IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 2 Issue 11, November 2015.
www.ijiset.com
ISSN 2348 – 7968

Mathematical Model of Urban Taxi Subsidy Scheme

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Abstract
The problem of having difficulty to take a taxi is a hot issue in modern society. The current research on reasonable planning of urban taxi layout are mostly based on traditional way, However there are few researches related to the third party platform. In order to effectively improve the efficiency of waiting for a taxi, the subsidy scheme mathematical model is studied in this paper. Used two indicators of vehicle mileage utilization and load factor to measure whether the balance of supply and demand determines taxi introduced balance correction coefficient, the degree "matching supply and demand" is discussed. And analysis of the balance between supply and demand of taxi and taxi operators establish regional balance model based on the company's. The study shows that the existing taxi subsidy programs can alleviate the current situation to a certain extent. Finally, we use the evolutionary game model, and determine the weights for each part, then get the new subsidy program that cooperates with the taxi company. Subsidies will be part of the profits to the taxi company. The subsidy scheme helps taxi companies advocate the implementation of reasonable scheme.
Keywords: *Balance of the supply and demand; Reasonable scale; Subsidy scheme;* evolutionary game model

1. Introduction

According to some investigations on the traffic status, urban traffic problems become increasingly serious. Especially the problems of having difficulty take a taxi urgently need to solve. As the coming of Internet age, there are lots of companies set up the software service platforms of taking a taxi relying on the mobile Internet, which achieves the information exchanging between the passengers and the taxi driver. The previous studies about a reasonable taxi plan are most based on the traditional taxi ways, however there's few research on the way in which using the third party platform to take a taxi. In order to help the taxi firms propose reasonable plans and increase the efficiency that people take a taxi, this paper mainly study on the matching degree of supply and demand of taxi resources among different time and space in the Internet age, and explores whether the taxi software and subsidy programs of those taxi companies are helpful to ease the problem of "Having difficulty to take a taxi", which can provide a reference for urban traffic management departments.

2. The matching model of supply and demand

There are three relationships between supply and demand of taxi: basic balance of supply and demand, supply exceeding demand or demand exceeding supply. So to tell whether there is a balance between supply and demand of taxi, we can measure mainly through the mileage utilization ratio and the vehicle load ratio[1] two indicators.

2.1 The reasonable scale model based on the balance between supply and demand

By choosing model parameters from the above supply and demand angle, we can find that the total effective mileage of a taxi has a direct influence on taxi passenger demand and taxi supply. Therefore we can choose the total effective mileage of a taxi as a model variable[2] which tells whether there is a balance between supply and demand of taxi, so that we can set up the urban taxi's reasonable scale model based on the balance between supply and demand.

2.1.1 Measurement and calculation of the total effective traveling mileage of a taxi from a demand angle

（1）Measurement and calculation of the traveling turnover which taxi undertakes
The traveling turnover of urban residents and floating population which taxi undertakes are partly expressed by formula （4）and formula （5）.

$$Q_1 = P_1 \cdot \lambda_1 \cdot pro(Taxi)_1 D_1 \qquad (4)$$

$$Q_2 = P_2 \cdot \lambda_2 \cdot pro(Taxi)_2 D_2 \qquad (5)$$

In the two formulas:
Q_1, Q_2 ——Respectively represents the traveling turnover of urban residents and floating population which taxi undertakes;
P_1, P_2 ——Respectively represents the urban resident population and the total floating population;
λ_1, λ_2 ——Respectively represents the per capita daily travel times of urban resident and floating population;

$pro(Taxi)_1, pro(Taxi)_2$ ——Respectively represents the proportion of taking a taxi for traveling in all traffic modes among urban resident population and floating population;
D_1, D_2 ——Respectively represents the average travel distance of urban resident and floating population when traveling by a taxi.
（2）Measurement and calculation of the total effective traveling mileage of a taxi from a demand angle
When a taxi is in the operation process, the number of passengers is not the same each time. From demand angle, the total traveling mileage of a taxi is equal to the traveling turnover which the taxi undertakes divided by the average effective number of passengers per traveling of a taxi[3]. Then the total effective traveling mileage of a taxi measured and calculated from a demand angle can be expressed by formula （6）.

$$L_h = \frac{Q_1}{S_1} + \frac{Q_2}{S_2} \quad (6)$$

In the formula:
L_h ——the total effective traveling mileage of a taxi measured and calculated from a demand angle;
S_1, S_2 ——Respectively represents the average effective number of passengers per traveling of urban resident and floating population when taking a taxi.

