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A Non Linear Filtering and Tracking of a Ballistic Missile for Terminal Phase Interception

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Abstract— **Tracking a ballistic missile in its reentry phase is one of the missile defence challenges as these reentry vehicles penetrate with large aerodynamic loads and sudden decelerations from exoatmospheric to endoatmospheric phase. With the advent of anti-ballistic missiles, the main goal of missile defence systems is to track the reentry vehicle to locate them precisely for allowing midair altitude interception. When these ballistic missiles suffer large aerodynamic loads and sudden decelerations the motion of the reentry vehicle is undoubtedly a nonlinear and forms a complex dynamic phenomenon. In this deceleration phase where the aerodynamic drag becomes predominant one parameter is used to characterize the deceleration: the ballistic coefficient $'\beta'$. Therefore the knowledge and accurate estimation of this ballistic coefficient is used to relax the interceptor guidance, guidance and fire control purpose in stressing engagement geometries. With these insights, the paper deals with the implementation of Extended Kalman Filter (EKF) based on the linearization of the estimated state. Also this paper presents how EKF's are current efficient and classical solution to nonlinear filtering for ballistic coefficient estimation. This paper also deals with simulation of decelerating ballistic trajectory by accurately estimating the ballistic coefficient using EKF and this trajectory information is necessary to determine the impact point on the ground and accurate intercept point prediction.**

Keywords— Ballistic Missile, Aerodynamic Drag, Deceleratoin, Ballictic Coefficient and Extended Kalman Filter.

I. INTRODUCTION

Ground-based missile systems having range capabilities varying from a few miles to several thousand miles are called ballistic or non-ballistic type depending on their mission requirements. The trajectory of a ballistic missile is composed of three segments. These segments are powered flight, free flight (or free fall) and reentry flight. The terms that are used to denote the conditions before the free-fall are cut off and burnout in powered flight. At lift off the missile acceleration from this thrust is between 1.1 g to 1.5 g; upon the fuel consumption and staging of missile, the acceleration increases which ranges between 5 g to 10 g. As the mass of the vehicle decreases the high acceleration can be achieved. Typically an ICBM will burnout at about 264.4 nm (490 km) altitude and 420.9 nm (780 km) downrange from its target. The position that constitutes most of the trajectory is a free-flight (or free fall) which is also called as "vacuum flight". The missile will reenter the atmosphere as the missile converges on the target. The segment where the atmosphere drag becomes predominant force in determining the missile path and lasts until impact i.e. the target on the surface of the earth is called reentry segment. The reentry phase begins at an altitude of about 100,000 ft. (30,480 m), where the dynamic pressure starts to significantly affect the motion of the missile. [1]

Anti-Ballistic Interceptors are encountered by various challenging tracking problems. Among which the problem of tracking a ballistic vehicle in its reentry phase still remains a topic of active research. Different ballistic vehicle interceptions presently pursued are Airborne Laser (ABL) in boost-phase interception, Exoatmospheric kill vehicle concept in mid-course defence, intercepting incoming ballistic missile in the earth's atmosphere is another missile defence challenge in terminal or reentry phase. One such an example is a Theater High Altitude Area Defence (THAAD) and the joint Israeli-US Arrow system. Therefore interceptions of these reentry vehicles are confronted with difficult challenges as these ballistic vehicles penetrate through the atmosphere from higher altitudes and subsequently increased speed. With these thoughts, this paper focuses on techniques and algorithms implemented for a reentry vehicle tracking in this case it is a ballistic missile. Also this paper focuses on performing an estimated trajectory using an Extended Kalman Filter (EKF) required for terminal phase interception. [2]

The organization of the paper is as follows: Section 2 outlines the related work on reentry vehicle (RV) tracking. Section 3 derives the target RV dynamics and motion characteristics. Section 4 focusses on state estimation of ballistic coefficients using EKF. Section 5 discusses on experimental results on ballistic coefficients and estimated trajectories for various cases.

II. RELATED WORK

Ballistic Vehicles penetrating the earth's atmosphere with varying rates are tracked with various sensors adopted by missile defence systems, one such an example is Millimeter Wave (MMW) radar in which the wavelengths ranges from 10 to one millimeter. A measurement is generated whenever the radar manages to detect the target in terms of target angular direction (elevation and azimuth), the target range and possibly the Doppler velocity. However, with possible bias and noises the measurements are imprecise. Usually the measurement noises are assumed to be White and Gaussian noise [3]. To guide the interceptor towards the ballistic target by successfully tracking the target is one of the main aims of Reentry Vehicle (RV) tracking. In the presence of dynamic non linearity, tracking should result in precise location of the target and to update the prediction of the future location of the target. To accomplish this task the track file i.e. the target position, first two derivatives its velocity and acceleration should be established. Yet due to the non-linear aerodynamic characteristics and non-linear harmonic motion properties [4] the state estimation formulation is inherently a highly non-linear problem. Aerodynamic drag becomes predominant in the deceleration phase where the acceleration vector and the velocity vector become collinear. Here ballistic coefficient $'\beta'$ is one of the parameter that can characterize the deceleration [3]. Evidently the knowledge of $'\beta'$ is indirect and its estimation becomes depends on the estimation of both the velocity and the acceleration. In stressing engagement geometries the prior knowledge of ballistic coefficient is used to establish advance guidance laws there by relaxing interceptor acceleration requirements. In fact the ballistic coefficient $'\beta'$ is necessary for fire control by predicting accurate intercept point when the interceptor is on a collision course. Eventually the concept of Extended Kalman Filter (EKF) can be applied for ballistic coefficient estimation [5]. However the insignificance of atmospheric density in Exoatmospheric phase causes indetermination of $'\beta'$ that would result in negligible acceleration. The estimation of $'\beta'$ can be found quite significant and meaningful only when the target penetrates the dense layers of the atmosphere. Then the determination of $'\beta'$ is essential to qualify the deceleration with which it can be integrated to yield velocity and position. The determination of $'\beta'$ also depends upon the quality and rate of radar measurements and tracking methods.

