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Posinormal and * Paranormal Composition Operators on the Fock Space

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Abstract

In this paper, posinormal, * paranormal, quasi posinormal and qusi * paranormal composition operators on Fock space are charecterised.

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1. INTRODUCTION

Composition operators on spaces of analytic functions have been studied in many settings. Much has been written about the properties of these operators on the Hardy, Bergman, and Bloch spaces on the unit disk in the complex plane or unit ball in C^n.(see, for example [2],[3] and [7]). Already the paper [1] discussed bounded and compact composition operator on Fock space. In paper [6]. We have discussed some classes of composition operators on the Fock space. In this paper posinormal, * paranormal, quasi posinormal and qusi * paranormal composition operators on Fock space are charecterised.

The Fock space F is the Hilbert space of all holomorphic functions on C^n with inner product

$$\langle f,g\rangle = \frac{1}{(2\pi)^n}\int_{C^n} f(z)\overline{g(z)}e^{\frac{-1}{2}|z|^2}\,dv(z)$$

Where v denotes Lebesgue measure on C^n. (refer [1] and [7]).

Let $e_n(z)=\sqrt{\frac{1}{n!}}\,z^n$ for a positive integer n. Then the $\{e_n\}$ forms an orthonormal basis for F. Since each point evaluation is a bounded linear functional on F, for $w\in C^n$ there exists a unique function $k_w \in F$ such that $\langle f, k_w \rangle = f(w)$ which holds for all $f \in F$. The reproducing kernel functions for the Fock space are given by $k_w(z) = e^{\langle z,w \rangle /2}$ where $\langle z, w \rangle = \sum_1^n z_j \overline{w_j}$. Note that the substitution $f = k_w$ into the reproducing formula $\langle f, k_w \rangle = f(w)$ which holds for all $f \in F$ and $w \in C^n$ leads to the identity $\|k_w\| = \exp(|w|^2/4)$. Throughout this paper we use $f = k_w$ is the reproducing kernel function for the Fock space F and $k_0 = 1$ be the point evaluation on F. (Refer [3], [4], [5] and [7])

For a given holomorphic mapping $\varphi : C^n \to C^n$, the composition operator $C_\varphi : F \to F$ is given by $C_\varphi(f) = f \circ \varphi$ The paper [4] already proved that if the operator C_φ is bounded ,then φ must be of the form $\varphi(z) = Az + B$ where A is an $n \times n$ matrix and B is an $n \times 1$ vector . Furthermore it will follow that $\|A\| \le 1$ for bounded C_φ and that B will restricted by the condition that $\langle A\varsigma, B \rangle = 0$ for any ς in C^n with $|A\varsigma| = |\varsigma|$. In paper [1] Theorem 1 shows that if C_φ is compact, then $\|A\| < 1$ with no restriction on B.

2. PRILIMINARIES

Let F be a Fock space and C_φ be a composition operator on F . Then C_φ is posinormal iff $C_\varphi C_\varphi^* \le c^2 C_\varphi^* C_\varphi$, quasiposinormal iff $(C_\varphi C_\phi^*)^2 \le c^2 C_\phi^{*2} C_\phi^2$, for some c > 0 ,* paranormal iff $C_\varphi^{*^2} C_\varphi^2 + 2\lambda C_\varphi C_\varphi^* + \lambda^2 \ge 0$ for $\lambda \ne 0$, quasi * paranormal if and only if $C_\varphi^{*^3} C_\varphi^3 + 2\lambda (C_\varphi^* C_\varphi)^2 + \lambda^2 C_\varphi^* C_\varphi \ge 0, \lambda \in R,$ C_φ is of (M,k) class if $C_\varphi^{*k} C_\varphi^k \ge (C_\varphi^* C_\varphi)^k$ for $k \ge 2$. It is known that the $(M,2)$ class coincides with the class of quasi hyponormal operators. But the class of hyponormal operators does not coincide with the (M,k) class for any k. However, if we define a class $(M,k)^*$ as $C_\varphi : C_\varphi^{*k} C_\varphi^k \ge (C_\varphi C_\varphi^*)^k$.

