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Regular Elements of the Semigroup $B_X(D)$ defined by Semilattices of the Class $\Sigma_3(X,8)$ when $Z_7 \neq \varnothing$

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Abstract

In this paragraph we give a full description of regular elements of the semigroup $B_X(D)$, which are defined by semilattices of the class $\Sigma_3(X,8)$. For the case where X-is a finite set we derive formulas by means of which we can calculate the numbers of regular elements of the respective semigroups. In this subsection it is assume that $Z_7 \neq \varnothing$

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1. We call an element α taken from the semigroup $B_X(D)$ a regular element of the semigroup $B_X(D)$ if in $B_X(D)$ there exists an element β such that $\alpha \circ \beta \circ \alpha = \alpha$.

Definition 1.1. The one-to-one mapping φ between the complete X – semilattices of unions D' and D'' is called a complete isomorphism if the condition $\varphi(\cup D_1) = \bigcup_{T' \in D_1} \varphi(T')$ is fulfilled for each nonempty subset D_1 of the semilattice D' (see Definition 6.3.2 of [1] or Definition 6.3.2 of [2]).

Definition 1.2. Let α be some binary relation of the semigroup $B_X(D)$. We say that the complete isomorphism φ between the complete semilattices of unions Q and D' is a complete $\alpha-$isomorphism if

(a) $Q = V(D,\alpha)$;

(b) $\varphi(\varnothing) = \varnothing$ for $\varnothing \in V(D,\alpha)$ and $\varphi(T)\alpha = T$ for eny $T \in V(D,\alpha)$ (see Definition 6.3.3 of [1] or Definition 6.3.3 of [2]).

Now assume that $D \in \Sigma_3(X,8)$. We introduce the following notation:

We denoted Q_i i=$(1,2,......16)$ the following semitattices by symbols:

1) $Q_1 = \{T\}$, where $T \in D$

2) $Q_2 = \{T, T'\}$, where $T, T' \in D$, $T \subset T'$;

3) $Q_3 = \{T, T', T''\}$, where $T, T', T'' \in D$, $T \subset T' \subset T''$;

4) $Q_4 = \{T, T', T'', T'''\}$, where $T, T', T'', T''' \in D$, $T \subset T' \subset T'' \subset T'''$,

5) $Q_5 = \{Z_7, T, T', T'', \breve{D}\}$, where $Z_7, T, T', T'', \breve{D} \in D$, $Z_7 \subset T \subset T' \subset T'' \subset \breve{D}$,

6) $Q_6 = \{T, T', T'', T' \cup T''\}$, where $T, T', T'' \in D$, $T \subset T'$, $T \subset T''$, $T' \setminus T'' \neq \varnothing$, $T'' \setminus T' \neq \varnothing$,

7) $Q_7 = \{T, T', T'', T''', T'' \cup T'''\}$, where, $T \subset T' \subset T''$, $T \subset T' \subset T'''$, $T'' \setminus T''' \neq \varnothing$, $T''' \setminus T'' \neq \varnothing$

8) $Q_8 = \{Z_7, T, Z_4, Z_2, Z_1, \breve{D}\}$, where $T \in \{Z_6, Z_5\}$

9) $Q_9 = \{Z_7, Z_5, Z_4, Z_3, Z_1, \breve{D}\}$,

10) $Q_{10} = \{T, T', T'', T' \cup T'', T'''\}$, where $T \subset T'$, $T \subset T''$, $T' \setminus T'' \neq \varnothing$, $T'' \setminus T' \neq \varnothing$, $T' \cup T'' \subset T'''$

11) $Q_{11} = \{Z_7, Z_6, Z_5, Z_4, T, \breve{D}\}$, where $T \in \{Z_2, Z_1\}$,

12) $Q_{12} = \{T, T', T'', T' \cup T'', T''', T' \cup T'' \cup T'''\}$, where, $T \subset T'$, $T \subset T''$, $T' \setminus T'' \neq \varnothing$, $T'' \setminus T' \neq \varnothing$, $T'' \subset T'''$, $(T' \cup T'') \setminus T''' \neq \varnothing$, $T''' \setminus (T' \cup T'') \neq \varnothing$,

13) $Q_{13} = \{Z_7, Z_5, Z_4, Z_3, Z_2, Z_1, \breve{D}\}$,

14) $Q_{14} = \{Z_7, Z_6, Z_5, Z_4, Z_3, Z_1, \breve{D}\}$,

15) $Q_{15} = \{Z_7, Z_6, Z_5, Z_4, Z_2, Z_1, \breve{D}\}$,

16) $Q_{16} = \left\{ Z_7, Z_6, Z_5, Z_4, Z_3, Z_2, Z_1, \breve{D} \right\}$

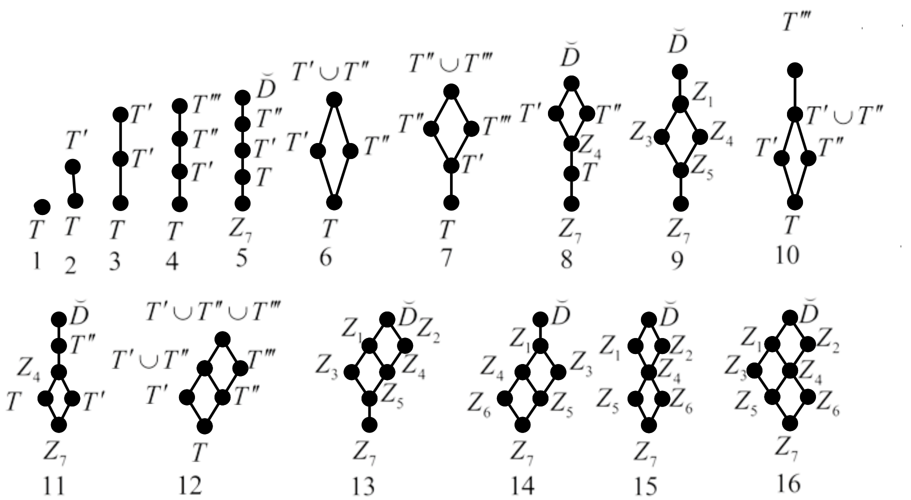

Figure.2

Denote by the symbol $\Sigma(Q_i)$ $(i = 1, 2, ..., 16)$ the set of all XI -subsemilattices of the semilattice D isomorphic to Q_i. Assume that $D' \in \Sigma Q_i \theta_{XI}$ and denote by the symbol $R(D')$ the set of all regular elements α of the semigroup $B_X(D)$, for which the semilattices $V(D,\alpha)$ and Q_i are mutually α isomorphic and $V(D,\alpha) = D'$.

Definition 1.3. Let the symbol $\Sigma'_{XI}(X, D)$ denote the set of all XI -subsemilattices of the semilattice D.

Let, further, $D, D' \in \Sigma'(X, D)$ and $\vartheta_{XI} \subseteq \Sigma'_{XI}(X, D) \times \Sigma'_{XI}(X, D)$. It is assumed that $D \vartheta_{XI} D'$ if and only if there exists some complete isomorphism φ between the semilattices D and D'. One can easily verify that the binary relation ϑ_{XI} is an equivalence relation on the set $\Sigma'_{XI}(X, D)$.

Let the symbol $Q_i \vartheta_{XI}$ denote the ϑ_{XI} -class of equivalence of the set $\Sigma'_{XI}(X, D)$, where every element is isomorphic to the X -semilattice Q_i and

$$R^*(Q_i) = \bigcup_{D' \in Q_i \vartheta_{XI}} R(D')$$

(see Definition 6.3.5 of [1] or Definition 6.3.5 of [2]).