III. TARGET DYNAMICS AND ITS MOTION CHARACTERISTICS

In order to understand the target dynamics and its motion characteristics a flat-earth model is considered and the ballistic target is taken as a point mass the geometry of which is shown in Fig. 1

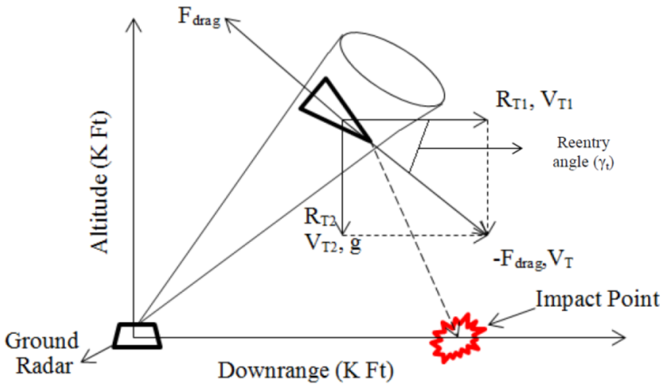

Fig.1 Point mass geometry of a target

From the above flat earth model, the acceleration components of the ballistic target along downrange and altitude directions can be shown in terms of $\dot{V}_{T1}$, $\dot{V}_{T2}$, target weight W, cross sectional area S_{ref}, zero lift drag $C_{D,0}$ gravity g. The equations are

$$\dot{V}_{T1}=\frac{-F_{drag}}{m}cos\gamma_t=\frac{-QS_{ref}C_{D,0}g}{W}=\frac{-Qg}{\beta}cos\gamma_t \quad (1)$$

$$\dot{V}_{T2}=\frac{-(-F_{drag})}{m}sin\gamma_t-g=\frac{QS_{ref}C_{D,0}g}{W}-g=\frac{Qg}{\beta}sin\gamma_t-g \quad (2)$$

Where Q is the dynamic pressure taken as $\frac{1}{2}\rho_\infty {V_T}^2$ in which V_T is the target total velocity. From the above two equations the $S_{ref}, C_{D,0}$ and weight W can be expressed in terms of ballistic coefficient β as

$$\beta=\frac{W}{S_{ref}C_{D,0}} \quad (3)$$

There by

$$\dot{V}_{T1}=\frac{-Qg}{\beta}cos\gamma_t \quad (4)$$

$$\dot{V}_{T2}=\frac{-Qg}{\beta}sin\gamma_t-g \quad (5)$$

Also the targets total velocity can also be expressed as

$$V_T=\sqrt{{V_{T1}}^2+{V_{T2}}^2} \quad (6)$$

And ρ is the air density approximated exponentially as [6]

$$\rho=0.0034e^{\frac{-R_{T2}}{22000}} \text{ and above 30,000 ft} \quad (7)$$

$$\rho=0.002378e^{\frac{-R_{T2}}{30000}} \quad (8)$$

Here target altitude and range are measured in feet (ft). For the point mass geometry mentioned in Fig.1, the concept of extended kalman filter can be illustrated by considering one dimensional tracking problem where two scenarios are considered. In the first case the target reentry angle is taken 45^0 and in the second case the reentry angle is 90^0. Assuming ballistic target coefficient is constant, the three differential equations associated with one dimensional ballistic target are

$$\dot{R}_{T2}=V_{T2} \quad (9)$$

$$\dot{V}_{T2}=\frac{0.0034e^{\frac{-R_{T2}}{22000}}}{2\beta}g{V_{T2}}^2-g \quad (10)$$

$$\dot{\beta}=0 \quad (11)$$

In the second scenario, referring to Fig. 1 the reentry angle is taken as $\gamma=45^0$ and the corresponding equations are given by

$$\dot{R}_{T2}=V_{T2} \quad (12)$$

$$\dot{V}_{T2}=\frac{0.0034e^{\frac{-R_{T2}}{22000}}}{2\beta}g{V_T}^2sin\delta_t-g \quad (13)$$

$$\dot{\beta}=0 \quad (14)$$

IV. ESTIMATION OF BALLISTIC COEFFICIENT USING EXTENDED KALMAN FILTER

Many dynamic systems and sensors are not absolutely linear but they are not far from it. In order to apply extended kalman filter for non-linear dynamic systems a set of non-linear differential equations has to be described in a real world. One standard nonlinear dynamic model is given by

$$X_k=f_{k-1}(X_{k-1})+W_{k-1} \quad (15)$$

$$\text{Where } W_k\sim N(0,Q_k) \quad (16)$$

Once after representing the differential equations governing one dimensional target in terms of R_{T2}, V_{T2} and ballistic coefficient β then the state vector X is given by

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$$X = \begin{bmatrix} R_{T2} \\ V_{T2} \\ \beta \end{bmatrix} \quad (17)$$

The Covariance matrix Q_k of process noise for zero mean random process noises W is given by

$$Q_k = \phi_s \begin{bmatrix} 0 & 0 & 0 \\ 0 & ({f_{23}}^2)\frac{{T_s}^3}{3} & (f_{23})\frac{{T_s}^2}{2} \\ 0 & (f_{23})\frac{{T_s}^2}{2} & T_s \end{bmatrix} \quad (18)$$

Where the values of f_{21}, f_{22} and $f_{23}$are given below.