3. MAIN RESULTS:

Theorem 3.1

If C_φ on F is posinormal if and only if $M_{k_B \circ \varphi} C_{\tau \circ \varphi} \le c^2 M_{k_B} C_{\varphi \circ \tau}$

Proof: C_φ is posinormal if and only if $C_\varphi C_\varphi^* \le c^2 C_\varphi^* C_\varphi$

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$$C_\varphi C_\varphi^* - c^2 C_\varphi^* C_\varphi \leq 0 \text{ for c} > 0$$

hence $M_{k_B \circ \varphi} C_{\tau \circ \varphi} \leq c^2 M_{k_B} C_{\varphi \circ \tau}$

Theorem 3.2

If C_φ^* on F is posinormal if and only if $M_{k_B} C_{\varphi\circ\tau} \leq c^2 M_{k_B \circ\varphi} C_{\tau\circ\varphi}$

Proof: C_φ^* is posinormal iff $C_\varphi^* C_\varphi - c^2 C_\varphi C_\varphi^* \leq 0$ for c > 0

$$M_{k_B} C_{\varphi\circ\tau} - c^2 M_{k_B\circ\varphi} C_{\tau\circ\varphi} \leq 0$$

We have $M_{k_B} C_{\varphi\circ\tau} \leq c^2 M_{k_B\circ\varphi} C_{\tau\circ\varphi}$

Corollary 3.3

From theorems 3.1 and 3.2 and if c = 1 we get C_φ and C_φ^* are posinormal operators if and only if C_φ is normal.

Theorem 3.4

C_φ on F is quasiposinormal if and only if $\left(M_{k_B\circ\varphi} C_{\tau\circ\varphi}\right)^2 \leq c^2 M_{k_B} M_{k_B o \tau} C_{\phi^{(2)} o \tau^{(2)}}$ where $\varphi^{(2)} = \varphi\circ\varphi$, $\tau^{(2)} = \tau\circ\tau$.

Proof: C_φ is quasiposinormal if $(C_\varphi C_\phi^*)^2 \leq c^2 C_\phi^{*2} C_\phi^2$ for some c > 0.

Now $\left(C_\varphi C_\varphi^*\right)^2 = \left(C_\varphi M_{k_B} C_\tau\right)^2 = \left(M_{k_B\circ\varphi} C_{\tau\circ\varphi}\right)^2$

And

$$\begin{aligned} C_\phi^{*2} C_\phi^2 &= C_\phi^* (M_{k_B} C_{\phi o \tau}) C_\phi \\ &= C_\phi^* M_{k_B} C_{\phi o \phi o \tau} \\ &= M_{k_B} C_\tau M_{k_B} C_{\phi o \phi o \tau} \\ &= M_{k_B} M_{k_B o \tau} C_{\phi^{(2)} o \tau^{(2)}} \end{aligned}$$

So C_φ is quasiposinormal iff

$$\left(M_{k_B\circ\varphi} C_{\tau\circ\varphi}\right)^2 \leq c^2 M_{k_B} M_{k_B o\tau} C_{\phi^{(2)} o \tau^{(2)}}$$

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Theorem 3.5

C_φ on F is * paranormal if and only if $\left(M_{k_B\circ\varphi}C_{\tau\circ\varphi}\right)^2 = M_{k_B\circ\varphi\circ\tau}\,C_{(\varphi\circ\tau)^2}$

Proof: C_φ is * paranormal if and only if

$$C_\varphi^{*^2}C_\varphi^{\;2} + 2\lambda C_\varphi C_\varphi^{\;*} + \lambda^2 \geq 0 \text{ for } \lambda \succ 0$$

$$M_{k_B}M_{k_B\circ\tau}C_{\varphi^{(2)}\circ\tau^{(2)}} + 2\lambda M_{k_B\circ\varphi}C_{\tau\circ\varphi} + \lambda^2 \geq 0$$