2.1.2. Measurement and calculation of the total effective traveling mileage of a taxi from a supply angle

The empty driving rate of a taxi is an important parameter with which governments can adjust the supply of taxis, besides, there is a direct correlation between the total effective traveling mileage of a taxi and its empty driving rate[4].And the empty driving rate of a taxi can be expressed by formula （7）.

$$K = 1 - \frac{L_h^{'}}{T \cdot \bar{v} \cdot n} \quad (7)$$

In the formula:
$L_h^{'}$ ——the total effective traveling mileage of a taxi measured and calculated from a supply angle;
n ——the total supply quantities of urban taxis;
T ——the average operation time of taxis;
$\bar{v}$ ——the average operation speed of taxis;
K ——the average empty driving rate of taxis.
From this we can see that the computational formula of a taxi's total effective traveling mileage measured and calculated from a supply angle is

$$L_h^{'} = n \cdot (1-K) \cdot T \cdot \bar{v} \quad (8)$$

According to the balance of a taxi's supply and demand, when the supply and demand of a taxi come to balance, it's the best state of urban taxi scales. Here we regard the total effective traveling mileage as a equilibrium point, that is to say when the total effective traveling mileage of a taxi measured and calculated from a demand angle L_h is equal to the total effective traveling mileage of a taxi measured and calculated from a supply angle $L_h^{'}$, the passenger transport demand and supply of taxis come to a relative equilibrium.

$$\frac{Q_1}{S_1} + \frac{Q_2}{S_2} = n \cdot (1-K) \cdot T \cdot \bar{v} \quad (9)$$

When deformed we can get

$$n' = \frac{Q_1 / S_1 + Q_2 / S_2}{T \cdot \bar{v} \cdot (1-K)} \quad (10)$$

2. 2 The correction of urban taxi's reasonable scale model based on the balance between supply and demand

Actually there are many factors which can influence the reasonable scale of taxis. But in the modeling, only the strongly related factors that can be operated and can be easily acquired from the data are considered, so there are certain limitations. So we need to make some proper corrections to the model, and then the reliability and practicability of our model can be improved. In this paper, we draw into an equilibrium coefficient η to correct the model. Its purpose is to make taxis fully meet people's travel needs as well as to meet the profits of the taxi business. The value of η should be a value more than 1,and the larger its value adjusts to, the more that taxi's supply will be able to adapt to the passenger traffic demand of taxis under an emergency. But in the other hand, the value can't be too large, or there will be a high empty driving rate of a taxi, which will finally bring pressure to urban traffics. In this paper, we take η from 1.1 to 1.5, so the urban taxi's reasonable scale model is

$$N = \frac{\eta \cdot \beta \cdot L_h}{T^{'} \cdot (1-K) \cdot \bar{v} \cdot \alpha} \quad (11)$$

In the formula:
N ——the reasonable scale of a taxi;
β ——the proportion of travel amount matched the taxi's main operation period in the total daily travel amount;
T' ——the whole time matched the taxi's main operation period in a day;
α ——the taxi driving rate, in this paper, we take it as 90%.

Taking in consideration that there are different taxi situation in different level cities and in different time period, we choose the representative first-tier city Shanghai and the representative second-tier city Harbin as our research subjects, respectively discussing the different time period. For the sake of the fact that third and four-tier cities have relatively small population simple traffic situation and isn't representative, here we don't discuss them. Finally we reflect the taxi supply and demand phenomenon among our whole nation.

2. 3Analysis of first-tier cities

The problem of "Having difficulty to take a taxi" is particularly acute in the first-tier cities. In order to explore the matching degree of supply and demand in those first-tier cities, we first choose Shanghai as the research subject to discuss with. The data is chosen from "the fifth survey report about residents' travel in Shanghai", referring to the results of the fifth comprehensive traffic survey in Shanghai at the same time.

The travel intensity and average daily travel times of residents in Shanghai

From the survey report we can see, the total amount of residents' daily travel times in downtown Shanghai is 5550×10^4 times, and per capita daily travel times come to 2.37 times.

The distribution of residents' travel time

From the results of the fifth comprehensive traffic survey in Shanghai: the difference between the flat peak and the high peak has shrunk. The travel amount of morning high peak continues to grow, and the travel amount increases by 10%.The evening high peak has an advanced trend, and its travel amount increases by 5%.The rising range of the flat peak's travel amount is a bit large. The travel amount increases by 11% from 10:00 to 15:00.Both the travel amount of the afternoon flat peak (from 15:00 to 17:00) and the flat peak after the evening high peak(from 19:00 to 21:00) increase by 15%. The rising range of the flat peak's travel amount is large, which shrinks the gap between it and the high peak. The distribution of daily travel time of residents in Shanghai is expressed as Chart1.