The nonlinear measurement model required by extended kalman filter is given by

$$Z_k = h_k(X_k) + V_k \quad (19)$$

$$\text{Where } V_k \sim N(0, R_k) \quad (20)$$

Here the measurement equation is taken as linear function of the states represented by

$$Z_k = R_{T2} + V_k = [1 \quad 0 \quad 0] \begin{bmatrix} R_{T2} \\ V_{T2} \\ \beta \end{bmatrix} + V_k \quad (20)$$

Where the position measurement uncertainty is a scalar variance of $R_K = {\sigma_K}^2$

A first order linear approximation [6] is used in Ricatti equation for system dynamic coefficient matrix $'F'$ and measurement sensitivity matrix $'H'$ defining the linear relationship between state of the dynamic system and measurements that can be made.

Therefore

$$F = \frac{\partial f_k}{\partial x} \Big| X = \hat{X}_k(-) \quad (21)$$

$$H_k = \frac{\partial h_k}{\partial x} \Big| X = \hat{X}_k \quad (22)$$

The dynamic coefficient matrices for two different scenarios are obtained from three differential equations given by

$$F = \frac{\partial f_k}{\partial x} \Big| X = \hat{X}_k(-) \quad (23)$$

Upon taking partial derivative of three differential equations, F is given by

$$F = \begin{bmatrix} 0 & 1 & 0 \\ f_{21} & f_{22} & f_{23} \\ 0 & 0 & 0 \end{bmatrix} \quad (24)$$

Where f_{21}, f_{22} and $f_{23}$are defined in terms of state estimates as $f_{21} = \frac{-\hat{\rho} g {\hat{V}_{T2}}^2}{44000\hat{\beta}}$; $f_{22} = \frac{-\hat{\rho} g \hat{V}_{T2}}{\hat{\beta}}$ and $f_{23} = \frac{-\hat{\rho} g {\hat{V}_{T2}}^2}{2\hat{\beta}^2}$ respectively.

The state transition matrix of discrete linear dynamic system is given by

$$\phi_k \approx I + FT_s = \begin{bmatrix} 1 & T_s & 0 \\ f_{21}T_s & 1 + f_{22}T_s & f_{23}T_s \\ 0 & 0 & 1 \end{bmatrix} \quad (25)$$

Whereas the discrete process noise matrix is given by

$$Q_k = \begin{bmatrix} 0 & 0 & 0 \\ 0 & {f_{23}}^2 \frac{{T_s}^3}{3} & F_{23}\frac{{T_s}^2}{2} \\ 0 & F_{23}\frac{{T_s}^2}{2} & T_s \end{bmatrix} \quad (26)$$

The governing equations for computation of priori covariance matrix, Kalman gain matrix and posteriori covariance matrix is given by

$$P_k(-) = \phi_k P_{k-1}(+) {\phi^T}_k + Q_{k-1} \quad (27)$$

$$\overline{K}_k = P_k(-) {H_k}^T [H_k P_k(-) {H_k}^T + R_k]^{-1} \quad (28)$$

$$P_k(+) = [I - \overline{K}_k H_k] P_k(-) \quad (29)$$

The actual extended filter equations for conditioning the predicted estimation on the measurement can be written in terms of the nonlinear measurement equation where the new estimate is old estimate plus a gain time a residual and it is expressed as

$$\hat{X}_k(+) = \hat{X}_k(-) + \overline{K}_k[Z_k - \hat{Z}_k] \quad (30)$$

The above equation is also called as state estimate observational update. In Eq. 30 the residual is the difference between the actual measurement and nonlinear measurement equation. Here the new state estimate do not have to be propagated forward from the old estimate with state transition matrix but instead can be obtained directly by integrating the actual nonlinear differential equation. Therefore the new extended kalman filter states will simply the old estimate propagated forward by Euler integration plus gain times a residual. The estimates are expressed as

$$\text{Residual} = {R_{T2}}^* - (\hat{R}_{T2_{k-1}} + \overline{\dot{R}_{T2} T_s}) \quad (31)$$

$$\hat{R}_{T2_k} = \hat{R}_{T2_{k-1}} + \overline{\dot{R}_{T2} T_s}) + K_1 * \text{Residual} \quad (32)$$

$$\hat{V}_{T2_k} = \hat{V}_{T2_{k-1}} + \overline{\dot{V}_{T2} T_s}) + K_2 * \text{Residual} \quad (33)$$

$$\hat{\beta}_k = \hat{\beta}_{k-1} + K_3 * \text{Residual} \quad (34)$$

The barred quantities from the above equations represent the derivatives required by Euler integration and are obtained directly from the nonlinear system equation as

$$\overline{\dot{R}_{T2}} = \hat{V}_{T2_{k-1}} \quad (35)$$

$$\overline{\dot{V}_{T2}} = \frac{0.0034 e^{\frac{-\hat{R}_{T2_{k-1}}}{22000}} g (\hat{V}_{T2_{k-1}})^2}{2\hat{\beta}_{k-1}} - g \quad (36)$$

The above equations are necessary to simulate Extended Kalman Filter for one-dimensional tracking problem.

