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A Study of Prevalent Disease Ebola based on the SIR Infect Mathematics Model

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Abstract

Ebola is spreading in three countries in west Africa, we analyed the three countries's data statistical since Ebola outbreak, it is concluded that the early spread of the virus obeied population growth model, based on the geographical location of the three annotations as well as the number of cases, roughly concluded that epidemic distribution of three units in the provinces in west Africa, through the study the growth and characteristics of every province in the past cases through the use of MATLAB fitting out the number of cases and the number of new cases, transmission speed, the number of health, the new number of rehabilitation, the death toll of the relationship between the equation.Data that can be used by preliminary study late new cases, to determine the required distribution, the number of different countries in different parts of the combined with the characteristics of local transportation system will be drug delivery to site, to avoid uneven and dispensing pharmacy not timely phenomenon; At the same time, governments should strengthen the epidemic protection propaganda, strictly implement isolation and resistance increase in the spread of the epidemic.

Keywords:*On-demand Distribution; Relation Equation; The Effective Control ;Data Fitting*

1. Introduction

Infectious Disease Model has been researching for a long time [3].In 1927, Kermack and McKendrick built the famous SIR Model for spread of disease when they researching the spread regularity of London in 1655 to 1666 and an epidemic of Bombay in 1906.It played an important role of the research of infectious diseases. And then they came up with the “Threshold theory” to distinguish weather a kind of disease is epidemic or not, laying a solid foundation to the Epidemic Dynamics. Herbert and others have researched a infectious disease model with quarantine item. When entering constants, the model will provide SIQS and SIQR model with bilinear and standard occurrence rate, which discuss the stability of Balance Point.Abroad, in 1760 Bernoull had studied the spread of smallpox problem in mathematical model, 1906 Hamer repeated in order to make the measles popular reasons clearly, structured and analyzed a discrete time model, Below a critical value, the prevalence of malaria will be under control. 1927 Kermack and Mckendrick in 1665 ~ 1666 in order to research the epidemic law of bubonic plague in London and epidemic law of the plague in mumbai in 1906, constructed the famous SIR bin model, throughout the year .SIS bin model is put forward in 1932. Analysis of the established mathematical model, put forward the disease is popular "threshold theory", which laid a foundation for research on mathematics model of infectious diseases. Nearly, over the past 20 years, the international research progress on the dynamics of infectious diseases is extremely rapid, a large number of mathematical models are used to analyze all kinds of infectious diseases, the mathematical model related to contact transmission, vertical transmission , the spread of insect-borne different mode of transmission, such as, also considering the incubation period of the disease, isolation are discussed , inoculation and cross infection, age structure, space, factors related to migration and proliferation.

Class, remove mathematical model related to contact transmission, such as vertical transmission, the spread of insect-borne spread in different ways, also consider the incubation period of the disease, to explore prevention and cross infection isolation, vaccination, age structure, space, factors related to migration and proliferation

In our Model, we build an epidemic model that considers treatment and quarantine measures. After studying it, we conclude that governments should pay attention to propaganda for the Ebola epidemic, strengthen the treatment measures and implement the policy of isolation strictly. Only by that way can the Ebola virus epidemic of three West Africa countries be controlled.

Through the analyzing, the result shows that the drug-dependence has been improved by the bad habits (washing hands in one basin, eating together, etc.), unhealthy (burial) custom and poverty (water-lacking, semi-primitive living habits). It's a terrible news because unless they change it efficiently, or they would still suffering the higher risk of being infected by other kinds of epidemics

2. Model Building

This paper tries to build a model with treatment and quarantine in the case that we already have the medicine to cure Ebola virus disease and the vaccine to prevent it. In our model we effectively controlled the Ebola virus epidemic of Guinea, Liberia and Sierra Leone, and then we successfully stop the Ebola virus epidemic. At the end, we eradicating Ebola virus of West Africa, the three countries of West Africa don't exist in the list of affected areas of WHO.

The following model is established based on the above analysis,

$$\begin{cases} S' = \Lambda - \beta I_u S - \mu S + \delta(1-p)\sigma(1-q) I_u + u_2 I_t, \\ E' = \beta I_u - (\mu+\sigma)E, \\ I_u{}' = \delta p E - (\mu+\sigma+u_1) I_u, \\ I_t' = \sigma q I_u + u_1 I_u - (\mu+u_2) I_t. \end{cases} \quad (1)$$

We assume that all new import enter into S. From the model we can see that when $t \geq 0$, all non-negative initial values has non-negative solutions. We use reproducing matrix[4] to get the basic reproduction number:

$$R_0 = \frac{\beta \Lambda \delta P}{\mu(\mu+\delta)(\mu+\delta+u_1)}. \quad (2)$$

It is easy to get the disease free equilibrium:

$$P_0 = (S_0, E_0, I_{u0}, I_{t0}) = \left(\frac{\Lambda}{\mu}, 0, 0, 0\right) \quad (3)$$