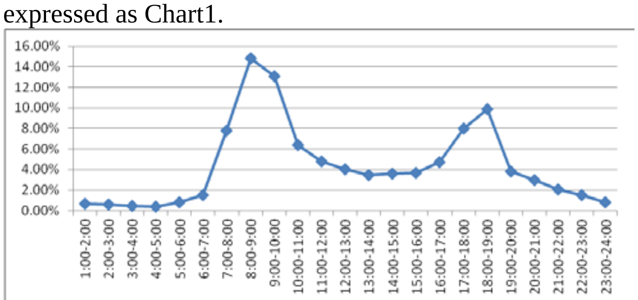

Fig.1 Shanghai residents travel time distribution of full-time

(3)the time distribution of residents' travel mode

Residents rely on public transits and walking as the main travel mode, whose proportion respectively is 3.46% and 26.4% in all daily travel modes. And among those travel modes, the proportion of residents who choose a taxi is 6.6%.The proportion of residents' travel mode in Shanghai is expressed as Chart2.

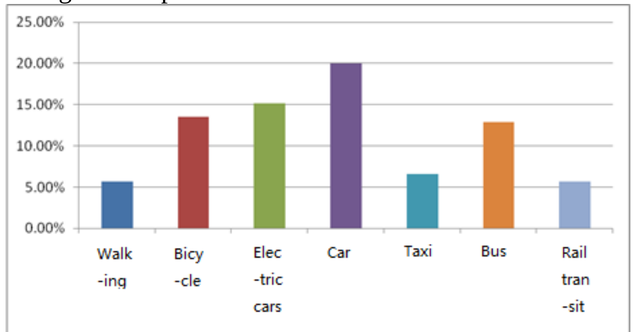

Fig.2 Shanghai residents travel mode scale map

(4) the average travel distance

The average travel distance of urban residents population and floating population when traveling by a taxi is 6.9 kilometers.

(5) the average empty driving rate of taxis

From the statistics, the average empty driving rate of taxis is K=0.3.

Calculation of related basic data

Measurement and calculation of taxis' passenger transport turnover:

From formula (6) and the survey data in the year of 2015, we can acquire the taxis' passenger transport turnover in the downtown Shanghai is

$$Q_t = 5550\times6.6\%\times69 = 2527.47(10^4\ person\cdot km)$$

In the downtown Shanghai in 2015, the average number of passengers per taxi is 1.78.From formula (6) we acquire that the total effective traveling mileage of a taxi in downtown Shanghai is:

$$L_h = \frac{Q_t}{S} = \frac{2527.47}{1.78} = 1419.93(10^4 km)$$

Measurement and calculation of the reasonable taxi number:

Since Shanghai is the economy center of our country and a very big city, so there are many floating people and all kinds of public transits in the city, and there is a big elastic demand of taxis. So we should take η as a bigger value and here we take it as η=1.55 .

As the result of the fact that both residents have different demand of taxis and road conditions are not the same in different time periods. Following we will discuss on morning and evening high peak together with leisure time separately:

（1）The time period of morning and evening high peak

From 7:00am to 9:00am and from 16:00 to 18:00 are the two main time periods when residents travel out, the proportion of which respectively comes to 26.3% and 17.9%.During this time period, the demand of taxis is pretty large, and at the same time, because the pressure of road traffic increases, the average speed of a taxi runs down. According to the results of the fifth comprehensive traffic survey in Shanghai, the average speed of a taxi during the high peak period is $\overline{v_1}$=20km/h . From formula (11), then the taxi's reasonable scale during morning and evening high peak period in downtown Shanghai in 2015 is:

$$N_1 = \frac{\eta \cdot \beta_1 \cdot L_h}{T' \cdot (1-K) \cdot \overline{v_1} \cdot \alpha} = \frac{1.55 \times 0.444 \times 10000 \times 1419.93}{10 \times 0.7 \times 20 \times 0.9} = 77555 veh$$

At present, the number of taxis operating among the whole Shanghai is 5.06 ten thousand .Because now we are considering a center city and the fact that taxi can't operate all day, the actual number of taxis operating during the high peak period is far less than the supply of taxis we calculate during the high peak period. There is a very large breach of taxi supply. At this time, passengers will have worse travel experience, for example, having difficulty in taking a taxi, waiting for a long time, or even coming across a phenomenon in which the taxi driver refuses to take you. Now there is a shortage of demand.

（2）the leisure time period

To remove the morning and evening high peak time one day, we get the leisure time period. During this time period, the road conditions are preferable as well as the number of residents traveling out is less. According to the results of the fifth comprehensive traffic survey in Shanghai,

The average speed of a taxi during the leisure time period is $\overline{v_2}$=30km/h .From formula (11), and then the taxi's reasonable scale model during the leisure time period in downtown Shanghai in 2015 is:

$$N_1 = \frac{\eta \cdot \beta \cdot L_h}{T' \cdot (1-K) \cdot \overline{v_1} \cdot \alpha} = \frac{1.55 \times 0.556 \times 10000 \times 1419.93}{19 \times 0.7 \times 30 \times 0.9} = 34076 veh$$

During the leisure time period, the number of taxis operating among the whole Shanghai is a little more than the taxi demand. At this time, the degree of supply and demand balance is basically reached. So residents traveling out by taxi have a higher degree of satisfaction, and at the same time, the empty driving rate of taxis is within a reasonable range.

