two integers. If a divides b means that there is a positive integer k such that b = ka. It is denoted by a | b. If a does not divide b, then we denote $a \nmid b$.

Definition : 2.4

Let G = (V(G), E(G)) be a simple graph and $f : V(G) \rightarrow \{1,2,...,|V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if f(u) | f(v) or f(v) | f(u) and the label 0 otherwise. The function f is called a divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph with a divisor cordial labeling is called a divisor cordial graph.

Definition : 2.5

Let G = (V(G), E(G)) be a simple graph with n vertices and $f : V \rightarrow \{1,3,\ldots,2n-1|\}$ be a bijection. For each edge uv, assign the label 1 if either f(u) | f(v) or f(v) | f(u) and the label 0 otherwise. f is called a vertex odd divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph with vertex odd divisor cordial labeling is called a vertex odd divisor cordial graph.

Definition : 2.6

The join $G = G_1+G_2$ of graphs G_1 and G_2 with disjoint vertex sets V_1 and V_2, and edge sets E_1 and E_2, is the graph union $G_1\cup G_2$ together with all the edges joining

V_1 and V_2. A wheel graph W_n is defined as C_n+K_1, where C_n denotes the cycle with n vertices.

Definition : 2.7

For a graph G the splitting graph S′(G) of a graph G is obtained by adding a new vertex v′ corresponding to each vertex v of G such that N(v) = N(v′).

Definition : 2.8

Let G = (V(G), E(G)) be a graph with vertex set $V = S_1 \cup S_2 \cup \ldots \cup S_i \cup T$ where each S_i is a set of vertices having at least two vertices of the same degree and $T = V \backslash \cup S_i$. The degree splitting graph of G denoted by DS(G) is obtained from G by adding vertices $w_1, w_2, w_3, \ldots, w_t$ and joining to each vertex of S_i for $1 \le i \le t$.

Definition : 2.9

For a simple connected graph G the square of graph G is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in G.

Definition : 2.10

Bistar $B_{n,n}$ is the graph obtained by joining the center (apex) vertices of two copies of $K_{1,n}$ by an edge.

Definition : 2.11

A subdivision of the edge e = uv of a graph G is the replacement of the edge e by a new vertex w and two new edges uw and wv. This operation is also called an elementary subdivision of G. A graph H obtained by a sequence of elementary subdivisions from a graph G is said to be a subdivision graph of G. It is denoted by S(G).

Definition : 2.12

A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition : 2.13

Consider t copies of stars namely $K_{1,n}^{(1)}, K_{1,n}^{(2)}, \ldots, K_{1,n}^{(t)}$. Then $G = < K_{1,n}^{(1)}, K_{1,n}^{(2)}, \ldots, K_{1,n}^{(t)} >$ is the graph obtained by joining apex vertices of each $K_{1,n}^{(m-1)}$ and $K_{1,n}^{(m)}$ to a new vertex x_{m-1} where $2 \le m \le t$ and G has $t(n+2)-1$ vertices and $t(n+2)-2$ edges.

3. Main Results

Theorem 3.1

The wheel graph $W_n = C_n + K_1$ is vertex odd divisor cordial graph, where $n \ge 3$.

Proof.

Let G be a wheel graph $W_n = C_n + K_1$. Let v be the central vertex and $v_1, v_2, \ldots, v_n$ be the vertices of C_n.

Then $|V(G)| = n+1$ and $|E(G)| = 2n$.

Define vertex labeling $f : V(G) \to \{1,3,5,\ldots,2n+1\}$ as follows.

$f(v) = 1$,

Case 1 : $n \equiv 1 \pmod 3$

$f(v_i) = 2i+1, \quad 1 \le i \le n-2.$

$f(v_{n-1}) = 2n+1$,

$f(v_n) = 2n-1$.

Then $e_f(1) = e_f(0) = n$.

Case 2 : $n \equiv 0,2 \pmod 3$

$f(v_i) = 2i+1, \quad 1 \le i \le n-1.$

Then $e_f(1) = e_f(0) = n$.

In both the cases, we have $|e_f(0) - e_f(1)| \le 1$.

Therefore, W_n is vertex odd divisor cordial graph.