Lemma 1.1. If X be a finite set and $|\Omega(Q)| = m_0$, then the following equalities are true:

a) $|R(Q_1)| = 1$;

b) $|R(Q_2)| = m_0 \cdot \left(2^{|T' \setminus T|} - 1\right) \cdot 2^{|X \setminus T'|}$;

c) $|R(Q_3)| = m_0 \cdot \left(2^{|T' \setminus T|} - 1\right) \cdot \left(3^{|T'' \setminus T'|} - 2^{|T'' \setminus T'|}\right) \cdot 3^{|X \setminus T''|}$;

d) $|R(Q_4)| = m_0 \cdot \left(2^{|T' \setminus T|} - 1\right) \cdot \left(3^{|T'' \setminus T'|} - 2^{|T'' \setminus T'|}\right) \cdot \left(4^{|T''' \setminus T''|} - 3^{|T''' \setminus T''|}\right) \cdot 4^{|X \setminus T'''|}$;

e) $|R(Q_5)| = m_0 \cdot \left(2^{|T \setminus Z_7|} - 1\right) \cdot \left(3^{|T' \setminus T|} - 2^{|T' \setminus T|}\right) \cdot \left(4^{|T'' \setminus T'|} - 3^{|T'' \setminus T'|}\right) \cdot \left(5^{|\breve{D} \setminus T''|} - 4^{|\breve{D} \setminus T''|}\right) \cdot 5^{|X \setminus \breve{D}|}$;

f) $|R(Q_6)| = 2 \cdot m_0 \cdot \left(2^{|T' \setminus T''|} - 1\right) \cdot \left(2^{|T'' \setminus T'|} - 1\right) \cdot 4^{|X \setminus (T' \cup T'')|}$;

g) $|R(Q_7)| = 2 \cdot m_0 \cdot \left(2^{|T' \setminus T|} - 1\right) \cdot 2^{|(T'' \cap T''') \setminus T'|} \cdot \left(3^{|T'' \setminus T'''|} - 2^{|T'' \setminus T'''|}\right) \cdot \left(3^{|T''' \setminus T''|} - 2^{|T''' \setminus T''|}\right) \cdot 5^{|X \setminus (T'' \cup T''')|}$;

h) $|R(Q_8)| = 2 \cdot m_0 \cdot \left(2^{|T \setminus Z_7|} - 1\right) \cdot \left(3^{|Z_4 \setminus T|} - 2^{|Z_4 \setminus T|}\right) \cdot 3^{|(Z_2 \cap Z_1) \setminus Z_4|} \cdot \left(4^{|Z_1 \setminus Z_2|} - 3^{|Z_1 \setminus Z_2|}\right) \cdot \left(4^{|Z_2 \setminus Z_1|} - 3^{|Z_2 \setminus Z_1|}\right) \cdot 6^{|X \setminus \breve{D}|}$;

i) $|R(Q_9)| = 2 \cdot \left(2^{|Z_5 \setminus Z_7|} - 1\right) \cdot 2^{|(Z_3 \cap Z_4) \setminus Z_5|} \cdot \left(3^{|Z_3 \setminus Z_4|} - 2^{|Z_3 \setminus Z_4|}\right) \cdot \left(3^{|Z_4 \setminus Z_3|} - 2^{|Z_4 \setminus Z_3|}\right) \cdot \left(6^{|\breve{D} \setminus (Z_3 \cup Z_4)|} - 5^{|\breve{D} \setminus (Z_3 \cup Z_4)|}\right) \cdot 6^{|X \setminus \breve{D}|}$;

j) $|R(Q_{10})| = 2 \cdot m_0 \cdot \left(2^{|T' \setminus T''|} - 1\right) \cdot \left(2^{|T'' \setminus T'|} - 1\right) \cdot \left(5^{|T''' \setminus (T' \cup T'')|} - 4^{|T''' \setminus (T' \cup T'')|}\right) \cdot 5^{|X \setminus T'''|}$;

k) $|R(Q_{11})| = 2 \cdot m_0 \cdot \left(2^{|T \setminus T'|} - 1\right) \cdot \left(2^{|T' \setminus T|} - 1\right) \cdot \left(5^{|T'' \setminus Z_4|} - 4^{|T'' \setminus Z_4|}\right) \cdot \left(6^{|\breve{D} \setminus T''|} - 5^{|\breve{D} \setminus T|''}\right) \cdot 6^{|X \setminus \breve{D}|}$;

l) $\left|R(Q_{12})\right| = m_0 \cdot \left(2^{|T'\setminus T''|}-1\right)\cdot\left(2^{|T''\setminus T'|}-1\right)\cdot\left(3^{|T'''\setminus(T'\cup T'')|}-2^{|T'''\setminus(T'\cup T'')|}\right)\cdot 6^{|X\setminus(T'\cup T''\cup T''')|}$;

m) $\left|R(Q_{13})\right| = \left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_3\cap Z_2)\setminus Z_5|}\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot\left(3^{|Z_4\setminus Z_2|}-2^{|Z_4\setminus Z_2|}\right)\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 7^{|X\setminus\breve{D}|}$;

n) $\left|R(Q_{14})\right| = \left(2^{|Z_5\setminus Z_4|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot\left(7^{|\breve{D}\setminus Z_1|}-6^{|\breve{D}\setminus Z_1|}\right)\cdot 7^{|X\setminus\breve{D}|}$;

o) $\left|R(Q_{15})\right| = 4\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot 4^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot\left(5^{|Z_2\setminus Z_1|}-4^{|Z_2\setminus Z_1|}\right)\cdot\left(5^{|Z_1\setminus Z_2|}-4^{|Z_1\setminus Z_2|}\right)\cdot 7^{|X\setminus\breve{D}|}$

p) $\left|R(Q_{16})\right| = \left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|Z_5\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot\left(5^{|Z_2\setminus Z_1|}-4^{|Z_2\setminus Z_1|}\right)\cdot 8^{|X\setminus\breve{D}|}$

Theorem 1.1 Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. Then a binary relation α of the semigroup $B_X(D)$ that has a quasinormal representation of the form to be given below is a regular element of this semigroup iff there exist a complete α – isomorphism φ of the semilattice $V(D,\alpha)$ on some subsemilattice D' of the semilattice D that satisfies at least one of the following conditions:

a) $\alpha = X\times T$, where $T\in D$;

b) $\alpha=(Y_T^\alpha\times T)\cup(Y_{T'}^\alpha\times T')$, where $T,T'\in D$, $T\subset T'$, $Y_T^\alpha,Y_{T'}^\alpha\notin\{\varnothing\}$, and satisfies the conditions: $Y_T^\alpha\supseteq\varphi(T)$, $Y_{T'}^\alpha\cap\varphi(T')\neq\varnothing$;

c) $\alpha=(Y_T^\alpha\times T)\cup(Y_{T'}^\alpha\times T')\cup(Y_{T''}^\alpha\times T'')$, where $T,T',T''\in D$, $T\subset T'\subset T''$, $Y_T^\alpha,Y_{T'}^\alpha,Y_{T''}^\alpha\notin\{\varnothing\}$, and satisfies the conditions: $Y_T^\alpha\supseteq\varphi(T)$, $Y_T^\alpha\cup Y_{T'}^\alpha\supseteq\varphi(T')$, $Y_{T'}^\alpha\cap\varphi(T')\neq\varnothing$, $Y_{T''}^\alpha\cap\varphi(T'')\neq\varnothing$;

d) $\alpha=(Y_T^\alpha\times T)\cup(Y_{T'}^\alpha\times T')\cup(Y_{T''}^\alpha\times T'')\cup(Y_{T'''}^\alpha\times T''')$, where $T,T',T'',T'''\in D$, $T\subset T'\subset T''\subset T'''$, $Y_T^\alpha,Y_{T'}^\alpha,Y_{T''}^\alpha,Y_{T'''}^\alpha\notin\{\varnothing\}$, and satisfies the conditions: $Y_T^\alpha\supseteq\varphi(T)$, $Y_T^\alpha\cup Y_{T'}^\alpha\supseteq\varphi(T')$, $Y_T^\alpha\cup Y_{T'}^\alpha\cup Y_{T''}^\alpha\supseteq\varphi(T'')$, $Y_{T'}^\alpha\cap(T')\neq\varnothing$, $Y_{T''}^\alpha\cap\varphi(T'')\neq\varnothing$, $Y_{T'''}^\alpha\cap\varphi(T''')\neq\varnothing$;

e) $\alpha=(Y_7^\alpha\times Z_7)\cup(Y_T^\alpha\times T)\cup(Y_{T'}^\alpha\times T')\cup(Y_{T''}^\alpha\times T'')\cup(Y_0^\alpha\times\breve{D})$, where $Z_7\subset T\subset T'\subset T''\subset\breve{D}$, $Y_7^\alpha,Y_T^\alpha,Y_{T'}^\alpha,Y_{T''}^\alpha,Y_0^\alpha\notin\{\varnothing\}$, and satisfies the conditions: $Y_7^\alpha\supseteq\varphi(Z_7)$, $Y_7^\alpha\cup Y_T^\alpha\supseteq\varphi(T)$, $Y_7^\alpha\cup Y_T^\alpha\cup Y_{T'}^\alpha\supseteq\varphi(T')$, $Y_7^\alpha\cup Y_T^\alpha\cup Y_{T'}^\alpha\cup Y_{T''}^\alpha\supseteq\varphi(T'')$, $Y_T^\alpha\cap\varphi(T)\neq\varnothing$, $Y_{T'}^\alpha\cap\varphi(T')\neq\varnothing$, $Y_{T''}^\alpha\cap\varphi(T'')\neq\varnothing$, $Y_0^\alpha\cap\breve{D}\neq\varnothing$;