In order to analyze taxi situations of different level cities, we choose Harbin from second-tier cities as our research subject. The data is chosen from the survey report about residents' travel and the road traffic volume in Harbin.

The research method is the same as the above. According to the survey report, we can acquire that Harbin's total amount of residents' daily travel out is 1032×10^4 .The proportion of residents who choose a taxi traveling out is 5.59%. The average travel distance of urban residents' population and floating population by a taxi is 6.2 kilometers.

Calculation of related basic data

Measurement and calculation of taxis' passenger transport turnover:

From formula (4) and the survey data in the year of 2015, we can acquire the taxis' passenger transport turnover in Harbin is

$$Q_t = 1032 \times 5.59\% \times 6.2 = 357.67(10^4\, person \cdot km)$$

In Harbin in 2015, the average number of passengers per taxi is 1.76.From formula (6) we acquire that the total effective traveling mileage of a taxi in Harbin is:

$$L_h = \frac{Q_t}{S} = \frac{357.67}{1.76} = 203.22(10^4\, km)$$

2. 4 Analysis of second-tier cities

Harbin is a main city in the north, and it's the representatives of second-tier cities. Although its public transits and urban population flows isn't as high as them of Shanghai, they are still relatively high in the whole country. So here we take the equilibrium coefficient η as η =1.35.From formula (11), then the taxi's reasonable scale in Harbin is:

$$N_1 = \frac{\eta \cdot \beta \cdot L_h}{T' \cdot (1-K) \cdot \overline{v_1} \cdot \alpha} = \frac{1.35 \times 0.85 \times 10000 \times 203.22}{15 \times 0.7 \times 26 \times 0.9} = 9491 veh$$

At present, the whole Harbin has the total amount of taxis as 12300, which is more than the reasonable scale. From the statistical data we can acquire that Harbin's taxi ownership amount is ranked ninth among the whole nation. As we can see, Harbin is a city where it's not difficult to take a taxi. However, there is a slight phenomenon of supply exceeding demand in Harbin.

Synthesize the solving results of the first and second tier representative cities Shanghai and Harbin, and we will acquire Table1.

Table1 The comparison table of the solving results of the first and the second tier cities

City	Time period	The reasonable scale of taxis' (veh)	The amount of taxi operation (veh)
The first-tier city——Shanghai	Morning and evening high peak period	77555	50600
	Leisure time period	34076	
The second-tier city——	All day	9491	12300

Harbin			

From the table above we can acquire that in those domestic first cities taking Shanghai as the representative, during the morning and evening high peak period, in other words from 7:00 to 9:00 and from 16:00 to 18:00,there is a large demand of taxis. So the number of taxis operating now can far from meet the demand, and the phenomenon of demand exceeding supply arises. In the other hand, during the leisure time period, there is a balance between supply and demand. However, taking overall consideration, now the amount of taxis in the first-tier cities in our country can hardly meet passengers' demand, and the problem of "Having difficulty to taking a taxi" is particularly severe. Comparing to first-tier cities, in those second-tier cities, the total amount of taxis now can better satisfy the needs of passengers, and even the phenomenon of supply exceeding demand arises. The phenomenon of "Having difficulty to taking a taxi" seldom appears.

In summary, measure whether taxis have a supply and demand balance through the mileage utilization ratio and the vehicle load ratio two indicators, draw into a equilibrium coefficient η to correct the model, set up the urban taxi's reasonable scale model based on the balance between supply and demand, and then research and analyze the specific city Shanghai——the representative of the first-tier cities, Harbin——the representative of the second cities in real life. Finally we get the matching degree of supply and demand of taxi resources among different time and space. In this paper, we mainly research on the mathematical model of subsidy programs of those taxi companies. Following we will measure whether subsidy programs of those taxi companies are helpful to ease the problem of" Having difficulty to take a taxi" by setting up the taxi operations balance model based on regions and analyze specifically whether it can save the driver's driving time and the waiting time of the passengers using a taxi software combined with concrete reality.

3. The matching model of supply and demand based on the mileage utilization ratio and the vehicle load ratio

In the face of an increasingly competitive urban road traffic system, especially the countries vigorously promote energy conservation and emissions reduction, encouraged public transit, taxi operation by a great impact, therefore the company's taxi subsidy scheme came into being, especially when a taxi introduced a variety of software solutions preferential promoted the dependence on the taxi passengers, not only makes the taxi driver reduced the rate of empty, but also make the passengers got on the bus fare is favorable and save the passengers' valuable time. By establishing the model based on the regional balance of taxi operation under the specific analysis and combined with the actual use a taxi driving software whether to save the driver's time and the passengers' waiting time to determine the taxi companies whether subsidy scheme are helpful to "difficult to alleviate a taxi".