Example 3.1

The vertex odd divisor cordial labeling of W_4 is shown in figure 3.1.

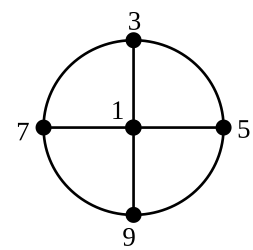

Figure 3.1

Theorem 3.2

Switching of a vertex in cycle C_n admits vertex odd divisor cordial labeling.

Proof.

Let $v_1, v_2, \ldots, v_n$ be the successive vertices of C_n.

G_v denotes graph is obtained by switching of vertex v of $G = C_n$.

Without loss of generality let the switched vertex be v_1.

Then $|V(G_{v_1})| = n$ and $|E(G_{v_1})| = 2n - 5$.

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Define vertex labeling $f : V(G_{v_1}) \rightarrow \{1,3,5,\ldots,2n-1\}$ as follows.

$f(v_1) = 1,$

$f(v_i) = 2i-1 \quad \text{for } 2 \le i \le n$

Then, $e_f(1) = n - 3$ and $e_f(0) = n - 2$.

Thus, $|e_f(0) - e_f(1)| \le 1$.

Hence, the graph obtained by switching of a vertex in cycle C_n is a vertex odd divisor cordial graph.

Example 3.2

The vertex odd divisor cordial labeling of switching of a vertex in cycle C_8 is shown in figure 3.2.

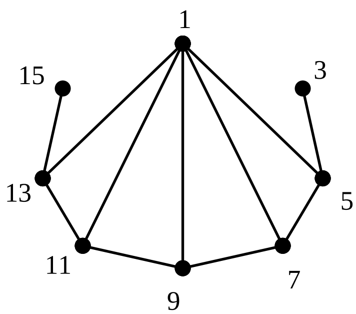

Figure 3.2

Theorem 3.3

Switching of a pendent vertex in path P_n is vertex odd divisor cordial graph.

Proof.

Let $v_1, v_2, \ldots, v_n$, be the vertices of path P_n.

The graph G is obtained by switching of a pendent vertex in path P_n. v_1 and v_n are pendent vertex of path P_n.

Without loss of generality, let the switched vertex be v_1.

Then $|V(G)| = n$ and $|E(G)| = 2n - 4$.

Define vertex labeling $f : V(G) \rightarrow \{1,3,5,\ldots,2n-1\}$ as follows.

$f(v_i) = 2i - 1, \quad 1 \le i \le n.$

Then $e_f(0) = e_f(1) = n - 2$.

Therefore, $|e_f(0) - e_f(1)| \le 1$.

Hence, G is vertex odd divisor cordial graph.

Example 3.3

The vertex odd divisor cordial labeling of switching of a pendent vertex in path P_6 is shown in figure 3.3.

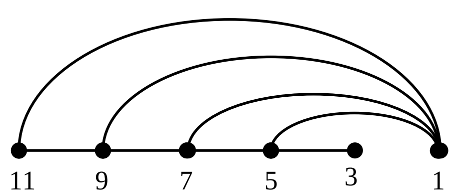

Figure 3.3

Theorem 3.4

The Bistar $B_{n,n}$ is vertex odd divisor cordial graph, where $n \ge 2$.

Proof.

Let $B_{n,n}$ be a graph with vertex set $\{u,v,u_i,v_i,\ 1 \le i \le n\}$, where u_i, v_i are pendant vertices.

Let G be the graph $B_{n,n}$.

The vertex set $V(G) = \{u, w, u_i, v_i : 1 \le i \le n\}$ and the edge set $E(G) = \{uw, uu_i, vv_i, :1 \le i \le n\}$.

Then $|V(G)| = 2n+2$ and $|E(G)| = 2n+1$.

p is the largest prime number such that $p \le 4n+3$.

Define vertex labeling $f : V(G) \rightarrow \{1,3,\ldots,4n+3\}$ as follows.

$f(u) = 1,$

$f(v) = p$ and assign the remaining labels to the remaining vertices $v_1,v_2,\ldots,v_n,u_1,u_2,\ldots,u_n$.

