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# LA – Semirings Satisfying The Identity a.b = a + b + 1

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This paper contains some results on LA– Semirings involving two variables. We consider a LA - Semiring (S, +, .) satisfying the identity a.b = a + b + 1, for all a, b inS then it is proved that (S, +,.) is commutative, medial and permutable. For this LA - Semiring we also determine the additive and multiplitive structures. It is proved that if (S, +,.) is a LA – semiring in which (S, .) is a regular semigroup satisfying the identity a.b = a + b + 1 for all a,b in S then (S, +) is regular , left (right) regular semigroup and completely regular semigroup. Again we consider the LA-Semiring satisfying the same identity with (S, +) be regular semigroup then (S,.) is also regular semigroup, left (right) regular semigroup and completely regular semigroup. It is also proved that in this LA- Semiring if (S,.) is separative then (S, +) is weakly separative, quasi – seperative and separative.

**Introduction** : LA- semirings are naturally developed by the concepts of LA-semigroup. The concepts of LA – semigroup was introduced by M.A. Kazim andM. Naseeruddin [1] in 1972. Since then lot of papers has been presented on LA - semigroups like. Mushtaq, Q and Khan [02], M Mustaq, Q. and yousuf, S.M. [03], Qaiser Mushtaq[04].

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In this paper we describe some results on LA- semirings the identities of two variables. The motivation to prove the theorems in this paper is due to the results of P. Srinivasulu reddy and G. Shobhalatha [5].

Keywords : LA-Semigroup, LA-semirings, permutable, Separative, quasi separative, regular semigroup, completely regular semigroup, left (right) semigroup.

**Preliminaries** : **Definition 1:** A left almost semigroup (LA-semigroup) or Abel-Grassmanns groupoid (AG-groupoid) is a groupoid S with left invertive law: (ab)c = (cb)a for all $a, b, c \in S$

Example:- Let S = {a, b, c} the following multiplication table shows that S is a LA-Semigroup

| . | a | b | c |
|---|---|---|---|
| a | a | a | a |
| b | a | a | c |
| c | a | a | a |

**Definition 2**: A semiring (S, +, .) is said to be LA-Semiring if

1. (S, +) is a LA-Semigroup
2. (S, .) is a LA-Semigroup  3. Distributive Laws hold in S.

**Example :** Let S = {a, b, c} is a mono semiring with the following tables 1, 2 which is a LA-Semiring

| + | a | b | c |
|---|---|---|---|
| a | a | a | a |
| b | a | a | c |
| c | a | a | a |

(1)

| . | a | b | c |
|---|---|---|---|
| a | a | a | a |
| b | a | a | c |
| c | a | a | a |

(2)

**Definition 3:** An element a of a semigroup (S,.) is left (right) regular if there exists an element x in S such that $xa^2 = a$( $a^2 x = a$)**.**

**Definition 4 :** An element a of a semigroup (S,.) is said to be regular if there exist x in S such that a x a = a**.**

**Definition 5:** A semigroup S is called left(right) permutable if every a,b,c in S, abc = bac(abc = acb)**.**

**Definition 6:** A semigroup S is called permutable if it is both left and right permutable**.**

**Definition 7:** A semigroup is called weakly separative if for any x,y $\in$S $x^2$ =xy= $y^2$ implies x=y**.**

**Definition 8:** A semigroup S is called quasi separative if $x^2$ =xy=yx= $y^2$ implies x=y for all x,y in S**.**

**Definition 9:** A semigroup S is called separative $\left.\begin{matrix} x^2 = xy \\ y^2 = yx \end{matrix}\right\} \Rightarrow x = y$

and $\left.\begin{matrix} x^2 = yx \\ y^2 = yx \end{matrix}\right\} \Rightarrow x = y$ for all x,y in S.

**Definition 10:** A semigroup S is called completely regular semigroup if it is left and right regular semi group satisfies the identity ax = xa for any a,x $\in$S**.**

**Theorem 1:** Let (S, +, .) be a LA- semiring with satisfying the identity a.b = a+b+1 $\forall a,b \in S$ then (S, +, .) is

1) Commutative

2) Medial

3) Permutable

**Proof:** Let (S, +, .) be a LA-semi ring with the identity a.b = a+b+1,

$\forall a,b \in S$

Consider a+b = a.1+b

$= (a+1+1)+b$

$= 1+(1+a)+b$

$= b+(1+a+1)$

$= b+1.a$

$= b+a$

$\therefore\ a+b = b+a$

Again $a.b = a+b+1$

$= a+b.1+1$

$= a+(b+1+1)+1$

$= (a+(1+1)+b)+1$

$= b+(1+1)+a+1$

$= b+(1+1+a)+1$

$= b+a+1+1+1$

$= b+a+1.1$

$= b+a+1$

$= b.a$

$\therefore$ a.b = b.a

Hence (S, .) is commutative

Therefore (S, +, .) is a commutative LA-semiring.

Since (S, +, .) is commutative. It is easy to prove (S, +, .) is medial and permutable.