3.1 The balance of supply and demand model

(1) The relation between supply and demand of conservati on

Assumes that the road network, including as traffic area se t, directed road set, and respectively starting area for passen gers and reaches sets,. In the condition of the balance of su pply and demand, passenger vehicles from village to villag e, in addition to the village to meet the needs of the destinati on for the community, and in the community into the empt y state, namely

$$\sum_{i\in I} D_{ij} = D_j \tag{12}$$

Formulation: from village to village of passenger's deman d, to produce the empty of traffic.

Passenger vehicles in the village to complete transportatio n into empty, and then through the software search the passengers from the community, in empty state, to the next village in probability and received the passengers, tax i from village to village is empty of traffic

$$q^v{}_{ji} = D_j P_{ji} = \sum_{i\in I} D_{ij} P_{ji} \tag{13}$$

Taxi from the village to empty state to village in the shorte st way and in the community into a passenger. From villag e to village empty vehicles to travel the sum total, namely

$$O_i = \sum_{j\in J} q^v{}_{ji} = \sum_{j\in J}\sum_{i\in I} D_{ij} P_{ji} \tag{14}$$

(2)Operating time of conservation

Taxi total operating time is total passenger time and total e mpty time. The total passenger time can be expressed as

$$q^0 = \sum_{i\in I}\sum_{j\in J} D_{ij} t_{ij} \tag{15}$$

Total empty time can be expressed as

$$q^v = \sum_{i\in I}\sum_{j\in J} (q^v{}_{ji} t_{ji} + \omega_i) \tag{16}$$

So the total operation time can be expressed as

$$q = \sum_{i\in I}\sum_{j\in J} D_{ij} t_{ij} + \sum_{i\in I}\sum_{j\in J} (q^v{}_{ji} t_{ji} + \omega_i) \tag{17}$$

In a fixed time, according to the operating time of conserv ation, it is

$$q=N \tag{18}$$

ISSN 2348 – 7968

In expression:
Nis taxi scale, namely the scope of a taxi to the total numb er of operating vehicles in the research.

3.2 The establishment of the model

United expression (14) and expression (18), a taxi have eq uilibrium using software to search operation balance mode l for passenger's taxi, and the model is

$$\begin{cases} \sum_{j\in J} q^{v}{}_{ji} = \sum_{j\in J}\sum_{i\in I} D_{ij}P_{ji} = O_i \\ \sum_{i\in I}\sum_{j\in J} D_{ij}t_{ij} + \sum_{i\in I}\sum_{j\in J}(q^{v}{}_{ji}t_{ji} + \omega_i) = N \end{cases} \quad (19)$$

From (19), under the established network conditions and taxi demand, the model variable is the average search time for a taxi in every district, and the number is equal to the starting area.

3.3 Passengers waiting time

Passengers waiting time should be started from the agreem ent with drivers to the arrival of taxi. For taxi, on the avera ge travel time from j village to i village and the sum of th e average search time in i village constitute the passengers waiting time[6]. So the average passenger waiting time is ω_i

$$W_i = \omega_i + \frac{\sum_j t_{ji}}{n_I - 1}$$

(20)

In expression: n_I is the number of traffic village.

3.3 The analysis of taxi software subsidy scheme

Taxi software company offering subsidy scheme is mainly for the purpose of quickly occupying the market, improving product awareness, letting more people experience the Internet age and bring convenience to life, cultivating customer's using habit. The figure 3 shows that after 2013 years in our country accumulative total number of registered users increased sharply. Taxi software shows a booming trend. By figure 4, Click a taxi and fast taxi share. The national market as high as 90.81%. The two taxi companies which use subsidy scheme have monopolized the domestic software market. Businesses possess the market quickly through subsidies; expand the market for the purpose of basic implementation.