Then $e_f(0) = n$ and $e_f(1) = n+1$.

Therefore, $|e_f(0) - e_f(1)| \le 1$.

Hence G is vertex odd divisor cordial graph.

Example 3.4

The vertex odd divisor cordial labeling of $B_{4,4}$ is shown in figure 3.4.

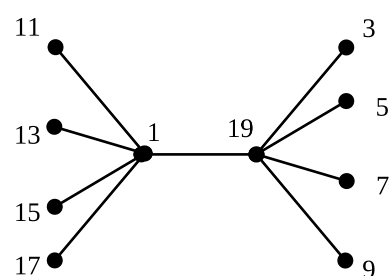

Figure 3.4

Theorem 3.5

The graph $S(K_{1,n})$ is a vertex odd divisor cordial graph.

Proof

Let G be a subdivision of the star $K_{1,n}$.

Let $V(G) = \{v, v_i, u_i : 1 \le i \le n\}$ and $E(G) = \{vv_i, v_iu_i : 1 \le i \le n\}$. Then $G = S(K_{1,n})$.

Define vertex labeling $f : V(G) \rightarrow \{1,3,5,\ldots,4n+1\}$ as follows

$f(v) = 1,$

$f(v_i) = 3 + 4(i-1) \quad \text{for } 1 \le i \le n,$

$f(u_i) = 5 + 4(i-1) \quad \text{for } 1 \le i \le n.$

Then, $e_f(0) = e_f(1) = n$.
Therefore $|e_f(0) - e_f(1)| \leq 1$.
Hence $S(K_{1,n})$ is a vertex odd divisor cordial graph.

Example 3.5

The vertex odd divisor cordial labeling of $S(K_{1,5})$ is shown in figure 3.5.

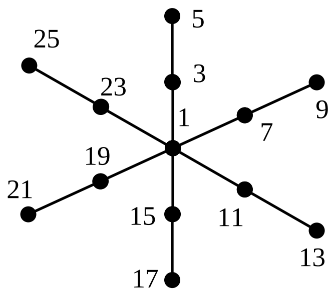

Figure 3.5

Theorem 3.6

$B^2_{n,n}$ is a vertex odd divisor cordial graph.

Proof.

Let $B_{n,n}$ be a graph with vertex set $\{u,v,u_i,v_i,\ 1\leq i \leq n\}$, where u_i, v_i are pendant vertices.

Let G be the graph $B^2_{n,n}$

Then $|V(G)| = 2n+2$ and $|E(G)| = 4n+1$.

Let p be the largest prime number $p \leq 4n+3$.

Define vertex labeling $f : V(G) \rightarrow \{1,3,5, \ldots, 4n+3\}$ as follows.

$f(u) = 1$,

$f(v) = p$,

Assign the remaining labels to the remaining vertices $u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n$.

Then, $e_f(0) = 2n$, $e_f(1) = 2n + 1$.

Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, $B^2_{n,n}$ is a vertex odd divisor cordial graph.

Example 3.6

The vertex odd divisor cordial labeling of $B^2_{5,5}$ is shown in figure 3.6.

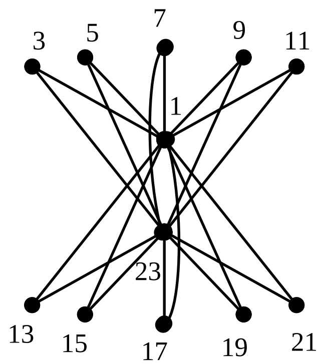

Figure 3.6

Theorem 3.7

$S'(B_{n,n})$ is a vertex odd divisor cordial graph.

Proof.

Let $B_{n,n}$ be a graph with vertex set $\{u,v,u_i,v_i,\ 1\leq i \leq n\}$, where u_i, v_i are pendant vertices.

In order to obtain $S'(B_{n,n})$, add u', v', u'_i, v'_i vertices corresponding to u, v, u_i, v_i, where $1 \leq i \leq n$.

Let G be a graph $S'(B_{n,n})$.

Then $|V(G)| = 4(n+1)$ and $|E(G)| = 6n + 3$.