f) $\alpha=(Y_T^\alpha\times T)\cup(Y_{T'}^\alpha\times T')\cup(Y_{T''}^\alpha\times T'')\cup(Y_{T'\cup T''}^\alpha\times(T'\cup T''))$, where $T,T',T''\in D$, $T\subset T'$, $T\subset T''$, $T'\setminus T''\neq\varnothing$, $T''\setminus T'\neq\varnothing$, $Y_T^\alpha,Y_{T'}^\alpha,Y_{T''}^\alpha\notin\{\varnothing\}$ and satisfies the conditions: $Y_T^\alpha\cup Y_{T'}^\alpha\supseteq\varphi(T')$, $Y_T^\alpha\cup Y_{T''}^\alpha\supseteq\varphi(T'')$, $Y_{T'}^\alpha\cap\varphi(T')\neq\varnothing$, $Y_{T''}^\alpha\cap\varphi(T'')\neq\varnothing$;

g) $\alpha=(Y_T^\alpha\times T)\cup(Y_{T'}^\alpha\times T')\cup(Y_{T''}^\alpha\times T'')\cup(Y_{T'''}^\alpha\times T''')\cup(Y_{T''\cup T'''}^\alpha\times(T''\cup T'''))$, where, $T\subset T'\subset T''$, $T\subset T'\subset T'''$, $T''\setminus T'''\neq\varnothing$, $T'''\setminus T''\neq\varnothing$ $Y_T^\alpha,Y_{T'}^\alpha,Y_{T''}^\alpha,Y_{T'''}^\alpha\notin\{\varnothing\}$ and satisfies the conditions: $Y_T^\alpha\supseteq\varphi(T)$ $Y_T^\alpha\cup Y_{T'}^\alpha\supseteq\varphi(T')$, $Y_T^\alpha\cup Y_{T'}^\alpha\cup Y_{T''}^\alpha\supseteq\varphi(T'')$, $Y_T^\alpha\cup Y_{T'}^\alpha\cup Y_{T'''}^\alpha\supseteq\varphi(T''')$ $Y_{T'}^\alpha\cap\varphi(T')\neq\varnothing$, $Y_{T''}^\alpha\cap\varphi(T'')\neq\varnothing$, $Y_{T'''}^\alpha\cap\varphi(T''')\neq\varnothing$.

h) $\alpha=(Y_7^\alpha\times Z_7)\cup(Y_T^\alpha\times T)\cup(Y_4^\alpha\times Z_4)\cup(Y_2^\alpha\times Z_2)\cup(Y_1^\alpha\times Z_1)\cup(Y_0^\alpha\times\breve{D})$, where $T\in\{Z_6,Z_5\}$, $Y_7^\alpha,Y_T^\alpha,Y_4^\alpha,Y_2^\alpha,Y_1^\alpha\notin\{\varnothing\}$ and satisfies the conditions: $Y_7^\alpha\supseteq\varphi(Z_7)$, $Y_7^\alpha\cup Y_T^\alpha\supseteq\varphi(T)$, $Y_7^\alpha\cup Y_T^\alpha\cup Y_4^\alpha\supseteq\varphi(Z_4)$, $Y_7^\alpha\cup Y_T^\alpha\cup Y_4^\alpha\cup Y_2^\alpha\supseteq\varphi(Z_2)$, $Y_7^\alpha\cup Y_T^\alpha\cup Y_4^\alpha\cup Y_1^\alpha\supseteq\varphi(Z_1)$, $Y_T^\alpha\cap\varphi(T)\neq\varnothing$, $Y_4^\alpha\cap\varphi(Z_4)\neq\varnothing$ $Y_2^\alpha\cap\varphi(Z_2)\neq\varnothing$, $Y_1^\alpha\cap\varphi(Z_1)\neq\varnothing$;

i) $\alpha=(Y_7^\alpha\times Z_7)\cup(Y_5^\alpha\times Z_5)\cup(Y_4^\alpha\times Z_4)\cup(Y_3^\alpha\times Z_3)\cup(Y_1^\alpha\times Z_1)\cup(Y_0^\alpha\times\breve{D})$, where $Z_7\subset Z_5\subset Z_3$, $Z_7\subset Z_5\subset Z_4$, $Z_3\setminus Z_4\neq\varnothing$, $Z_4\setminus Z_3\neq\varnothing$ $Y_7^\alpha,Y_5^\alpha,Y_4^\alpha,Y_3^\alpha,Y_1^\alpha,Y_0^\alpha\notin\{\varnothing\}$ and satisfies the conditions: $Y_7^\alpha\supseteq\varphi(Z_7)$, $Y_7^\alpha\cup Y_5^\alpha\supseteq\varphi(Z_5)$ $Y_7^\alpha\cup Y_5^\alpha\cup Y_3^\alpha\supseteq\varphi(Z_3)$ $Y_7^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\supseteq\varphi(Z_4)$, $Y_5^\alpha\cap\varphi(Z_5)\neq\varnothing$, $Y_3^\alpha\cap\varphi(Z_3)\neq\varnothing$, $Y_4^\alpha\cap\varphi(Z_4)\neq\varnothing$, $Y_0^\alpha\cap\varphi(\breve{D})\neq\varnothing$;

j) $\alpha=(Y_T^\alpha\times T)\cup(Y_{T'}^\alpha\times T')\cup(Y_{T''}^\alpha\times T'')\cup(Y_{T'\cup T''}^\alpha\times(T'\cup T''))\cup(Y_{T'''}^\alpha\times T''')$, where $T\subset T'$, $T\subset T''$, $T'\setminus T''\neq\varnothing$, $T''\setminus T'\neq\varnothing$, $T'\cup T''\subset T'''$ $Y_T^\alpha,Y_{T'}^\alpha,Y_{T''}^\alpha,Y_{T'''}^\alpha\notin\{\varnothing\}$ and satisfies the conditions: $Y_T^\alpha\cup Y_{T'}^\alpha\supseteq\varphi(T')$, $Y_T^\alpha\cup Y_{T''}^\alpha\supseteq\varphi(T'')$, $Y_{T'}^\alpha\cap\varphi(T')\neq\varnothing$, $Y_{T''}^\alpha\cap\varphi(T'')\neq\varnothing$, $Y_{T'''}^\alpha\cap\varphi(T''')\neq\varnothing$;

$k)$ $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_6^\alpha\times Z_6\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_T^\alpha\times T\right)\cup\left(Y_0^\alpha\times\breve{D}\right)$, where $T\in\{Z_2,Z_1\}$, $Y_7^\alpha,Y_6^\alpha,Y_5^\alpha,Y_T^\alpha,Y_0^\alpha\notin\{\varnothing\}$ and satisfies the conditions:, $Y_7^\alpha\cup Y_6^\alpha\supseteq\varphi(Z_6)$, $Y_7^\alpha\cup Y_5^\alpha\supseteq\varphi(Z_5)$ $Y_7^\alpha\cup Y_6^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\cup Y_T^\alpha\supseteq\varphi(T)$, $Y_6^\alpha\cap\varphi(Z_6)\neq\varnothing$, $Y_5^\alpha\cap\varphi(Z_5)\neq\varnothing$ $Y_T^\alpha\cap\varphi(T)\neq\varnothing$, $Y_0^\alpha\cap\varphi\left(\breve{D}\right)\neq\varnothing$;

$l)$ $\alpha=\left(Y_T^\alpha\times T\right)\cup\left(Y_{T'}^\alpha\times T'\right)\cup\left(Y_{T''}^\alpha\times T''\right)\cup\left(Y_{T'\cup T''}^\alpha\times(T'\cup T'')\right)\cup\left(Y_{T'''}^\alpha\times T'''\right)\cup\left(Y_{T'\cup T''\cup T'''}^\alpha\times(T'\cup T''\cup T''')\right)$, where $T\subset T'$, $T\subset T''$, $T'\setminus T''\neq\varnothing$, $T''\setminus T'\neq\varnothing$, $T''\subset T'''$, $(T'\cup T'')\setminus T'''\neq\varnothing$, $T'''\setminus(T'\cup T'')\neq\varnothing$, , $Y_T^\alpha,Y_{T'}^\alpha,Y_{T''}^\alpha,Y_{T'''}^\alpha,Y_{T'\cup T''\cup T'''}^\alpha\notin\{\varnothing\}$ and satisfies the conditions:, $Y_T^\alpha\cup Y_{T'}^\alpha\supseteq\varphi(T')$, $Y_T^\alpha\cup Y_{T''}^\alpha\supseteq\varphi(T'')$ $Y_T^\alpha\cup Y_{T''}^\alpha\cup Y_{T'''}^\alpha\supseteq\varphi(T''')$, $Y_{T'}^\alpha\cap\varphi(T')\neq\varnothing$, $Y_{T''}^\alpha\cap\varphi(T'')\neq\varnothing$ $Y_{T'''}^\alpha\cap\varphi(T''')\neq\varnothing$;

13) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_3^\alpha\times Z_3\right)\cup\left(Y_2^\alpha\times Z_2\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times\breve{D}\right)$, where $Z_5\subset Z_3$, $Z_5\subset Z_4$, $Z_3\setminus Z_4\neq\varnothing$, $Z_4\setminus Z_3\neq\varnothing$, $Z_4\subset Z_2$, $Z_1\setminus Z_2\neq\varnothing$, $Z_2\setminus Z_1\neq\varnothing$, , $Y_7^\alpha,Y_5^\alpha,Y_4^\alpha,Y_3^\alpha,Y_2^\alpha,Y_1^\alpha,Y_0^\alpha\notin\{\varnothing\}$ and satisfies the conditions: $Y_7^\alpha\supseteq\varphi(Z_7)$, $Y_7^\alpha\cup Y_5^\alpha\supseteq\varphi(Z_5)$ $Y_7^\alpha\cup Y_5^\alpha\cup Y_3^\alpha\supseteq\varphi(Z_3)$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\supseteq\varphi(Z_4)$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\cup Y_1^\alpha\supseteq\varphi(Z_1)$, $Y_5^\alpha\cap\varphi(Z_5)\neq\varnothing$, $Y_3^\alpha\cap\varphi(Z_3)\neq\varnothing$, $Y_4^\alpha\cap\varphi(Z_4)\neq\varnothing$ $Y_1^\alpha\cap\varphi(Z_1)\neq\varnothing$;

14) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_6^\alpha\times Z_6\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_3^\alpha\times Z_3\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times\breve{D}\right)$, where, $Z_6\subset Z_4$, $Y_7^\alpha,Y_6^\alpha,Y_5^\alpha,Y_4^\alpha,Y_3^\alpha,Y_1^\alpha,Y_0^\alpha\notin\{\varnothing\}$ and satisfies the conditions: $Y_7^\alpha\cup Y_5^\alpha\supseteq\varphi(Z_5)$, $Y_7^\alpha\cup Y_6^\alpha\supseteq\varphi(Z_6)$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_3^\alpha\supseteq\varphi(Z_3)$, $Y_5^\alpha\cap\varphi(Z_5)\neq\varnothing$, $Y_6^\alpha\cap\varphi(Z_6)\neq\varnothing$, $Y_3^\alpha\cap\varphi(Z_3)\neq\varnothing$ $Y_0^\alpha\cap\kappa\left(\breve{D}\right)\neq\varnothing$;

15) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_6^\alpha\times Z_6\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_2^\alpha\times Z_2\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times\breve{D}\right)$, where $Y_7^\alpha,Y_6^\alpha,Y_5^\alpha,Y_4^\alpha,Y_2^\alpha,Y_1^\alpha\notin\{\varnothing\}$ and satisfies the conditions: $Y_7^\alpha\cup Y_6^\alpha\supseteq\varphi(Z_6)$, $Y_7^\alpha\cup Y_5^\alpha\supseteq\varphi(Z_5)$, $Y_7^\alpha\cup Y_6^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\cup Y_2^\alpha\supseteq\varphi(Z_2)$ $Y_7^\alpha\cup Y_6^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\cup Y_1^\alpha\supseteq\varphi(Z_1)$ $Y_6^\alpha\cap\varphi(Z_6)\neq\varnothing$, $Y_5^\alpha\cap\varphi(Z_5)\neq\varnothing$, $Y_2^\alpha\cap\varphi(Z_2)\neq\varnothing$ $Y_1^\alpha\cap\varphi(Z_1)\neq\varnothing$;

16) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_6^\alpha\times Z_6\right)\cup\left(Y_5^\alpha\times Z\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_3^\alpha\times Z_3\right)\cup\left(Y_2^\alpha\times Z_2\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times\breve{D}\right)$, where, $Y_7^\alpha,Y_6^\alpha,Y_5^\alpha,Y_4^\alpha,Y_3^\alpha,Y_2^\alpha,Y_1^\alpha,Y_0^\alpha\notin\{\varnothing\}$ and satisfies the conditions: $Y_7^\alpha\supseteq\varphi(Z_7)$ $Y_7^\alpha\cup Y_5^\alpha\supseteq\varphi(Z_5)$, $Y_7^\alpha\cup Y_6^\alpha\supseteq\varphi(Z_6)$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_3^\alpha\supseteq\varphi(Z_3)$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_6^\alpha\cup Y_4^\alpha\cup Y_2^\alpha\supseteq\varphi(Z_2)$, $Y_5^\alpha\cap\varphi(Z_5)\neq\varnothing$, $Y_6^\alpha\cap\varphi(Z_6)\neq\varnothing$, $Y_3^\alpha\cap\varphi(Z_3)\neq\varnothing$, $Y_2^\alpha\cap\varphi(Z_2)\neq\varnothing$;

a′) **Lemma 1.2.** Let $D=\left\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\right\}\in\Sigma_3(X,8)$ and $Z_8\neq\varnothing$. Then $\left|R^*(Q_1)\right|=8$.

b′) Now let binary relation α of the semigroup $B_X(D)$ satisfying the condition $b)$ of the Theorem 1.1 In this case we have $Q_2=\{T,T'\}$, where $T,T'\in D$ and $T\subset T'$. By definition of the semilattice D follows that

$$\begin{aligned}Q_2\vartheta_{XI}=\Big\{&\{Z_7,\breve{D}\},\{Z_6,\breve{D}\},\{Z_5,\breve{D}\},\{Z_4,\breve{D}\},\{Z_3,\breve{D}\},\{Z_2,\breve{D}\},\{Z_1,\breve{D}\},\{Z_7,Z_6\},\{Z_7,Z_5\},\{Z_7,Z_4\},\{Z_7,Z_3\},\{Z_7,Z_2\},\\&\{Z_7,Z_1\},\{Z_6,Z_4\},\{Z_6,Z_2\},\{Z_6,Z_1\},\{Z_5,Z_4\},\{Z_5,Z_3\},\{Z_5,Z_2\},\{Z_5,Z_1\},\{Z_4,Z_2\},\{Z_4,Z_1\},\{Z_3,Z_1\}\Big\}\end{aligned}$$

It is easy to see $\left|\Phi(Q_2,Q_2)\right|=1$ and $\left|\Omega(Q_2)\right|=23$. Assume that $D_1'=\left\{Z_7,\breve{D}\right\}$

Lemma 1.3. Let X be a finite set, $D=\left\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\right\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. Then $\left|R^*(Q_2)\right|=23\cdot\left(2^{\left|\breve{D}\setminus Z_7\right|}-1\right)\cdot2^{\left|X\setminus\breve{D}\right|}$.

$c')$ Now let binary relation α of the semigroup $B_X(D)$ satisfying the condition c) of the Theorem 1.1 In this case we have $Q_2=\{T,T',T''\}$, where $T,T',T''\in D$ and $T\subset T'\subset T''$. By definition of the semilattice D follows that

$$\begin{aligned}Q_3\vartheta_{XI}=\Big\{&\{Z_7,Z_1,\breve{D}\},\{Z_7,Z_2,\breve{D}\},\{Z_7,Z_3,\breve{D}\},\{Z_7,Z_4,D\},\{Z_7,Z_5,\breve{D}\},\{Z_7,Z_6,\breve{D}\},\{Z_7,Z_6,Z_4\},\{Z_7,Z_6,Z_2\},\\&\{Z_7,Z_6,Z_1\},\{Z_7,Z_5,Z_4\},\{Z_7,Z_5,Z_3\},\{Z_7,Z_5,Z_2\},\{Z_7,Z_5,Z_1\},\{Z_7,Z_4,Z_2\},\{Z_7,Z_4,Z_1\},\{Z_7,Z_3,Z_1\},\\&\{Z_6,Z_4,Z_2\},\{Z_6,Z_4,Z_1\},\{Z_6,Z_4,\breve{D}\},\{Z_6,Z_2,\breve{D}\},\{Z_6,Z_1,\breve{D}\},\{Z_5,Z_4,Z_2\},\{Z_5,Z_4,Z_1\},\{Z_5,Z_4,\breve{D}\},\\&\{Z_5,Z_3,Z_1\},\{Z_5,Z_3,\breve{D}\},\{Z_5,Z_2,\breve{D}\},\{Z_5,Z_1,\breve{D}\},\{Z_4,Z_2,\breve{D}\},\{Z_4,Z_1,\breve{D}\},\{Z_3,Z_1,\breve{D}\}\Big\}\end{aligned}$$