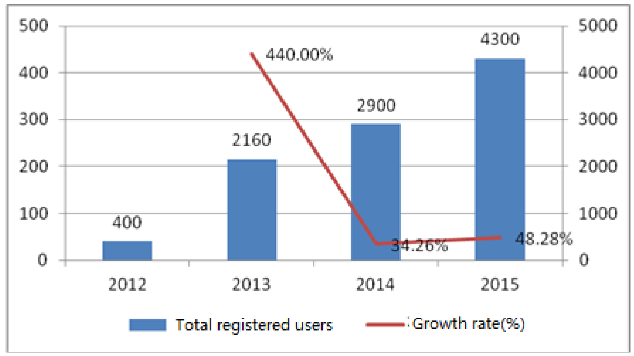

Figure3 from 2012 to 2015 taxi application registered users in China

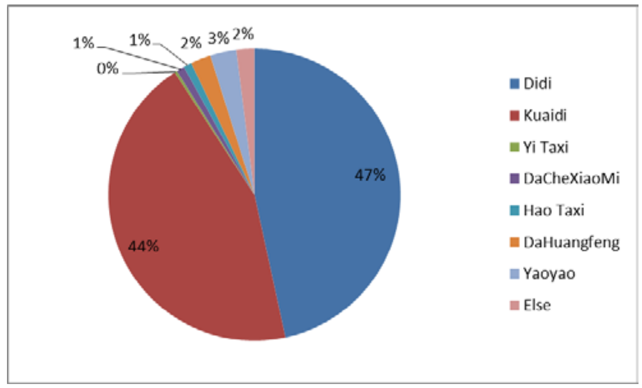

Figure4 the market share of a taxi software

Now using the established model to explore the way of using a taxi software whether can effectively shorten the passenger waiting time.(19)is systems of nonlinear equations, if directly use in a complex actual traffic network, the amount of calculation will be very large. Then reduce (19), the system of equations contains equations and variables. This model does not exist exact solution. (19) can be converted to

$$\begin{cases} \sum_{j\in J} q^{v}_{ji} - O_i = 0 \\ \sum_{i\in I}\sum_{j\in J} D_{ij}t_{ij} + \sum_{i\in I}\sum_{j\in J}(q^{v}_{ji}t_{ij} + \omega_i) - N = 0 \end{cases}$$

(21)

Because of the exact solutions of equations don't exist only; it should as far as possible narrow (20) equals the left items, making it approximate the right hand side value. Setting the absolute error value, then

$$e_{rr} = \alpha\left|\sum_{j\in J} q^{v}_{ji}t_{ij} - O_i\right| + \beta\left|\sum_{i\in I}\sum_{j\in J} D_{ij}t_{ij} + \sum_{i\in I}\sum_{j\in J}(q^{v}_{ji}t_{ij} + \omega_i) - N\right|$$

(22)

Among them: respectively are (18) and (21) the undetermined weight of absolute error to total error .Model in this paper from the conservation equations into unconstrained optimization problem, namely

$$\min e_{rr} = \alpha\left|\sum_{j\in J} q^{v}_{ji} t_{ij} - O_i\right| + \beta\left|\sum_{i\in I}\sum_{j\in J} D_{ij} t_{ij} + \sum_{i\in I}\sum_{j\in J}(q^{v}_{ji} t_{ij} + \omega_i) - N\right| \quad (23)$$

By using MATLAB genetic algorithm can solve.

Community	1	2	3	4	5
1		6	10	3	10
2	6		4	5	7
3	11	5		10	8
4	4	4	8		7
5	12	9	8	8	

According to simple network the feasibility of the model and algorithm are tested. In order to be more generally reflect the urban traffic [7]. Now create a traffic plot of five traffic models, shown in figure 5. Among them, circles with number represent point of each traffic area, connecting lines with arrow represent adjacency relations, and weight values are different between two points on the direction of the shortest travel time. Travel demand of taxi OD matrix are shown in table 2

Community	1	2	3	4	5	O_i
1		20	20	30	10	80
2	10		10	20	10	50
3	30	10		20	40	100
4	20	40	10		10	80
5	40	20	20	10		90
O_i	100	90	60	80	70	400

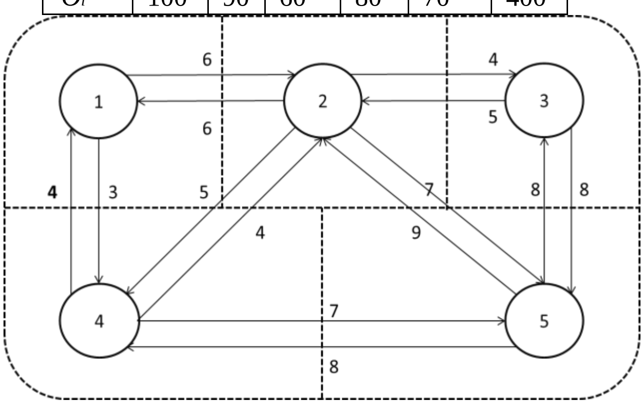

Figure5 simple traffic network

Table2 OD matrix way

Assume that taxi drivers' path in each traffic zone is the shortest path, we can determine the shortest travel time between the traffic areas, and the results are shown in table 3.