V. ESTIMATED BALLISTIC COEFFICIENT RESULTS

In order to perform the implementation of Extended Kalman Filter and how it estimates the value of ballistic coefficient let us consider a situation referring to Fig. 1 where an ICBM at an altitude of 100 Kilo Feet altitude with ballistic coefficient of 500 lb/ft^2 is travelling downward penetrating through the atmosphere at a speed of 6000 ft/s. A ground based radar having measurement variance of 500 ft^2 will track the target for every 0.05 s. Here the initial estimation of position, velocity and ballistic coefficient is taken as 100,025 ft, 6150

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ft/s and 800 lb/ft^2. The state error covariance matrix (P_0) has also been considered where the variances in the initial estimate of position, velocity and ballistic coefficient along the diagonal elements of the matrix are taken to be 500 ft^2, 20000 ft^2/s^2 and 90000 lb^2/ft^4. Also here a second order Runge-Kutta numerical integration is applied for solving actual nonlinear differential equations representing the ballistic target. With the same initial conditions mentioned above the implementation of EKF has also been done with varying ballistic coefficient as a ramp function of altitude. The flow chart representing the working of EKF has been given below

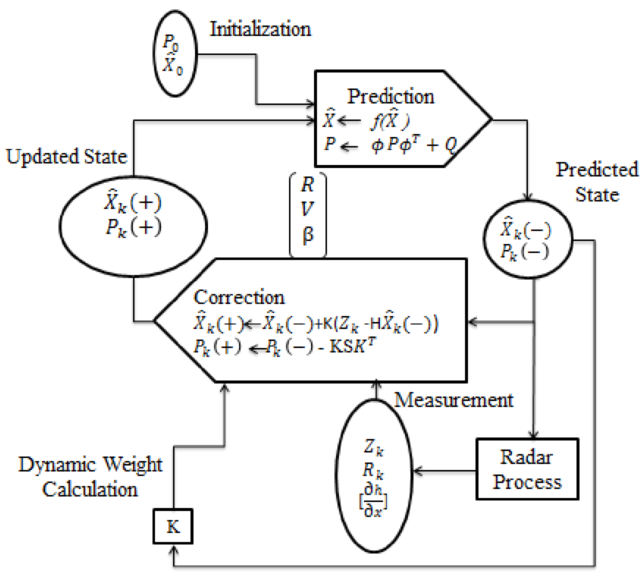

Fig. 2 Iterative and Recursive Filtering of EKF

In this section the ballistic trajectories for four different cases has been plotted. The Fig. 3 shows the result of a nominal case that was run to plot estimated and actual ballistic coefficient versus altitude. The initial estimate of ballistic coefficient is on the high side by 300 lb/ft^2 at 100 K ft altitude and when target descends to 60 K ft the EKF has an excellent estimate of the targets ballistic coefficient.

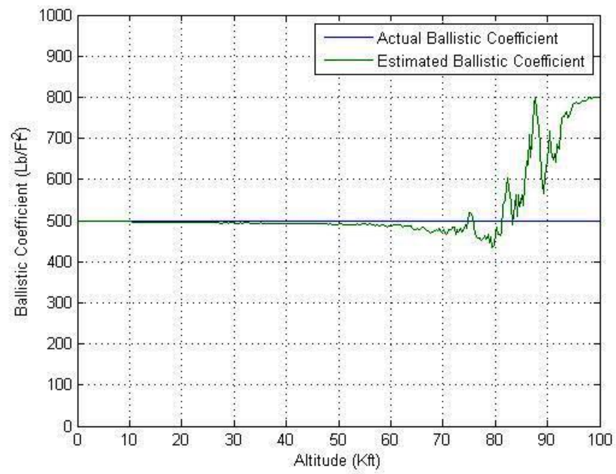

Fig. 3 Plot of Actual and Estimated $'\beta'$ with no uncertainty.

With the estimated ballistic coefficient another nominal case was run to plot actual ballistic trajectories and the trajectory that was generated with the estimated ballistic coefficient and is shown in Fig. 4. And the close shot of Fig. 4 is shown in Fig. 5

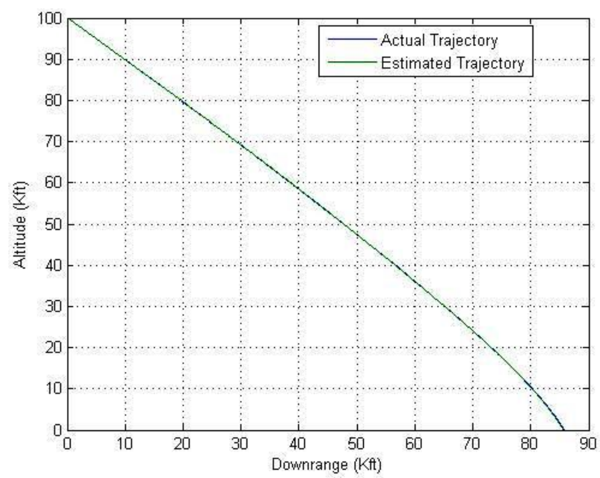

Fig. 4 Actual and Estimated Trajectory with no uncertainty in $'\beta'$

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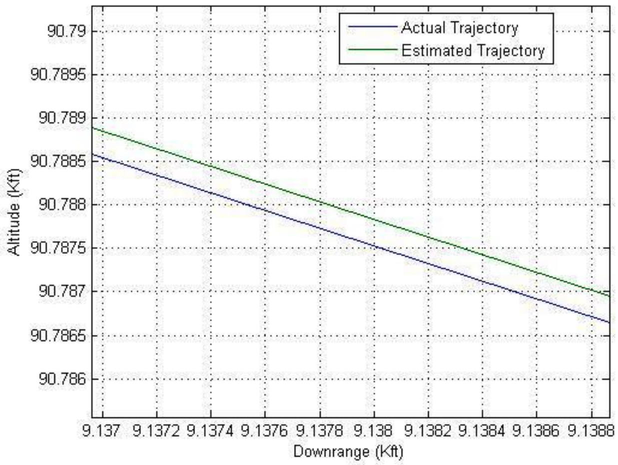