It is easy to see $\left|\Phi(Q_3,Q_3)\right|=1$ and $\left|\Omega(Q_3)\right|=31$. Asumme that

$D_1'=\left\{Z_7,Z_1,\breve{D}\right\}$, $D_2'=\left\{Z_7,Z_2,\breve{D}\right\}$, $D_3'=\left\{Z_7,Z_3,\breve{D}\right\}$, $D_4'=\left\{Z_7,Z_4,\breve{D}\right\}$, $D_5'=\left\{Z_7,Z_5,\breve{D}\right\}$, $D_6'=\left\{Z_7,Z_6,\breve{D}\right\}$

Lemma 1.4 Let X be a finite set, $D=\left\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\right\}\in\Sigma_1(X,8)$ and $Z_7\neq\varnothing$. Then

$$\left|R^*(Q_3)\right|=\sum_{i=1}^{6}\left|R(D_i')\right|+\left|R(D_1')\cap R(D_6')\right|+\left|R(D_1')\cap R(D_5')\right|-\left|R(D_1')\cap R(D_3')\right|$$
$$-\left|R(D_1')\cap R(D_4')\right|-\left|R(D_2')\cap R(D_4')\right|-\left|R(D_3')\cap R(D_5')\right|-\left|R(D_4')\cap R(D_5')\right|$$

Lemma 1.5 Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. then

$$\begin{aligned}\left|R^*(Q_3)\right|&=31\cdot\left(2^{|Z_1\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot3^{|X\setminus\breve{D}|}+31\cdot\left(2^{|Z_2\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_2|}-2^{|\breve{D}\setminus Z_2|}\right)\cdot3^{|X\setminus\breve{D}|}+\\&+31\cdot\left(2^{|Z_3\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_3|}-2^{|\breve{D}\setminus Z_3|}\right)\cdot3^{|X\setminus\breve{D}|}+31\cdot\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_4|}-2^{|\breve{D}\setminus Z_4|}\right)\cdot3^{|X\setminus\breve{D}|}+\\&+31\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_5|}-2^{|\breve{D}\setminus Z_5|}\right)\cdot3^{|X\setminus\breve{D}|}+31\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_6|}-2^{|\breve{D}\setminus Z_6|}\right)\cdot3^{|X\setminus\breve{D}|}+\\&+31\cdot2^{|Z_1\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot3^{|X\setminus\breve{D}|}-31\cdot2^{|Z_1\setminus Z_3|}\cdot\left(2^{|Z_3\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot3^{|X\setminus D|}\\&-31\cdot2^{|Z_1\setminus Z_4|}\cdot\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot3^{|X\setminus D|}-31\cdot2^{|Z_2\setminus Z_4|}\cdot\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_2|}-2^{|\breve{D}\setminus Z_2|}\right)\cdot3^{|X\setminus D|}\\&-31\cdot2^{|Z_3\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_3|}-2^{|\breve{D}\setminus Z_3|}\right)\cdot3^{|X\setminus D|}-31\cdot2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_4|}-2^{|\breve{D}\setminus Z_4|}\right)\cdot3^{|X\setminus D|}\\&-31\cdot2^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_4|}-2^{|\breve{D}\setminus Z_4|}\right)\cdot3^{|X\setminus D|}\end{aligned}$$

d′) Now let binary relation α of the semigroup $B_X(D)$ satisfying the condition $d)$ of the Theorem 1.1 In this case we have $Q_4=\{T,T',T'',T'''\}$ where $T,T',T'',T'''\in D$ and $T\subset T'\subset T''\subset T'''$. By definition of the semilattice D follows that

$$\begin{aligned}Q_4\vartheta_{XI}=\{&\{Z_7,Z_6,Z_4,\breve{D}\},\{Z_7,Z_6,Z_2,D\},\{Z_7,Z_6,Z_1,\breve{D}\},\{Z_7,Z_5,Z_4,\breve{D}\},\{Z_7,Z_5,Z_3,\breve{D}\},\{Z_7,Z_5,Z_2,\breve{D}\},\\&\{Z_7,Z_5,Z_1,\breve{D}\},\{Z_7,Z_4,Z_2,D\},\{Z_7,Z_4,Z_1,\breve{D}\},\{Z_7,Z_3,Z_1,\breve{D}\},\{Z_7,Z_5,Z_4,Z_2\},\{Z_7,Z_5,Z_4,Z_1\},\{Z_7,Z_5,Z_3,Z_1\},\\&\{Z_7,Z_6,Z_4,Z_2\},\{Z_7,Z_6,Z_4,Z_1\},\{Z_6,Z_4,Z_2,\breve{D}\},\{Z_6,Z_4,Z_1,\breve{D}\},\{Z_5,Z_4,Z_2,\breve{D}\},\{Z_5,Z_4,Z_1,\breve{D}\},\{Z_5,Z_3,Z_1,\breve{D}\}\}\end{aligned}$$

It is easy to see $\left|\Phi(Q_4,Q_4)\right|=1$ and $\left|\Omega(Q_4)\right|=20$. assume

$D_1'=\{Z_7,Z_6,Z_4,\breve{D}\}$, $D_2'=\{Z_7,Z_6,Z_2,\breve{D}\}$, $D_3'=\{Z_7,Z_6,Z_1,\breve{D}\}$, $D_4'=\{Z_7,Z_5,Z_4,\breve{D}\}$, $D_5'=\{Z_7,Z_5,Z_3,\breve{D}\}$,
$D_6'=\{Z_7,Z_5,Z_2,\breve{D}\}$, $D_7'=\{Z_7,Z_5,Z_1,\breve{D}\}$, $D_8'=\{Z_7,Z_4,Z_2,\breve{D}\}$, $D_9'=\{Z_7,Z_4,Z_1,\breve{D}\}$, $D_{10}'=\{Z_7,Z_3,Z_1,\breve{D}\}$,

Lemma 1.6 Let X be a finite set, $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. Then

$$\left|R^*(Q_4)\right|=\sum_{i=1}^{10}\left|R(D_i')\right|-\left|R(D_1')\cap R(D_2')\right|-\left|R(D_1')\cap R(D_3')\right|-\left|R(D_2')\cap R(D_8')\right|$$
$$-\left|R(D_3')\cap R(D_9')\right|-\left|R(D_4')\cap R(D_6')\right|-\left|R(D_4')\cap R(D_7')\right|-\left|R(D_5')\cap R(D_7')\right|-$$
$$-\left|R(D_6')\cap R(D_8')\right|-\left|R(D_7')\cap R(D_9')\right|-\left|R(D_7')\cap R(D_{10}')\right|$$

Lemma 1.7. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. Then

$$\begin{aligned}\left|R^*(Q_4)\right|&=20\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_4|}-3^{|\breve{D}\setminus Z_4|}\right)\cdot4^{|X\setminus\breve{D}|}+\\&+20\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_6|}-2^{|Z_2\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot4^{|X\setminus\breve{D}|}+\\&+20\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_6|}-2^{\|Z_1\setminus Z_6\|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot4^{|X\setminus\breve{D}|}+\\&+20\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_4|}-3^{|\breve{D}\setminus Z_4|}\right)\cdot4^{|X\setminus\breve{D}|}+\\&+20\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_3|}-3^{|\breve{D}\setminus Z_3|}\right)\cdot4^{|X\setminus\breve{D}|}+\\&+20\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_5|}-2^{|Z_2\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot4^{|X\setminus\breve{D}|}+\\&+20\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_5|}-2^{|Z_1\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot4^{|X\setminus\breve{D}|}+\\&+20\cdot\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot4^{|X\setminus\breve{D}|}+\\&+20\cdot\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot4^{|X\setminus\breve{D}|}+\\&+20\cdot\left(2^{|Z_3\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot4^{|X\setminus\breve{D}|}+\end{aligned}$$

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$$
\begin{aligned}
&-20\cdot 2^{|Z_6\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 3^{|Z_2\setminus Z_4|}\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}\\
&-20\cdot 2^{|Z_6\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_4|}\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}\\
&-20\cdot 2^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 3^{|Z_2\setminus Z_2|}\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}\\
&-20\cdot 2^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_1|}\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}\\
&-20\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_2\setminus Z_4|}\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}\\
&-20\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_4|}\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}\\
&-20\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_3|}\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}\\
&-20\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_2\setminus Z_2|}\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}\\
&-20\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_1|}\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}\\
&-20\cdot 2^{|Z_3\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_1|}\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}
\end{aligned}
$$

e′) Now let binary relation α of the semigroup $B_X(D)$ satisfying the condition $e)$ of the Theorem 1.1 .In this case we have $Q_5=\{Z_7,T,T',T'',\breve{D}\}$, where $T,T',T''\in D$ and $Z_7\subset T\subset T'\subset T''\subset\breve{D}$. By definition of the semilattice D follows that $Q_5\vartheta_{XI}=\{\{Z_7,Z_6,Z_4,Z_2,\breve{D}\},\{Z_7,Z_6,Z_4,Z_1,\breve{D}\},\{Z_7,Z_5,Z_4,Z_2,\breve{D}\},\{Z_7,Z_5,Z_4,Z_1,\breve{D}\},\{Z_7,Z_5,Z_3,Z_1,\breve{D}\}\}$.