Table3 the shortest travel time between c and D

Because the software to the driver side took the subsidy scheme, so the driver's income improved correspond, 《Penetration of the software , the development of the economic , social development and transportation links》, [8], according to the report in March 2014, the taxi driver income year-on-year increase of 2000-3000 yuan per month. With the taxi driver's income has increased, and the present situation of the first-tier cities in short supply, we can foresee that the number of city operation cab will continue to improve in the future with the popularity of a taxi software promotion. At the same time, software to take a taxi car way to constantly promote the city of the actual operation of passenger vehicles will continue to grow such as Uber. Following, we discuss taxi scale expansion's influence on the passenger waiting time. Set different N to calculate different area of the average search time. According to the example data and introduced above algorithm, using of genetic algorithm toolbox of Matlab programming, the global optimal solution can be obtained,(see appendix) .we can determine the average search time in each traffic area and five district overall average search time ,among them $\alpha = \beta = 0.5$. Finally results are shown in table 4

Table 4 the average search time

N	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	e_{rr}
50	0.630	1.282	0.498	1.130	0.840	0.876	3.780
100	0.612	1.280	0.498	1.070	0.560	0.806	4.210
200	0.589	1.280	0.320	0.820	0.350	0.692	2.470
400	0.380	1.030	0.293	0.700	0.260	0.533	3.290

Table 5 shows that with the taxi scale increasing, the average search time is on the decline of each district. Using genetic algorithm has better search ability, the absolute error of optimization problem under different N values are within the acceptable range. Using (20) to calculate the passengers' average waiting time, are shown in table 5, with the increase of number of the taxi, t the average waiting time shorten.

Table5 each district average passenger waiting time

N	ω_1	ω_2	ω_3	ω_4	ω_5
50	8.880	7.282	7.998	7.630	8.840
100	8.862	7.280	7.998	7.570	8.560
200	8.839	7.280	7.820	7.420	8.350
400	8.630	7.030	7.793	7.200	8.260

In conclusion, a taxi software search method compared with the traditional search method, it has the advantage of a short time and a better line. At the same time, along with the popularity of a taxi software, operating vehicle size increases. Under the influence of the two kinds of function, the average waiting time of the passenger's shortens. The passengers' average waiting time is the important measure of "difficult to take a taxi". When the passengers' waiting time is more than 10 minutes, the passenger will have the idea of "difficult to take a taxi".
So a taxi software company subsidy scheme makes use of software on the rise of the taxi, a taxi operation on the number of vehicles increases, under the action of the two, passenger bus waiting time reduced. Subsidy scheme indeed makes China eased the problem of "difficult to take a taxi". In the process of solving the above, however, is the premise of a taxi passenger taxi ride USES software, however, currently using a taxi software is only 15.61% of the proportion of the taxi ride. Taking a taxi software utilization rate is low. The current subsidy scheme for both passengers and drivers is not attractive enough, taking a taxi software's enthusiasm is not high. It helps ease China's present situation of "difficult to take a taxi" but it is limited. Aimed at this phenomenon, in this paper designed a taxi company directly, and subsidies will be part of the profits to the rental car company subsidy scheme, the following will be detailed in this paper. 4 Subsidy scheme design based on evolutionary game theory

4. The establishment of the evolutionary game model

4.1 The establishment of the evolutionary game model

Assuming that the taxi driver groups take strategy Sd1 ratio is X, the corresponding (1 - X) strategy for Sd2 probability. Passenger groups take strategy Sp1 ratio is Y, the corresponding (1 - Y) as the strategy of Sp2 probability [9].
When the taxi driver selection strategy of Sd1 expected return as follows:

$$U_{d1} = Y * 0 + \pi_2 * (1 - Y) = \pi_2 * (1 - Y) \quad (24)$$

Choosing a strategy Sd2 expected return for:

$$U_{d2} = Y * \pi_1 + \pi_1 * (1 - Y) = \pi_1 \quad (25)$$

The taxi driver group average expected return as follows:

$$R_d = X * \pi_2 * (1 - Y) + \pi_1 * (1 - X) \quad (26)$$

According to the above conclusions, the taxi driver group replicated dynamic equation:

$$Y = \frac{dY}{dt} = Y[U_{p1} - R_p] = Y(1 - Y)[XV_0 + V_1(1 - X) - XV_2 - (1 - X)V_0] \quad (27)$$

Placed for replication dynamic relationship in two-dimensional axis, as shown in the figure below:

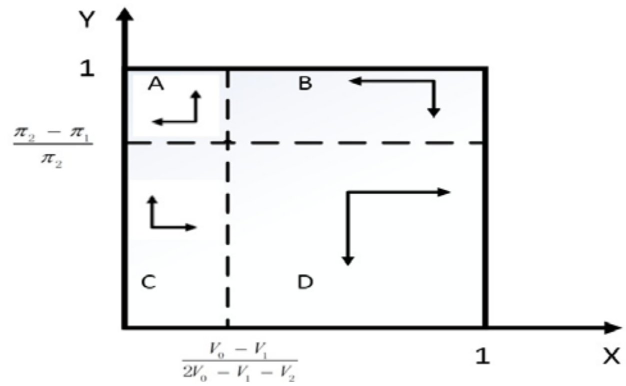

Figure 6 taxi drivers and passengers under the new strategy of taking a taxi is the software evolution game
According to the picture above, it can get a taxi driver and passengers in the game, $X^* = 0$、$Y^* = 1$ and $X^* = 1$、$Y^* = 0$ for the evolutionary stable strategy of the game. In this game the replication dynamic evolution game, when the initial X, Y fall in A region, it will eventually be stable in the evolutionary stable strategy $X^*=0$、$Y^*=1$ point, which is the driver groups don't insist on using the proposed tariffs as the new scheme, at the same time, the passenger groups insist to use existing car rental pricing rules. When the initial X, Y fall in the D area, it will eventually be stable in the evolutionary stable strategy point, which is the driver groups insist on using the proposed tariffs as the new scheme, at the same time, the passenger groups don't insist on using existing car rental pricing rules.