Let p_1 and p_2 be the largest and next largest prime numbers, such that $p_2 < p_1 \leq 8n+7$.

Define vertex labeling $f : V(G) \rightarrow \{1, 3, 5, \ldots, 8n+7\}$ as follows.

$f(u) = 1$,

$f(u') = 3$,

$f(v) = p_2$,

$f(v') = p_1$,

$f(u_i) = 9 + 6(i - 1)$ for $1 \leq i \leq n$

$f(u'_i) = f(u_i) - 2$ for $1 \leq i \leq n$

$f(v'_i) = f(u_i) - 4$ for $1 \leq i \leq n$

Assign the remaining labels to the remaining vertices $v_1, v_2, \ldots, v_n$.

Then, $e_f(0) = 3n+1$ and $e_f(1) = 3n+2$.

Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, $S'(B_{n,n})$ is a vertex odd divisor cordial graph.

Example 3.7

The vertex odd divisor cordial labeling of $S'(B_{3,3})$ is shown in figure 3.7.

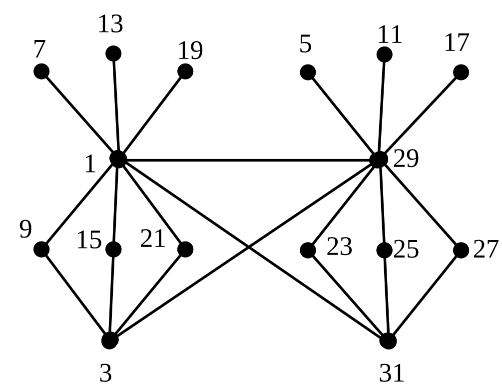

Figure 3.7

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Theorem 3.8

$DS(B_{n,n})$ is a vertex odd divisor cordial graph.

Proof.

Let $B_{n,n}$ be a graph with vertex set $\{u,v,u_i,v_i,\ 1\le i\le n\}$, where u_i, v_i are pendant vertices.

Here, $V(B_{n,n}) = V_1 \cup V_2$, where $V_1 = \{u_i, v_i : 1 \le i \le n\}$ and $V_2 = \{u, v\}$.

In order to obtain $DS(B_{n,n})$ from $B_{n,n}$, add w_1, w_2 corresponding to V_1 and V_2.

Let G be a graph $DS(B_{n,n})$.

Then, $|V(G)| = 2n+4$ and $E(G)| = 4n+3$.

Let p be the largest prime number such that $p \le 8n+7$.

Define vertex labeling $f : V(G) \to \{1,3,5, \ldots, 8n+7\}$ as follows.

$f(u) = 3$,
$f(v) = p$,
$f(w_1) = 1$,
$f(w_2) = 9$,

Case 1 : n is even and k = n/2

$f(u_i) = 5 + 6(i–1)$ for $1 \le i \le k$
$f(u_i) = 7 + 6(i–k–1)$ for $k+1 \le i \le n$.

Then, $e_f(0) = 2n+2$ and $e_f(1) = 2n+1$.

Case 2 : n is odd and k = (n+1)/2

$f(u_i) = 5 + 6(i–1)$ for $1 \le i \le k$
$f(u_i) = 7 + 6(i–k–1)$ for $k+1 \le i \le n$.

Then, $e_f(0) = 2n+2$ and $e_f(1) = 2n+1$.

In both cases, $|e_f(0) – e_f(1)| \le 1$.

Hence, $DS(B_{n,n})$ is a vertex odd divisor cordial graph.

Example 3.8

The vertex odd divisor cordial labeling of $DS(B_{5,5})$ is shown in figure 3.8.

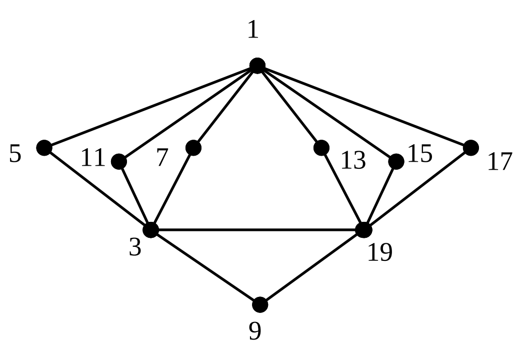

Figure 3.8

Theorem 3.9

The graph $G = \langle K^{(1)}_{1,n},\ K^{(2)}_{1,n},\ K^{(3)}_{1,n} \rangle$ is vertex odd divisor cordial.