It is easy to see $|\Phi(Q_5,Q_5)|=1$ and $|\Omega(Q_5)|=5$ .asuume that

$$D_1'=\{Z_7,Z_6,Z_4,Z_2,\breve{D}\},\ D_2'=\{Z_7,Z_6,Z_4,Z_1,\breve{D}\},\ D_3'=\{Z_7,Z_5,Z_4,Z_2,\breve{D}\},$$
$$D_4'=\{Z_7,Z_5,Z_4,Z_1,\breve{D}\},\ D_5'=\{Z_7,Z_5,Z_3,Z_1,\breve{D}\}.$$

Lemma 1.8. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_8\neq\varnothing$. Then

$$
\begin{aligned}
|I^*(Q_5)|=&\,5\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_2|}-4^{|\breve{D}\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&+5\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&+5\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_2|}-4^{|\breve{D}\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&+5\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&+5\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_3|}-3^{|Z_1\setminus Z_3|}\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}
\end{aligned}
$$

f′) Now let binary relation α of the semigroup $B_X(D)$ satisfying the condition $f)$ of the Theorem 1.1 In this case we have $Q_6=\{T,T',T'',T'\cup T''\}$, where $T,T',T''\in D$ and $T\subset T'$, $T\subset T''$, $T'\setminus T''\neq\varnothing$, $T''\setminus T'\neq\varnothing$. By definition of the semilattice D follows that

$$
\begin{aligned}
Q_6\vartheta_{XI}=&\{\{Z_7,Z_2,Z_1,\breve{D}\},\{Z_7,Z_6,Z_5,Z_4\},\{Z_7,Z_6,Z_3,Z_1\},\{Z_7,Z_4,Z_3,Z_1\},\{Z_7,Z_3,Z_2,\breve{D}\},\\
&\{Z_6,Z_2,Z_1,\breve{D}\},\{Z_5,Z_2,Z_1,\breve{D}\},\{Z_4,Z_2,Z_1,\breve{D}\},\{Z_5,Z_4,Z_3,Z_1\},\{Z_5,Z_3,Z_2,\breve{D}\}\}
\end{aligned}
$$

It is easy to see $|\Phi(Q_6,Q_6)|=2$ and $|\Omega(Q_6)|=10$. Assume that

$$D_1'=\{Z_7,Z_2,Z_1,\breve{D}\},\ D_2'=\{Z_7,Z_1,Z_2,\breve{D}\},\ D_3'=\{Z_7,Z_6,Z_5,Z_4\},\ D_4'=\{Z_7,Z_5,Z_6,Z_4\},D_5'=\{Z_7,Z_6,Z_3,Z_1\},$$
$$D_6'=\{Z_7,Z_3,Z_6,Z_1\},\ D_7'=\{Z_7,Z_4,Z_3,Z_1\},\ D_8'=\{Z_7,Z_3,Z_4,Z_1\},D_9'=\{Z_7,Z_3,Z_2,\breve{D}\},D_{10}'=\{Z_7,Z_2,Z_3,\breve{D}\},$$

Lemma 1.9. Let X be a finite set, $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. If by $R^*(Q_6)$ denoted all regular elements of the semigroup $B_X(D)$ satisfying the condition $f)$ of the Theorem 1, then

$$
\begin{aligned}
|R^*(Q_6)|=&\sum_{i=1}^{10}|R(D_i')|-|R(D_1')\cap R(D_{10}')|-|R(D_2')\cap R(D_9')|-|R(D_3')\cap R(D_5')|-|R(D_4')\cap R(D_6')|\\
&-|R(D_5')\cap R(D_7')|-|R(D_6')\cap R(D_8')|-|R(D_7')\cap R(D_{10}')|-|R(D_8')\cap R(D_9')|
\end{aligned}
$$

Lemma 1.10. Let $D=\{Z_8,Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_1(X,9)$ and $Z_8\neq\varnothing$. Then

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$$\begin{aligned}\left|R^*(Q_6)\right| &= 20\cdot\left(2^{|Z_2\setminus Z_1|}-1\right)\cdot\left(2^{|Z_1\setminus Z_2|}-1\right)\cdot 4^{|X\setminus\breve{D}|}+20\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 4^{|X\setminus Z_4|}+\\ &+20\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_6|}-1\right)\cdot 4^{|X\setminus Z_1|}+20\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot 4^{|X\setminus Z_1|}+\\ &+20\cdot\left(2^{|Z_2\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot 4^{|X\setminus\breve{D}|}-20\cdot\left(2^{|Z_7\setminus Z_3|}-1\right)\cdot 2^{|Z_3\setminus Z_4|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 4^{|X\setminus Z_1|}\\ &-10\cdot 2^{|Z_2\setminus\breve{D}|}\cdot\left(2^{|Z_2\setminus Z_1|}-1\right)\cdot 2^{|Z_1\setminus\breve{D}|}\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot 4^{|X\setminus\breve{D}|}-10\cdot 2^{|Z_1\setminus\breve{D}|}\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot 2^{|Z_2\setminus\breve{D}|}\cdot\left(2^{|Z_2\setminus Z_1|}-1\right)\cdot 4^{|X\setminus\breve{D}|}-\\ &-10\cdot 2^{|Z_6\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|Z_3\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot 4^{|X\setminus Z_1|}-10\cdot 2^{|Z_3\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot 2^{|Z_6\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 4^{|X\setminus Z_1|}-\\ &-10\cdot 2^{|Z_4\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|Z_3\setminus Z_1|}\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot 4^{|X\setminus Z_1|}-10\cdot 2^{|Z_3\setminus Z_1|}\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot 2^{|Z_4\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 4^{|X\setminus Z_1|}-\\ &-10\cdot 2^{|Z_2\setminus Z_1|}\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot 2^{|Z_3\setminus\breve{D}|}\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot 4^{|X\setminus\breve{D}|}-10\cdot 2^{|Z_3\setminus\breve{D}|}\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot 2^{|Z_2\setminus Z_1|}\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot 4^{|X\setminus\breve{D}|}\end{aligned}$$

g′) Now let binary relation α of the semigroup $B_X(D)$ satisfying the condition $g)$ of the Theorem 1.1 In this case we have $\{T,T',T'',T''',T''\cup T'''\}$, where $T,T',T'',T'''\in D$, $T\subset T'\subset T''$, $T\subset T'\subset T'''$, $T''\setminus T'''\neq\varnothing$ and $T'''\setminus T''\neq\varnothing$. By definition of the semilattice D follows that

$$\begin{aligned}Q_7\vartheta_{XI}=\big\{&\{Z_7,Z_4,Z_2,Z_1,\breve{D}\},\{Z_7,Z_6,Z_2,Z_1,\breve{D}\},\{Z_7,Z_5,Z_2,Z_1,\breve{D}\},\{Z_7,Z_5,Z_3,Z_2,\breve{D}\},\\ &\{Z_7,Z_5,Z_4,Z_3,Z_1\},\{Z_6,Z_4,Z_2,Z_1,\breve{D}\},\{Z_5,Z_4,Z_2,Z_1,\breve{D}\}\big\}\end{aligned}$$