By theorem (3.4)

$$\left(M_{k_B\circ\varphi}C_{\tau\circ\varphi}\right)^2 \leq M_{k_B}M_{k_B\circ\tau}C_{\varphi^{(2)}\circ\tau^{(2)}}$$

which reduces to

$$M_{k_B}M_{k_B\circ\tau}C_{\varphi^{(2)}\circ\tau^{(2)}} \geq M_{k_B\circ\varphi\circ\tau}C_{(\varphi\circ\tau)^2}$$

Hence we have

$$\left(M_{k_B\circ\varphi}C_{\tau\circ\varphi}\right)^2 = M_{k_B\circ\varphi\circ\tau}\,C_{(\varphi\circ\tau)^2}.$$

Theorem 3.6

C_φ on F is quasi * paranormal if and only if

$$M_{k_B\circ\tau^{(3)}}C_{\varphi^{(3)}\circ\tau^{(3)}} + 2\lambda M_{k_B\circ\varphi\circ\tau}C_{(\varphi\circ\tau)}^{\;\;2} + \lambda^2 C_{\varphi\circ\tau} \geq 0$$

Proof :

C_φ is quasi * paranormal if and only if

$$C_\varphi^{*^3}C_\varphi^{\;3} + 2\lambda\left(C_\varphi^{\;*}C_\varphi\right)^2 + \lambda^2 C_\varphi^{\;*}C_\varphi \geq 0, \lambda \in R$$

Using theorem(3.4) we have

$$C_\varphi^{*2}C_\varphi^{\;2} = M_{k_B}M_{k_B o\tau}C_{\phi^{(2)}o\tau^{(2)}}$$

Now

$$C_\varphi^{*3}C_\varphi^{\;3} = C_\varphi^{\;*}\left(C_\varphi^{*^2}C_\varphi^{\;2}\right)C_\varphi$$

$$= M_{k_B}C_\tau\left(M_{k_B}M_{k_B\circ\tau}C_{\varphi^{(2)}\circ\tau^{(2)}}\right)C_\varphi$$

$$= M_{k_B}M_{k_B\circ\tau^{(3)}}C_{\varphi^{(3)}\circ\tau^{(3)}}$$

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Since $C_{\varphi}^{*^3} C_{\varphi}^{\ 3} + 2\lambda\left(C_{\varphi}^{\ *} C_{\varphi}\right)^2 + \lambda^2 C_{\varphi}^{\ *} C_{\varphi} \geq 0, \lambda \in R$

$$M_{k_B} M_{k_B \circ \tau^{(3)}} C_{\varphi^{(3)} \circ \tau^{(3)}} + 2\lambda M_{k_B} M_{k_B \circ \varphi \circ \tau} C_{(\varphi \circ \tau)}^{\ 2} + \lambda^2 M_{k_B} C_{\varphi \circ \tau} \geq 0$$

Hence $M_{k_B \circ \tau^{(3)}} C_{\varphi^{(3)} \circ \tau^{(3)}} + 2\lambda M_{k_B \circ \varphi \circ \tau} C_{(\varphi \circ \tau)}^{\ 2} + \lambda^2 C_{\varphi \circ \tau} \geq 0$

Theorem 3.7

C_{φ} on F is class $(M,2)^*$ if and only if C_{φ} is * paranormal.

Proof

If C_{φ} on F is class $(M,2)^*$ then $C_{\varphi}^{*2} C_{\varphi}^{\ 2} \geq \left(C_{\varphi} C_{\varphi}^{\ *}\right)^2$.

Now $C_{\varphi}^{*2} C_{\varphi}^{\ 2} = M_{k_B} M_{k_B o \tau} C_{\phi^{(2)} o \tau^{(2)}}$

$$\left(C_{\varphi} C_{\varphi}^{\ *}\right)^2 = \left(C_{\varphi} M_{k_B} C_{\tau}\right)^2 = \left(M_{k_B \circ \varphi} C_{\tau \circ \varphi}\right)^2$$

Hence $M_{k_B} M_{k_B o \tau} C_{\phi^{(2)} o \tau^{(2)}} \geq \left(M_{k_B \circ \varphi} C_{\tau \circ \varphi}\right)^2$

By theorem (3.5) which reduces to * paranormal.

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