Proof.

Let $G = \langle K^{(1)}_{1,n},\ K^{(2)}_{1,n},\ K^{(3)}_{1,n} \rangle$ be a graph with 3n + 5 vertices and 3n + 4 edges.

Let $v^{(i)}_1, v^{(i)}_2, \ldots, v^{(i)}_n$ be the pendant vertices of $K^{(i)}_{1,n}$ and let c_i be the apex vertex of $K^{(i)}_{1,n}$ for i = 1, 2, 3 .

Then $|V(G)| = 3n+5$ and $|E(G)| = 3n+4$.

Here, c_1 and c_2 are adjacent to x_1 and c_2 and c_3 are adjacent to x_2.

Let p be the largest prime number such that $p \le 6n+9$.

Define vertex labeling $f : V(G) \to \{1,3,5,\ldots, 6n+9\}$ as follows.

$f(c_1) = p$,
$f(c_2) = 1$,
$f(c_3) = 3$,
$f(x_1) = 5$,
$f(x_2) = 7$,

Case 1 : n is even and k = n/2

$f(v^{(3)}_i) = 9 + 6(i–1)$ for $1 \le i \le k$

$f(v^{(3)}_i) = 11 + 6(i–k–1)$ for $k+1 \le i \le n$.

Assign the remaining labels to the remaining vertices $v^{(2)}_1, v^{(2)}_2, \ldots, v^{(2)}_n, v^{(3)}_1, v^{(3)}_2, \ldots, v^{(3)}_n$.

Then, $e_f(0) = e_f(1) = \dfrac{3n+4}{2}$.

Case 2 : n is odd and k = (n+1)/2

$f(v^{(3)}_i) = 9 + 6(i–1)$ for $1 \le i \le k$

$f(v^{(3)}_i) = 11 + 6(i–k–1)$ for $k+1 \le i \le n$.

Assign the remaining labels to the remaining vertices $v^{(2)}_1, v^{(2)}_2, \ldots, v^{(2)}_n, v^{(3)}_1, v^{(3)}_2, \ldots, v^{(3)}_n$.

Then, $e_f(0) = \dfrac{3n+3}{2}$ and $e_f(1) = \dfrac{3n+5}{2}$.

In both cases, $|e_f(0) - e_f(1)| \le 1$.

Hence, the graph $G = \langle K^{(1)}_{1,n},\ K^{(2)}_{1,n},\ K^{(3)}_{1,n} \rangle$ is vertex odd divisor cordial graph.

Example 3.9

The vertex odd divisor cordial labeling of the graph $\langle K^{(1)}_{1,5},\ K^{(1)}_{1,5},\ K^{(1)}_{1,5} \rangle$ is shown in figure 3.9.

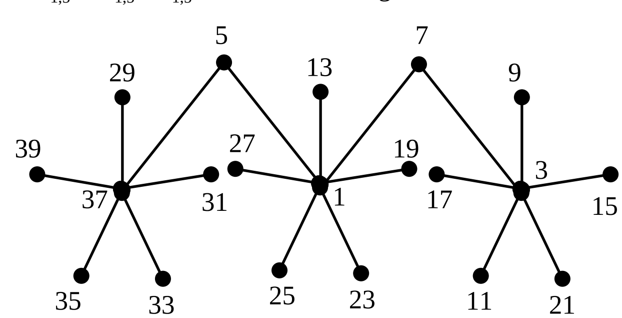

Figure 3.9

4. Conclusions

In this paper, the vertex odd divisor cordial labeling of wheel graph W_n, switching of a pendent vertex in path P_n, switching of a vertex in cycle C_n, Bistar $B_{n,n}$, $S(K_{1,n})$, $B^2_{n,n}$, $DS(B_{n,n})$, $S'(B_{n,n})$ and $< K^{(1)}_{1,n}, K^{(2)}_{1,n}, K^{(3)}_{1,n} >$ are proved.

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