It is easy to see $|\Phi(Q_7,Q_7)|=2$ and $|\Omega(Q_7)|=7$. Assume that

$$\begin{aligned}&D_1'=\{Z_7,Z_4,Z_2,Z_1,\breve{D}\},\ D_2'=\{Z_7,Z_4,Z_1,Z_2,\breve{D}\},\ D_3'=\{Z_7,Z_6,Z_2,Z_1,\breve{D}\},\ D_4'=\{Z_7,Z_6,Z_1,Z_2,\breve{D}\},\\ &D_5'=\{Z_7,Z_5,Z_2,Z_1,\breve{D}\},\ D_6'=\{Z_7,Z_5,Z_1,Z_2,\breve{D}\},\ D_7'=\{Z_7,Z_5,Z_3,Z_2,\breve{D}\},\ D_8'=\{Z_7,Z_5,Z_2,Z_3,\breve{D}\},\\ &D_9'=\{Z_7,Z_5,Z_4,Z_3,Z_1\},\ D_{10}'=\{Z_7,Z_5,Z_3,Z_4,Z_1\}\end{aligned}$$

Lemma 1.11. Let X be a finite set, $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_8\neq\varnothing$. If by $R^*(Q_7)$ denoted all regular elements of the semigroup $B_X(D)$ satisfying the condition $g)$ of the Theorem 1, then

$$\begin{aligned}\left|R^*(Q_7)\right|&=\sum_{i=1}^{10}\left|R(D_i')\right|-\left|R(D_1')\cap R(D_3')\right|-\left|R(D_1')\cap R(D_5')\right|-\left|R(D_2')\cap R(D_4')\right|-\left|R(D_2')\cap R(D_6')\right|\\ &-\left|R(D_2')\cap R(D_7')\right|-\left|R(D_5')\cap R(D_8')\right|-\left|R(D_7')\cap R(D_{10}')\right|-\left|R(D_8')\cap R(D_9')\right|\end{aligned}$$

Lemma 1.12. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. Then

$$\begin{aligned}\left|R^*(Q_7)\right|&=14\cdot\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\ &+14\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_6|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\ &+14\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_5|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\ &+14\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_2\cap Z_3)\setminus Z_5|}\cdot\left(3^{|Z_2\setminus Z_3|}-2^{|Z_2\setminus Z_3|}\right)\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\ &+14\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_4\cap Z_3)\setminus Z_5|}\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\ &-7\cdot 2^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}\\ &-7\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}\\ &-7\cdot 2^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}\\ &-7\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}\\ &-7\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}\\ &-7\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_5|}\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}\\ &-7\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_3\cap Z_2)\setminus Z_5|}\cdot 3^{|Z_3\setminus\breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 3^{|Z_2\setminus Z_1|}\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot 5^{|X\setminus\breve{D}|}\\ &-7\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_3\cap Z_2)\setminus Z_5|}\cdot 3^{|Z_2\setminus Z_1|}\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot 3^{|Z_3\setminus\breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}\end{aligned}$$

h′) Now let binary relation α of the semigroup $B_X(D)$ satisfying the condition $h)$ of the Theorem 1 In this case we have $Q_8=\{Z_7,T,Z_4,Z_2,Z_1,\breve{D}\}$, where $T\in\{Z_6,Z_5\}$. By definition of the semilattice D follows that $Q_8\vartheta_{XI}=\{\{Z_7,Z_6,Z_4,Z_2,Z_1,\breve{D}\},\{Z_7,Z_5,Z_4,Z_2,Z_1,\breve{D}\}\}$.It is easy to see $|\Phi(Q_8,Q_8)|=2$ and $|\Omega(Q_8)|=2$. If

$$D'_1=\{Z_7,Z_6,Z_4,Z_2,Z_1,\breve{D}\},\ D'_2=\{Z_7,Z_6,Z_4,Z_1,Z_2,\breve{D}\},$$
$$D'_3=\{Z_7,Z_5,Z_4,Z_2,Z_1,\breve{D}\},\ D'_4=\{Z_7,Z_5,Z_4,Z_1,Z_2,\breve{D}\}.$$

Lemma 1.13. Let X be a finite set, $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. If by $R^*(Q_8)$ denoted all regular elements of the semigroup $B_X(D)$ satisfying the condition $h)$ of the Theorem 1, then

$$|R^*(Q_8)|=|R(D'_1)|+|R(D'_2)|+|R(D'_3)|+|R(D'_4)|+$$

Lemma 1.14. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. Then

$$|R^*(Q_8)|=4\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot3^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\left(4^{|Z_2\setminus Z_1|}-3^{(Z_2\setminus Z_1)}\right)\cdot\left(4^{|Z_1\setminus Z_2|}-3^{(Z_1\setminus Z_2)}\right)\cdot6^{|X\setminus\breve{D}|}+$$
$$+4\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot3^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\left(4^{|Z_2\setminus Z_1|}-3^{(Z_2\setminus Z_1)}\right)\cdot\left(4^{|Z_1\setminus Z_2|}-3^{(Z_1\setminus Z_2)}\right)\cdot6^{|X\setminus\breve{D}|}+$$

i′) Let binary relation α of the semigroup $B_X(D)$ satisfying the condition $r)$ of the Theorem 1. In this case we have $Q_9=\{Z_7,Z_5,Z_4,Z_3,Z_1,\breve{D}\}$. By definition of the semilattice D follows that $Q_9\theta_{XI}=\{Z_7,Z_5,Z_4,Z_3,Z_1,\breve{D}\}$. It is easy to see $|\Phi(Q_9,Q_9)|=1$ and $|\Omega(Q_9)|=2$. If, $D'_1=\{Z_7,Z_5,Z_4,Z_3,Z_1,\breve{D}\}$ then $R^*(Q_9)=R(D'_1)$, $|R^*(Q_9)|=|R(D'_1)|$ and

$$|R^*(Q_9)|=\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot2^{|(Z_3\cap Z_4)\setminus Z_5|}\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot\left(6^{|\breve{D}\setminus Z_1|}-5^{|\breve{D}\setminus Z_1|}\right)\cdot6^{|X\setminus\breve{D}|}.$$

j′) Now let binary relation α of the semigroup $B_X(D)$ satisfying the condition $j)$ of the Theorem 1. In this case we have $Q_{10}=\{T,T',T'',T'\cup T'',T'''\}$, where $T\subset T'$, $T\subset T''$, $T'\setminus T''\neq\varnothing$, $T''\setminus T'\neq\varnothing$, $T'\cup T''\subset T'''$. By definition of the semilattice D follows that

$$Q_{10}\vartheta_{XI}=\left\{\{Z_7,Z_6,Z_5,Z_4,\breve{D}\},\{Z_7,Z_6,Z_3,Z_1,\breve{D}\},\{Z_7,Z_4,Z_3,Z_1,\breve{D}\},\{Z_7,Z_6,Z_5,Z_4,Z_2\},\right.$$
$$\left.\{Z_7,Z_6,Z_5,Z_4,Z_1\},\{Z_5,Z_4,Z_3,Z_1,\breve{D}\}\right\}.$$

It is easy to see $|\Phi(Q_{10},Q_{10})|=2$ and $|\Omega(Q_{10})|=6$. If

$$D'_1=\{Z_7,Z_6,Z_5,Z_4,\breve{D}\},\ D'_2=\{Z_7,Z_5,Z_6,Z_4,\breve{D}\},\ D'_3=\{Z_7,Z_6,Z_3,Z_1,\breve{D}\},\ D'_4=\{Z_7,Z_3,Z_6,Z_1,\breve{D}\},$$
$$D'_5=\{Z_7,Z_4,Z_3,Z_1,\breve{D}\},\ D'_6=\{Z_7,Z_3,Z_4,Z_1,\breve{D}\}$$

Lemma 1.15. Let X be a finite set, $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. If by $R^*(Q_{10})$ denoted all regular elements of the semigroup $B_X(D)$ satisfying the condition $j)$ of the Theorem 1, then

$$|R^*(Q_{10})|=\sum_{i=1}^{6}|R(D'_i)|-|R(D'_1)\cap R(D'_3)|-|R(D'_2)\cap R(D'_4)|-|R(D'_3)\cap R(D'_5)|-|R(D'_4)\cap R(D'_6)|$$