Fig. 5 Actual and Estimated Trajectory with no uncertainty in $'\beta'$

In the second case another nominal trajectory was run at the normal altitude of 100 K ft. However this time the initial estimate of ballistic coefficient was 1500 lb/ft^2 rather than 800 lb/ft^2. The uncertainty in $'\beta'$ in third diagonal element of the initial state error covariance matrix was increased to 1000^2 lb^2/ft^4. The estimated ballistic coefficient is shown in Fig. 6

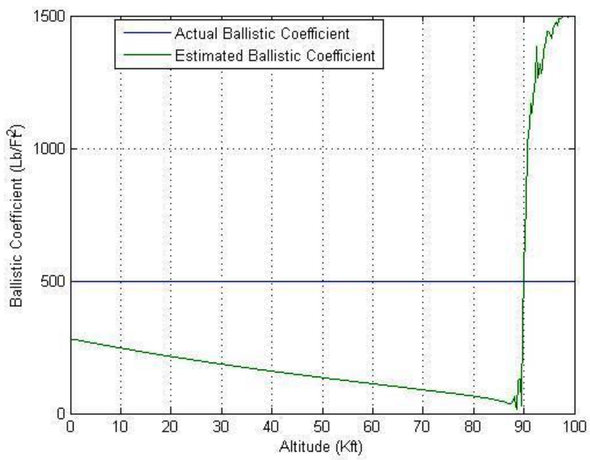

Fig. 6 Plot of Actual and Estimated $'\beta'$ with no uncertainty and far away from actual $'\beta'$

Under these circumstances the EKF was unable to estimate the ballistic coefficient and the resultant trajectory is shown in Fig. 7. Referring to Fig. 6 and Fig. 7 it shows when the initial estimate of $'\beta'$ is severely overestimated by 1000 lb/ft^2 the EKF was unable to estimate $'\beta'$ since the filters covariance matrix predictions indicate that the errors in the estimate of ballistic coefficient are not near zero and as a result the filter doesn't even realize when it is broken apparently. Also from the Fig. 7 the estimated trajectory has been deviated from the actual trajectory and the error of 15 K ft was observed in impact point.

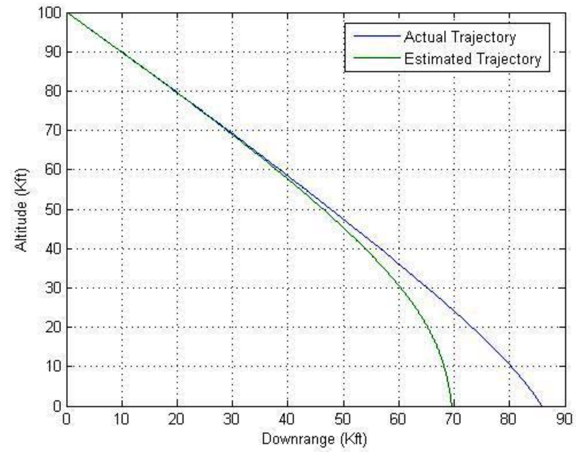

Fig. 7 Actual and Estimated Trajectory with no uncertainty in $'\beta'$ and initial estimate of $'\beta'$ far away from actual $'\beta'$.

In the third case the robustness of EKF has been shown in Fig. 8 where the process noise was added to the filter of value $300^2/3$ and from the Fig. 8 by adding process noise the estimated and actual ballistic coefficient converging fairly quickly can be observed and the estimated trajectory is also shown in Fig. 9