In regard to system, because when $t \to \infty$, $S + E + I_u + I_t \to \frac{\Lambda}{\mu}$, we use this limit equation and set $I_t = \frac{\Lambda}{\mu} - S - E - I_u$, so we can discuss the system (I') only about S, E and I_u in the invariant set Ω, and

$$\Omega = \left\{ (S, E, I_u) \middle| S, E, I_u \geq 0, S + E + I_u \leq \frac{\Lambda}{\mu} \right\}. \quad (4)$$

In the disease free equilibrium, the Jacobian matrix of the system (I') is

$$J = \begin{pmatrix} -\beta I_u - \mu - u_2 & \delta(1-p) - u_2 & -\beta S + \sigma(1-q) - u_2 \\ \beta I_u & -(\mu+\delta) & \beta S \\ 0 & \delta p & -(\mu+\sigma+u_1) \end{pmatrix}. \quad (5)$$

At first we analyses the stability of the disease free equilibrium. Set $\lambda_1, \lambda_2, \lambda_3$ as Jacobian matrix's three eigenvalues at P_0, we get that $\lambda_1 = -\mu - u_2$, and

$$\lambda_2 + \lambda_3 = -(\mu+\delta) - (\mu+\delta+u_1) < 0,$$

$$\lambda_2 \lambda_3 = (\mu+\delta)(\mu+\sigma+u_1)(1-R_0),$$

The three eigenvalues are negative, and at this time P_0 has local asymptotic stability. However, when $R_0 > 1$, there will be positive eigenvalues, P_0 is not stable. So we draw the conclusion that when $R_0 < 1$, the disease free equilibrium, P_0, has global stability.

We set former constant function, u_1 and u_2, as a function about treatment of time variable ,t. We suppose that $0 \leq u_i \leq 1$, and $u_i\ (i=1,2)$ is measurable function.

In order to minimize the quantity of patients infected by Ebola virus and minimize the social cost of the countries where the Ebola virus epidemic broke out in a certain time interval, we set our objective function as

$$J(u_1, u_2) = \int_{t_0}^{t_f} \left[E(t) + I_u(t) + I_t(t) + \frac{B_1}{2} u_1^2(t) + \frac{B_2}{2} u_2^2(t) \right] dt \quad (6)$$

Because the integrand of the objective function is convex about $u_i\ (i=1,2)$ and the right parts of the system(0) is bounded, we can see that u_1^* and u_2^* which optimize the objective function exist, and making that

$$\Gamma = \left\{ (u_1, u_2) \in L^1(t_0, t_f) \middle| 0 \leq u_1 \leq 1, i = 1, 2 \right\} \quad (7)$$

For the purpose of getting optimal solution, we use Lagrange multiplier. We make Lagrangian function: $L(S, E, I_u, I_t)$ of system:

$$L(S, E, I_u, I_t) = E(t) + I_u(t) + I_t(t) + \frac{B_1}{2} u_1^2(t) + \frac{B_2}{2} u_2^2(t) \quad (8)$$

And we define Hamiltonian function of this problem as H:

$$H = L(S,E,I_u,I_t) + \sum_{i=1}^{4} \lambda_i f_i = L(S,E,I_u,I_t) + \lambda_1\left[\Lambda - \beta I_u S - \mu S + \sigma(1-q)I_u + \delta(1-p)E + u_2(t)I_t\right] + \lambda_2\left[\beta I_u S - (\mu+\delta)E\right] + \lambda_3\left[\delta pE - (\mu+\delta+u_1(t))I_u\right] + \lambda_4\left[\sigma q I_u + u_1(t)I_u - (\mu+u_2(t))I_t\right] \quad (9)$$

In the function, $f_i\,(i=1,2,3,4)$ represents the right parts of system, $\lambda_i\,(i=1,2,3,4)$ is adjoint variable. Since $0 \le u_i \le 1\,(i=1,2)$, there will be three cases when considering control space Γ :

$(1)\, u_1^* = 0, if\ \frac{\partial H}{\partial u_1} < 0;$

$(2)\, 0 < u_1^* < 1, if\ \frac{\partial H}{\partial} = 0;$

$(3)\, u_1^* = 1, if\ \frac{\partial H}{\partial} > 0.$

So it is with $u_2^*(t)$ .So we can see that

$$u_1^* = \max\left\{\min\left\{\frac{(\lambda_3-\lambda_4)I_u^*}{B_1},1\right\},0\right\}, \quad u_2^* = \max\left\{\min\left\{\frac{(\lambda_4-\lambda_1)I_u^*}{B_2},1\right\},0\right\}. \quad (10)$$

It can be seen that u_1^* and u_2^* can be calculated. We can use our conclusion to minimize the quantity of patients infected by Ebola virus and minimize the social cost of the countries where the Ebola virus epidemic broke out.