Lemma 1.16 Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. Then

$$R^*(Q_{10})=12\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_4|}-4^{|\breve{D}\setminus Z_4|}\right)\cdot5^{|X\setminus\breve{D}|}+12\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_6|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot5^{|X\setminus\breve{D}|}+$$
$$+12\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot5^{|X\setminus\breve{D}|}-$$
$$-6\cdot2^{|Z_6\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot2^{|Z_3\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot5^{|X\setminus\breve{D}|}-$$
$$-6\cdot2^{|Z_3\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot2^{|Z_6\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot5^{|X\setminus\breve{D}|}-$$
$$-6\cdot2^{|Z_4\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot2^{|Z_3\setminus Z_1|}\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot5^{|X\setminus\breve{D}|}-$$
$$-6\cdot2^{|Z_3\setminus Z_1|}\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot2^{|Z_4\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot5^{|X\setminus\breve{D}|}$$

k′) Now let binary relation α of the semigroup $B_X(D)$ satisfying the condition $k)$ of the Theorem 1. In this case we have $\{Z_7,T,T',Z_4,T'',\breve{D}\}$, where $T'''\in\{Z_2,Z_1\}$. By definition of the semilattice D follows that $Q_{11}\vartheta_{XI}=\left\{\{Z_7,Z_6,Z_5,Z_4,Z_2,\breve{D}\},\{Z_7,Z_6,Z_5,Z_4,Z_1,\breve{D}\}\right\}$. It is easy to see $|\Phi(Q_{11},Q_{11})|=2$ and $|\Omega(Q_{11})|=2$. If

$$D'_1=\{Z_7,Z_6,Z_5,Z_4,Z_2,\breve{D}\},D'_2=\{Z_7,Z_5,Z_6,Z_4,Z_2,\breve{D}\},D'_3=\{Z_7,Z_6,Z_5,Z_4,Z_1,\breve{D}\},D'_4=\{Z_7,Z_5,Z_6,Z_4,Z_1,\breve{D}\}$$

Lemma 1.17. Let X be a finite set, $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. then

$$\left|R^*(Q_{11})\right| = 4\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(5^{|Z_2\setminus Z_4|}-4^{|Z_2\setminus Z_4|}\right)\cdot\left(6^{|\breve{D}\setminus Z_2|}-5^{|\breve{D}\setminus Z_2|}\right)\cdot 6^{|X\setminus\breve{D}|}$$
$$+4\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(5^{|Z_1\setminus Z_4|}-4^{|Z_1\setminus Z_4|}\right)\cdot\left(6^{|\breve{D}\setminus Z_1|}-5^{|\breve{D}\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}$$

$\mathrm{l}')$ Now let binary relation α of the semigroup $B_X(D)$ satisfying the condition $l)$ of the Theorem 1.1. In this case we have $\{T,T',T'',T'\cup T'',T''',T'\cup T''\cup T'''\}$, where $T\subset T'$, $T\subset T''$, $T'\setminus T''\neq\varnothing$, $T''\setminus T'\neq\varnothing$, $T''\subset Z$, $(T'\cup T'')\setminus T'''\neq\varnothing$, $T'''\setminus(T'\cup T'')\neq\varnothing$. By definition of the semilattice D follows that $Q_{12}\vartheta_{XI}=\{\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_1\},\{Z_7,Z_6,Z_3,Z_2,Z_1,\breve{D}\},\{Z_7,Z_4,Z_3,Z_2,Z_1,\breve{D}\},\{Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}.\}$

It is easy to see $\left|\Phi(Q_{12},Q_{12})\right|=1$ and $\left|\Omega(Q_{12})\right|=4$

$D_1'=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_1\}$, $D_2'=\{Z_7,Z_6,Z_3,Z_2,Z_1,\breve{D}\}$, $D_3'=\{Z_7,Z_4,Z_3,Z_2,Z_1,\breve{D}\}$,

Lemma 1.18. Let X be a finite set, $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. If by $R^*(Q_{12})$ denoted all regular elements of the semigroup $B_X(D)$ satisfying the condition $l)$ of the Theorem 1.1 then

$$R^*(Q_{12})=\left|R(D_1')\right|+\left|R(D_2')\right|+\left|R(D_3')\right|-\left|R(D_2')\cap R(D_3')\right|$$

Lemma 1.19. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. Then

$$\left|R^*(Q_{12})\right|=4\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot 6^{|X\setminus Z_1|}+$$
$$+4\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_6\setminus Z_2|}-1\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}+$$
$$+4\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}-$$
$$-4\cdot 2^{|Z_4\setminus(Z_6\cup Z_3)|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}$$

m) Let binary relation α of the semigroup $B_X(D)$ satisfying the condition $r)$ of the Theorem 1. In this case we have $\{Z_7,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}$. By definition of the semilattice D follows that $Q_{13}\vartheta_{XI}=\{\{Z_7,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\}$. It is easy to see $\left|\Phi(Q_{13},Q_{13})\right|=1$ and $\left|\Omega(Q_{13})\right|=1$. If $D_1'=\{Z_7,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}$, then $R^*(Q_{13})=R(D_1')$, $\left|R^*(Q_{13})\right|=\left|R(D_1')\right|$ and

$$\left|R^*(Q_{13})\right|=\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_3\cap Z_2)\setminus Z_5|}\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 7^{|X\setminus\breve{D}|}.$$

n) Let binary relation α of the semigroup $B_X(D)$ satisfying the condition $r)$ of the Theorem 1. In this case we have $\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_1,\breve{D}\}$. By definition of the semilattice D follows that $Q_{14}\vartheta_{XI}=\{\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_1,\breve{D}\}\}$. It is easy to see $\left|\Phi(Q_{14},Q_{14})\right|=1$ and $\left|\Omega(Q_{14})\right|=1$. If $D_1'=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_1,\breve{D}\}$, then $R^*(Q_{14})=R(D_1')$, $\left|R^*(Q_{14})\right|=\left|R(D_1')\right|$ and

$$\left|R^*(Q_{14})\right|=\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot\left(7^{|\breve{D}\setminus Z_1|}-6^{|\breve{D}\setminus Z_1|}\right)\cdot 7^{|X\setminus\breve{D}|}.$$

o) Let binary relation α of the semigroup $B_X(D)$ satisfying the condition $r)$ of the Theorem 1. In this case we have $\{Z_7,Z_6,Z_5,Z_4,Z_2,Z_1,\breve{D}\}$. By definition of the semilattice D follows that $Q_{15}\vartheta_{XI}=\{\{Z_7,Z_6,Z_5,Z_4,Z_2,Z_1,\breve{D}\}\}$. It is easy to see $\left|\Phi(Q_{15},Q_{15})\right|=4$ and $\left|\Omega(Q_{15})\right|=1$. If $D_1'=\{Z_7,Z_6,Z_5,Z_4,Z_2,Z_1,\breve{D}\}$, then $R^*(Q_{14})=R(D_1')$, $\left|R^*(Q_{14})\right|=\left|R(D_1')\right|$ and

$$\left|R^*(Q_{15})\right|=4\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot 4^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\left(5^{|Z_2\setminus Z_1|}-4^{|Z_2\setminus Z_1|}\right)\cdot\left(5^{|Z_1\setminus Z_2|}-4^{|Z_1\setminus Z_2|}\right)\cdot 7^{|X\setminus\breve{D}|}.$$

$\mathbf{r}')$ Let binary relation α of the semigroup $B_X(D)$ satisfying the condition $r)$ of the Theorem 1. In this case we have $\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}$. By definition of the semilattice D follows that $Q_{16}\vartheta_{XI}=\{\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\}$. It is easy to see $\left|\Phi(Q_{16},Q_{16})\right|=1$ and $\left|\Omega(Q_{16})\right|=1$. If $D_1'=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}$, then $R^*(Q_{16})=R(D_1')$, $\left|R^*(Q_{16})\right|=\left|R(D_1')\right|$ and

$$\left|R^*(Q_{16})\right|=\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|Z_5\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot\left(5^{|Z_2\setminus Z_1|}-4^{|Z_2\setminus Z_1|}\right)\cdot 8^{|X\setminus\breve{D}|}.$$

Let us assume that

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$$r_1 = |R^*(Q_1)| + |R^*(Q_2)| + |R^*(Q_3)| + |R^*(Q_4)| + |R^*(Q_5)| + |R^*(Q_6)| + |R^*(Q_7)| + |R^*(Q_8)| + |R^*(Q_9)| + \\ + |R^*(Q_{10})| + |R^*(Q_{11})| + |R^*(Q_{12})| + |R^*(Q_{13})| + |R^*(Q_{14})| + |R^*(Q_{15})| + |R^*(Q_{16})| + |R^*(Q_{17})|.$$

Theorem 2. Let $D \in \Sigma_1(X,9)$ and $Z_8 \neq \varnothing$. If X is a finite set and R_D is a set of all regular elements of the semigroup $B_X(D)$. Then $|R_D| = r_1$.

No	Set X	Semilattice D	Number of	
			elements of the Semi-group $B_X(D)$	regular elements of the semi-group $B_X(D)$
1	$X = \{1,2,3,4,5\}$	$D = \{\{1\},\{1,4\},\{1,5\},\{1,4,5\},\{1,3,5\},\{1,2,4,5\},\{1,3,4,5\},\{1,2,3,4,5\}\}$	32768	2761

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