**Theorem 2:** Let (S, +, . ) be a LA-Semiring in which (S, .) is a regular semigroup satisfying the identity a.b = a + b + 1, $\forall a,b \in S$ then (S, +) is

i) regular semigroup

ii) left (right) regular semigroup

iii) completely regular semigroup

**Proof:** Let (S, +, .) be a LA-Semiring in which (S, .) be a regular semigroup and satisfying the identity a.b = a + b + 1, $\forall a,b \in S$ where 'l' is the multiplicative identity of S.

1) Since (S, .) is regular, for any a $\in$ S, therexist x $\in$ S such that

a = axa

= (a.x)a

= (a+x+1)a

= a+x+1+a+1 $(\because a.b = a + b + 1)$

= a+x+(1+a+1)

= a+x+1.a

a = a+x+a

$\therefore$ a is regular element in (S, +)

Hence (S, +) is a regular semigroup

2) Let $a \in S$. Since (S, .) is regular semigroup and there exists an element x in (S, .) such that a x a = a

Let a + a + x = 1.a+a+x.1

= a+a+x+1+1

= a+(a+x+1)+1

= a+a.x+1 $(\because a.b = a + b + 1)$

= a+ax+1

= a ax

a+a+x= $a^2$x

= a (Since (S, .) is left regular

$\Rightarrow$ a is left regular element in (S, +) $\Rightarrow$ (S, +) is left regular

Similarly we can prove that (S, +) is right regular semigroup.

3) Since (S, +) and (S, .) are regular and left (right)regular semigroups then a+x+a = a = a+a+x = x+a+a

So a+x+a = a+a+x

x+(a+x+a) = x+(a+a+x)

x+a = (x+a+a)+x

x+a = a+x

$\therefore$ (S, +) is completely regular semigroup

**Theorem 3:** Let (S, +, .) be a LA-Semiring and (S, +) be a regular semigroup satisfying the identity a.b = a+b+1 for all $a, b \in S$ then (S, .) is

1. regular semigroup

2. left (right) regular semigroup

3. completely regular semigroup

**Proof :** Let (S, +, .) be an LA-semiring and (S, +) be the regular semigroup satisfying the identity a.b = a+b+1 for all $a, b \in S$

i) Since (S, +) is a regular semigroup,for any a $\in$ S therexist x $\in$ S such that a = a+x+a

consider a = a+x+a

= a+x+a.1

= a+(x+a+1)+1

= a+(x.a)+1

a = axa

$\therefore$ a is a regular element of (S, .)

Hence (S, .) is regular semigroup

(ii) Since (S, +) is regular semigroup

a = a+x+a

a = a+x+a.1

= a+x+a+1+1 $(\because a.b = a + b + 1)$

= a+(x+a+1)+1

= a+x.a+1

= a+ax+1 ($\because$ (S, .) is commutative by th. 3.1.3)

$= a^2x$

$\therefore$ a is left (right) regular element of (S, .)

Similarly we can prove that (S, .) is right regular

Therefore (S, .) is a left (right) regular semigroup.

iii) Since (S, .) is regular and left (right) regular semigroup

$axa = xa^2$

$axax = xa^2x$

$(axa)x = x(a^2x)$

$ax = xa$

$\therefore$ (S, .) is a completely regular semigroup

**Theorem 4:** Let (S, +, .) be a LA-Semiring. If (S, .) is separative satisfying the identity $a.b = a + b + 1\ \forall a, b \in S.$ then (S, +) is

(a) Weakly separative

(b) Quasi – separative

(c) Separative.

**Proof:** Let (S, +, .) be a semiring satisfies the identity $a.b = a + b + 1\ \forall a, b \in S.$

where 1 is the multiplicative identity of S.

1. To prove that (S, +) is weakly separative

i.e., $a+a=a+b=b+b \Rightarrow a=b$

Let a + a = a + b similary a + b = b + b

Adding 1 on bothsides a + b + 1 = b + b + 1

a + a + 1 = a + b + 1 $\Rightarrow a.b=b.b$

a.a = a.b $\Rightarrow ab=b^2$

$a^2 = ab$ …. (1) $ba = b^2$ ….. (2)

from (1) and (2) $a^2 = ab$, $ba = b^2$

since (S, .) is separative we have $a^2=ab, b^2=ba, \Rightarrow a=b$

$a+a=a+b=b+b \Rightarrow a=b$

$\therefore$ (S, + ) is weakly separative

2. To prove that (S, +) is quasi separative

i.e., $a+a=a+b=b+a=b+b \Rightarrow a=b$

by the theorem 3.1.3 (S, +) is commutative

Now a+a = a+b b+a = b+b

a+a+1 = a+b+1 b+a+1 = b+b+1

$a.a = a.b$ $\quad$ $b.a = b^2$

$a^2 = ab$ $\quad$ $ba = b^2 \Rightarrow$

$\Rightarrow a+a = a+b = b+a = b+b \Rightarrow a^2 = ab, ba = b^2$

Since (S .) is separative $\Rightarrow$ **a = b**

III) Since I and II, (S, +) is clearly separative semi group similarly we can prove that.

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