3.Results & Analysis

By assign values to the model according to the actual situation, we can see the trends of $R_0 < 1$ and $R_0 > 1$:

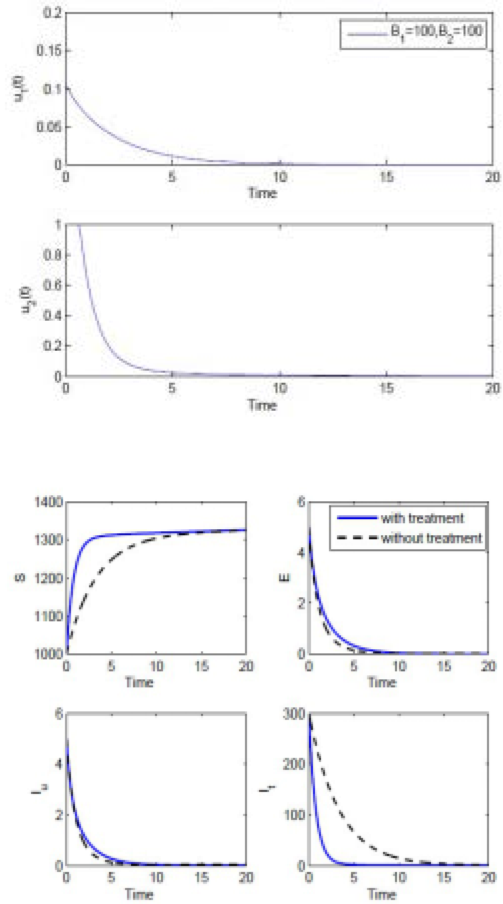
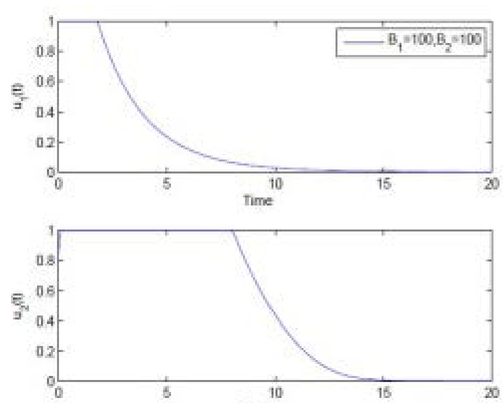


Figure 1 Control Function changes over time and its impact on the status of an epidemic when $R_0 < 1$.



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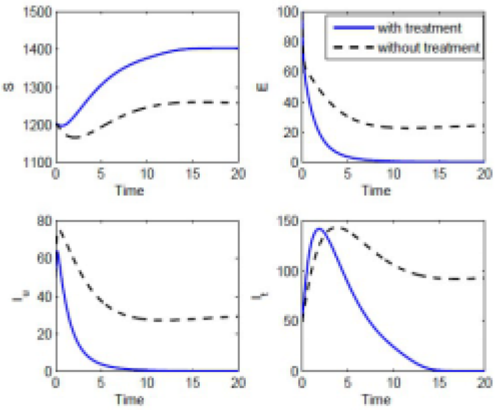

Figure 2 Control Function changes over time and its impact on the status of an epidemic when $R_0 > 1$

When $R_0 < 1$, the epidemic will tend to die no matter whether we take treatment and quarantine measures or not. But the extinction of epidemic will be sooner if we take treatment and quarantine measures. When $R_0 > 1$, if we don't take treatment and quarantine measures, the epidemic will continue to develop and become more and more serious. In this situation, only taking treatment and quarantine measures can we control the outbreak and eventually eradicating Ebola.
Coming down to West Africa, first of all we should make our best to have a situation that $R_0 < 1$. Through the study of the parameter of R_0, if patients who are already infected take screening and treatment measures proactively, the value of R_0 declines significantly.

If the situation that $R_0 < 1$ can't be reached, correspond to Figure 8, only by strengthening the treatment measures and strictly implementing the policy of isolation can we contain the outbreak and at last eradicate Ebola.
By the analysis above, we draw the conclusion. First of all, The Government should pay attention to propaganda for the Ebola epidemic, introduce the seriousness and harmfulness of the Ebola outbreak to people, as well as the importance of active treatment and screening in order to increase the proportion of patients who take screening and treatment measures proactively to reduce the value of R_0.So the probability that the epidemic can be strained will be increased. Subsequently, Governments should strengthen the treatment measures, implement the policy of isolation strictly ,optimize distribution scheme of drugs and vaccine to increase the intensity of treatment as well as take quarantine measures to eradicate the situation that infected persons escape from the i quarantine area and continue to infect others.

Only in this way, the Ebola virus epidemic of three West Africa countries can be controlled under a certain scope, and ultimately we can achieve the goal of eradication of Ebola.

4. Conclusions

Our suggestions are reasonable and objective, fully considered the common situation and the living atmosphere in West Africa. The solution is a prediction of the whole area, which has got a good result fitting well the information that are already known.However, the only problem troubles most is the lack of data, and the credibility about data which we have got. Something accord that in some days there are less total amount of people dead in Ebola than the day before, which confused us much in some time point. About the solution, there are still something have not been so reasonable in West Africa, and in spite of that, the model works well as if nothing happened.

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