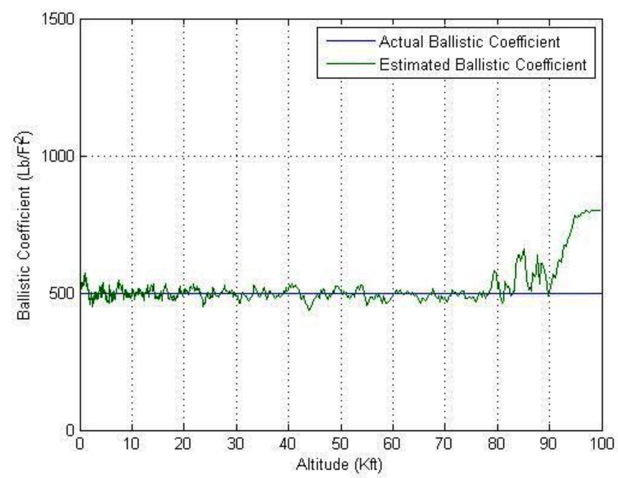

Fig. 8 Plot of Actual and Estimated $'\beta'$ with process noise

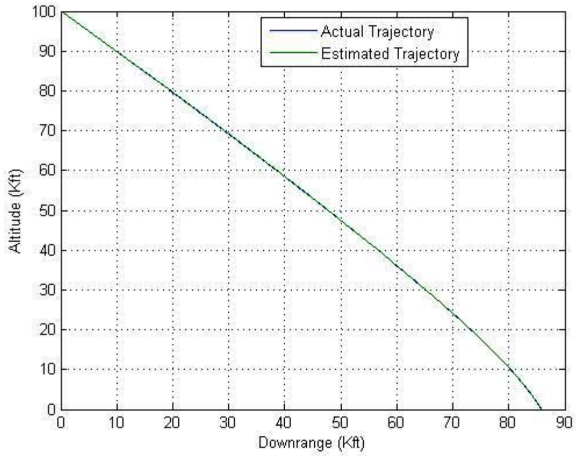

Fig. 9 Actual and Estimated Trajectory with uncertainty in $'\beta'$

Finally the fourth case was also run with same uncertainty of 1000^2, adding process noise of $300^2/3$ and taking β as 1500 lb/ft^2 that was shown in Fig. 10. The estimated trajectory of the ballistic missile under these situations was shown in Fig. 11

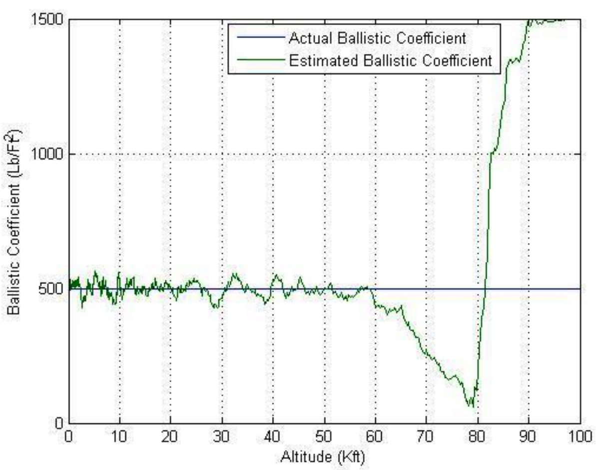

Fig. 10 Plot of Actual and Estimated $'\beta'$ with process noise and far away from actual $'\beta'$

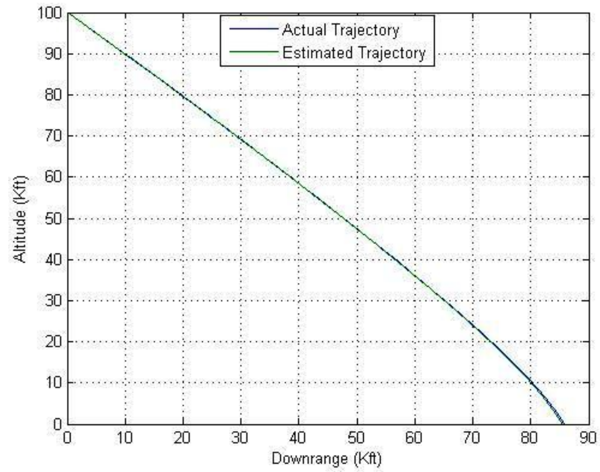

Fig. 11 Actual and Estimated Trajectory with uncertainty in $'\beta'$ and initial estimate of $'\beta'$ far away from actual $'\beta'$.

Here the Table. 1 show the typical values of the errors observed in impact point for each nominal case shown in the Fig.4, Fig. 7, Fig.9 and Fig. 11 for different initial estimates of ballistic coefficients and by including process noise.

Table 1

Sl No	Actual Ballistic coefficient (lb/ft^2)	Initial ballistic coefficient estimate (lb/ft^2)	Noise in Ballistic Coefficient Estimate $(\mathrm{lb/ft^2})^2$	Error in process noise covariance $(\mathrm{lb/ft^2})^2$	Error in impact point (Kft)
1	500	800	0	300*300	0.44
2	500	1500	0	1000*1000	15
3	500	800	3000	300*300	0.005
4	500	1500	3000	1000*1000	0.2

Another simulation analysis has been done by considering ballistic coefficient as a time varying model. In this case the actual ballistic coefficient is taken as

$$\beta = 300 + 0.002R_{T2} \quad (37)$$

The Eq. 37 shows the actual ballistic coefficient vary linearly with altitude and considered as a ramp function which change constantly with time. With the same initial conditions the estimation of ballistic coefficients and respective trajectories for the four cases are shown in following figures.

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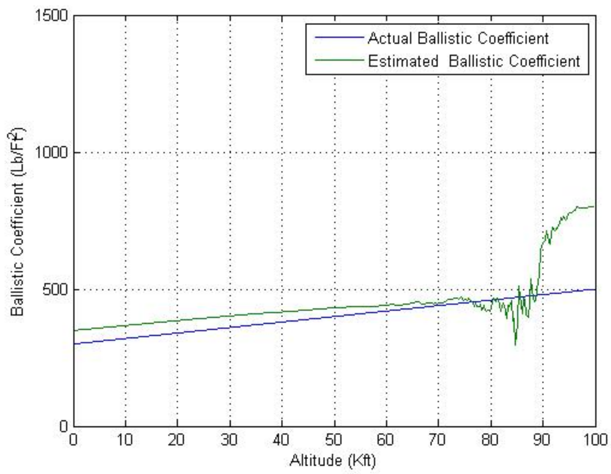

Fig. 12 Plot of Actual and Estimated $'\beta'$ with no uncertainty.