4.2 The analysis by evolution game theory

Through the final result of evolution game theory for area is more likely to fall to D, taxi drivers and passengers will choose a new scheme proposed in this paper.
From the perspective of a taxi service platform, cooperating with the taxi company means that more taxi drivers and passengers use the service of taxi. A taxi company accepted the legitimacy of the software. Cooperation means more software popularization, and taking a taxi service platform can be deposit payment for investment through online. In the software into some sponsorship means advertising revenue. Obviously, in cooperation with the taxi company, a taxi third-party software company will be much more profitable.
Above all, taken after the operation and subsidy scheme, a taxi will further enhance utilization rate of the software.

Go out by taxi software will become a kind of life habits. In view of the conclusion, higher proportion of taxi software using can shorten the waiting time of passengers. It has an important role for a taxi to alleviate the current "hard" situation. It can be seen this solution has the effect to improve urban traffic in China. At the same time, a taxi drivers and software company earnings are improving, and it has an important role for the healthy development of the taxi market in China.

4.3 Third party a taxi service platform for subsidy scheme to determine the taxi company

If electricity service expected profit is A, s_1, s_2 represents money feasible allocation. Two companies' feasible assignment collection is $S = \{(s_1, s_2) \mid 0 \le s_i \le A, s_1 + s_2 \le A\}$, i = 1, 2. And it assumes $d = (d_1, d_2)$ that the burst point by said. d_1, d_2 Respectively taxi companies and software companies burst profits. The problem of configuration utility , in conclusion $B = (S, d; u_1, u_2)$.

At the same time it satisfy the pare to efficiency, symmetry, linear transformation invariance, independent of the independent choice four axioms of unique solution for Nash solution, Nash solution for the following constrained optimization problem of the solution:

$$\max_{S1,S2}[(u_1(s) - u_1(d))(u_2(s) - u_2(d))] \quad (28)$$

$$S.T.(s_1, s_2) \in S \& (s_1, s_2) \ge (d_1, d_2)$$

This solution is Nash solution. For this problem, it needs to satisfy the following Nash by the constraints of the optimization problem

$$\max_{S1,S2}[(s_1-d_1)(s_2-d_2)]$$

$$S.T. \quad s_1+s_2=A \quad (29)$$

According to the constraint conditions to get into (29), change to a single variable optimization problem

$$\max_{S1}[(s_1-d_1)(A-s_1-d_2)]$$

Can be obtained finally

$$s_1^* = u_1^* = \frac{A-d_2+d_1}{2}$$

$$s_2^* = u_2^* = \frac{A+d_2-d_1}{2}$$

Through

$$s_1^* - d_1 = \frac{A-d_2-d_1}{2}$$

Available $s_1^* > d_1$, by the same token $s_2^* > d_2$, the inequality $s_1^* > d_1$ and $s_2^* > d_2$, and thus have proved to be a taxi service platform for the optimal values of the taxi company subsidies. Visible, in the face of reality, a taxi software company desires to the taxi company is the most optimal value subsidies, can achieve mutual benefit and win-win results, to alleviate the present situation of "difficult to take a taxi".

5 Conclusions

In this research, the mathematical models of subsidy programs of taxi are built. We make the urban taxi's reasonable scale model based on the balance between supply and demand, and we can get the matching degree of supply and demand of taxi resources among different time and space. Additionally, we take the difference of "Having difficulty to taking a taxi" between different cities in consideration, and reflects the situation of taxis' supply and demand in the whole country. Analyzing the balance of supply and demand, setting up the taxi operations balance model based on regions and explaining the problem using data seems to be pithier and clearer. Passengers' waiting time $W_i = \omega_i + \frac{\sum_j t_{ji}}{n_I - 1}$ offers reference for the optimization of passengers' taxi experience. the taxi's reasonable scale during morning and evening high peak period and during leisure time period in Shanghai is respectively 77555veh and 34076veh.While .the taxi's reasonable scale in Harbin is 9491veh.Through the formula $s_1^* - d_1 = \frac{A-d_2-d_1}{2}$ we can acquire that the optimization of which taxi service platforms give taxi companies subsidies is (s_1^*, s_2^*) ,which has a certain reference for the urban taxi resource's reasonable planning.

Acknowledgments

Supported by Hebei Natural Science Foundation of China (NO. A2015209229)

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