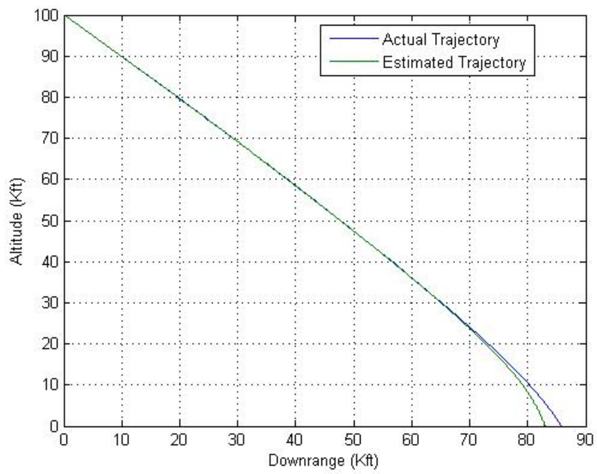

Fig. 13 Actual and Estimated Trajectory with no uncertainty in $'\beta'$

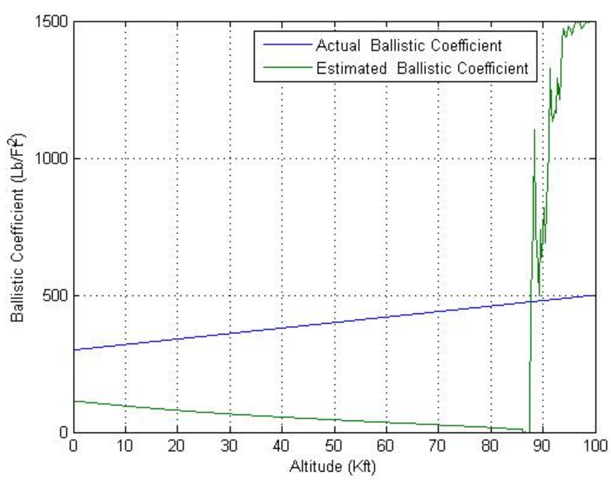

Fig. 14 Plot of Actual and Estimated $'\beta'$ with no uncertainty and far away from actual $'\beta'$

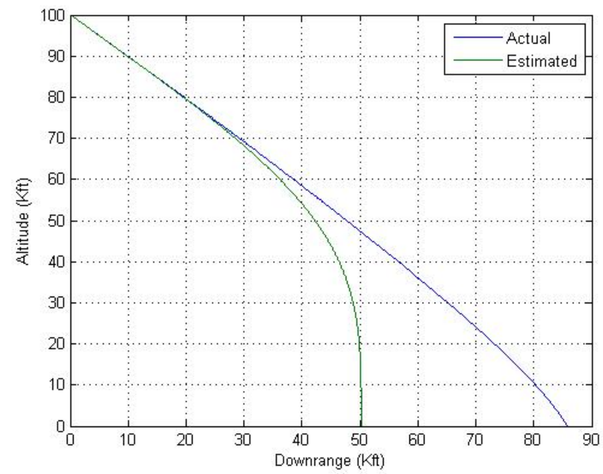

Fig. 15 Actual and Estimated Trajectory with no uncertainty in $'\beta'$ and initial estimate of $'\beta'$ far away from actual $'\beta'$.

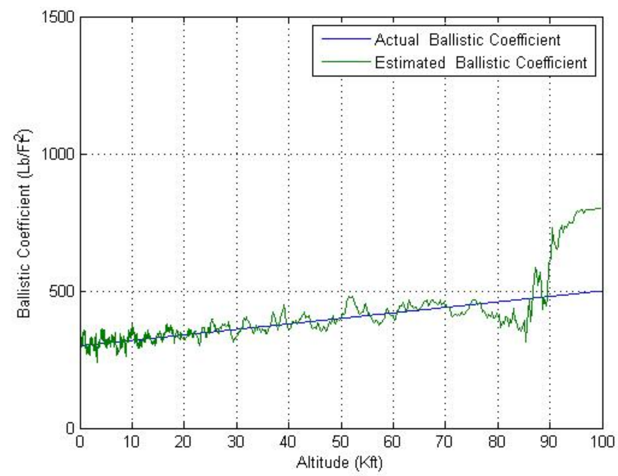

Fig. 16 Plot of Actual and Estimated $'\beta'$ with process noise

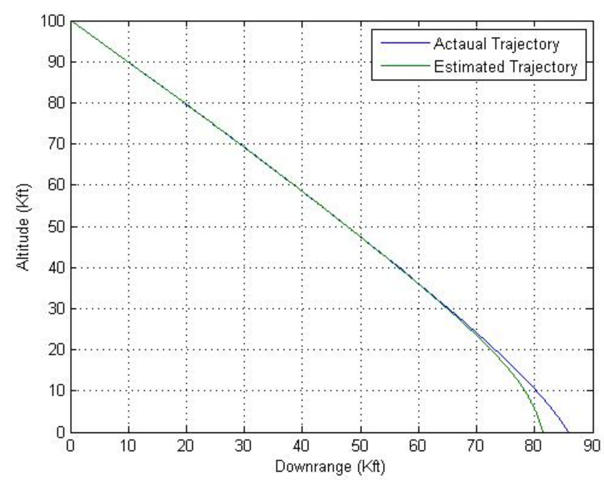

Fig. 17 Actual and Estimated Trajectory with uncertainty in $'\beta'$

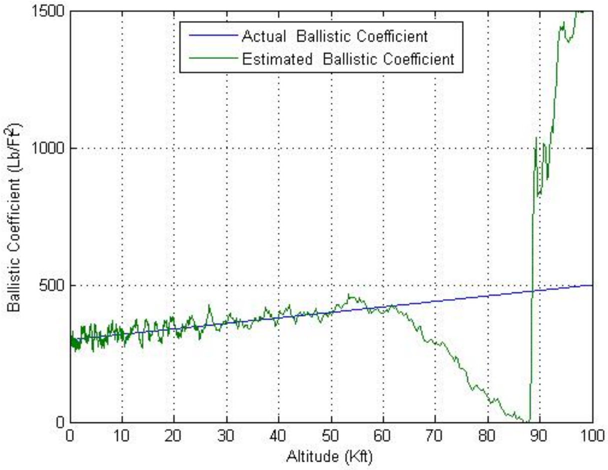

Fig. 18 Plot of Actual and Estimated $'\beta'$ with process noise and far away from actual $'\beta'$

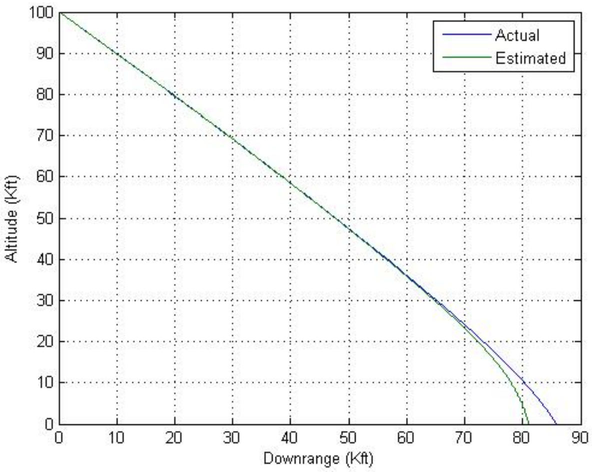

Fig. 19 Actual and Estimated Trajectory with uncertainty in $'\beta'$ and initial estimate of $'\beta'$ far away from actual $'\beta'$.

Here the Table. 2 show the typical values of the errors observed in impact point for each nominal case shown in the Fig.13, Fig. 15, Fig. 17 and Fig. 19 for different initial estimates of ballistic coefficients and by including process noise.

Table 2

Sl No	Actual Ballistic coefficient (lb/ft^2)	Initial ballistic coefficient estimate (lb/ft^2)	Noise in Ballistic Coefficient Estimate $(lb/ft^2)^2$	Error in process noise covariance $(lb/ft^2)^2$	Error in impact point (Kft)
1	Ramp function of Altitude	800	0	300*300	2.7
2	Ramp function of Altitude	1500	0	1000*1000	35
3	Ramp function of Altitude	800	3000	300*300	4.1
4	Ramp function of Altitude	1500	3000	1000*1000	4.2

VI. CONCLUSION

In this paper it is evident from the results that the Extended Kalman Filter paper estimates the Ballistic coefficient considering the initial estimate of the Ballistic Coefficient as 800 lb/ft^2 and 1500 lb/ft^2. In Fig. 3 with initial estimate of the Ballistic Coefficient as 800 lb/ft^2 , the EKF estimates the target's ballistic coefficient with error in Ballistic Coefficient estimation very near to zero below 60-kft altitude. Whereas by considering the initial ballistic coefficient estimate as 1500 lb/ft^2, the EKF overestimates the Ballistic Coefficient and the error in Ballistic coefficient persists with larger value till the target reaches the impact point. Hence the Trajectory estimated deviates far from the actual Trajectory and the impact points nearly shifted 15kft towards the left as shown in Fig 7.

Hence the EKF is much concerned about initial estimates and the performance of the filter can be achieved better by considering the initial estimates nearer to the actual value. With initial estimate far from the actual value, the estimated and actual ballistic coefficients converge fairly quickly only if process noise is added. Adding Process noise implies that we are specifying uncertainty in Ballistic Coefficient estimation. By adding Process noise the EKF estimates the Ballistic Coefficient properly and thus the estimated Trajectory converges with the actual one as shown in Fig. 8 and Fig. 19.

NOMENCLATURE

V_k - Measurement Noise
f_k - Non-linear function of states for state equation
h_k - Non-linear function of states for measurement equation
W_k - Process noise
$X(t)$ - State vector
$W(t)$ - Input vector
X_0 - Initial estimate of the state.
$\underline{\phi}$ - State estimation matrix
$\widehat{X}_k(+)$ - Smoothed estimate at time k
$\widehat{X}_k(-)$ - Predicting the state in $(k-1)^{th}$ interval for k^{th} interval
K_k - Kalman gain matrix
Q_k - Covariance matrix of state estimation uncertainty
R_k - Covariance matrix of observational uncertainty
$P_k(-)$ - Prediction of covariance matrix of state estimation uncertainty
$P_k(+)$ - Smoothing of covariance matrix of state estimation uncertainty
H_k - Measurement sensitivity matrix
V_T - Target velocity
γ_t - Missile Reentry Angle
β - Missile Ballistic Coefficient
F - System dynamic coefficient matrix
H - Measurement sensitivity matrix
ρ - Atmospheric air density
g - Acceleration due to gravity
T_s - Radar scanning interval

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