

# Buckling of thin walled long steel pipes subjected to external pressure in process industries

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## Abstract

The effect of geometric imperfections on the critical external pressure of thin walled pipes subjected to uniform external pressure are numerically investigated in this paper for diameter to thickness ratio range from 20 to 50. Geometric surveys was conducted on process piping applications in order to obtain the severest geometric imperfection accepted tolerances and its influence on the allowable critical buckling pressure based on ASME B31.3 design procedure. These imperfections were then modelled using finite element analysis computer package, namely ANSYS. This paper highlights the discrepancies found in ASME B31.3 design code where this situation is pointed out and validated by numerical studies. The results are interpreted and compared to the results of the FE model to predict the allowable critical buckling pressure acting on the pipes in order to provide an adopted design proposal for very long thin pipes subjected to external pressure. For example, the results of the current work showed that the code predictions of geometric imperfections influence on critical buckling pressure are almost 30% higher than the actual value and this valve increases by the increase of diameter to thickness ratio.

**Keywords:** Cylindrical shell, Buckling, External pressure, geometrically imperfect pipes, initial out-of-roundness, jacketed piping

## 1. Introduction

The process applications such as chemical and petrochemical industries includes several applications in which the piping system are subjected to an external pressure (especially in jacketed piping application). Interconnecting pipes are widely used to transfer the fluid products such as gas, oil or steam between various equipment in the process plants due to its simple geometry. The majority of the piping systems can be considered as

a long cylindrical shells which are acts in different manner than short cylindrical shells when subjected to external pressure. There is a lot of design code discussing the buckling of long cylindrical shell such as API BUL 5C3 [17], the DNV rules for the design, construction, and inspection of offshore structures, appendix C (1977) [18] and ASME process piping B31.3 [2].

ASME process piping B31.3 is one of the most widely used codes in the design of these piping systems therefore it is very important to investigate the design procedures stated in this code due to its direct impact on the process industries . According to ASME B31.3 when the piping system is subjected to external pressure it is considered as a pressure vessel and the design procedure shall be in accordance with ASME boiler and pressure vessel code section VIII division 1 [1] which was developed using general criteria by S.B. Kendrick [19] [20], a similar procedure to ASME B&PV code used is the British code PD500 (Specification for unfired fusion welded pressure vessels) [5]. The majority of the pressure vessels are considered as a short cylinder shells [29]. However , the majority of the piping systems considered a long cylinder shells as it exceed the minimum length where the strengthening influence of the ends can be ignored which is called the “critical length” [25] [26]. The existence of such a critical length was found experimentally by Carman [27] and Stewart [28] who made many tests on long pipes and tubes. In addition the criticality of pressure vessels equipment are very high when compared with the piping system which made ASME B&PV code very conservative in its design procedure for process piping systems. In-addition there is a lot of differences between piping and pressure vessels such as the diameter to

thickness ratio ranges, The length to diameter ratio ranges and the general boundary conditions [29] [30].

ASME B&PV procedure is conservative even for a critical equipment such as pressure vessel, that is why made code committee to approved a relatively new less conservative design procedure (code case 2286) [1] developed by Miller, C.D. [21] [22]. This procedure introduces an alternative rule for determining the allowable external pressure and both procedures are still valid for use. For example the calculated critical external pressure for a very long pipe with diameter to thickness ratio 30 according to ASME B&PV is 4 MPa, when using ASME code case 2286 critical pressure value increases to 5.3 MPa and by using FEA the critical pressure value is 11.4 MPa. Although the ASME code case 2286 procedure is valid in ASME B&PV code but it is not allowed to be used in ASME B31.3 code!?. Interpretation was sent to ASME Process Piping B31.3 committee to approve ASME B&PV code case 2286 as alternative rule for determining the allowable external pressure [1].

## 2. Background

### 2.1 Buckling Theories Review

External pressure may be applied either purely radially, as "external pressure loading, (Figure 1(a)) or it may be applied all-round the cylindrical shell, that is both radially and axially (Figure 1(b)) as "external hydrostatic pressure loading. However, the first case doesn't really exist as the external radial pressure generate axial force on the pipe which convert it to the second case without applying external axial [31] .

The developed compressive stress in the shell are :

$$\text{Circumferential stress } S\theta = P \left( \frac{r}{t} \right) \dots \dots \dots (1)$$

$$\text{Tangential stress } Sx = 0.5 S\theta = 0.5 P \left( \frac{r}{t} \right) \dots \dots \dots (2)$$

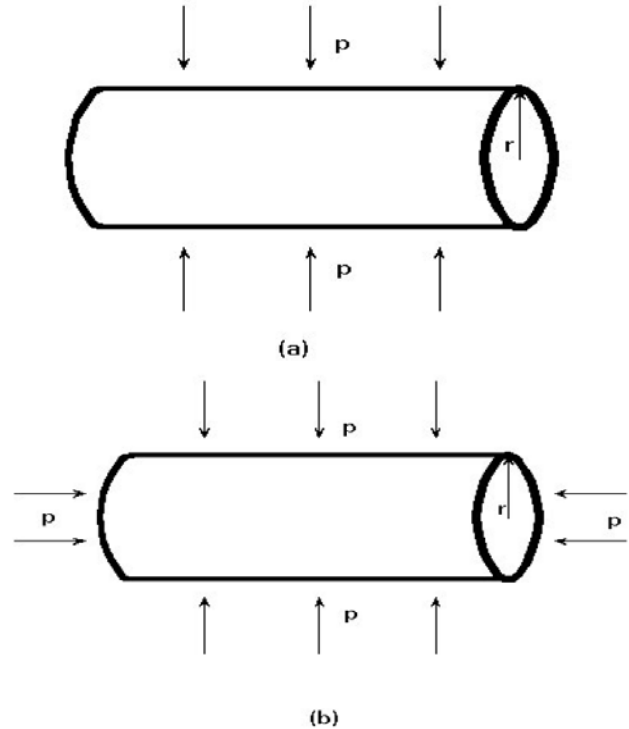


Figure 1 pressure loading.

When investigating pipes subjected to external pressure, they are generally classified to either short pipes governed by von-misses buckling equation [15] or long pipes governed by Euler buckling equation [16]. This is because the shell-buckling behavior of long cylindrical shells is considered as a column-buckling problem of a circular ring rather than a shell-buckling problem.

When an un-stiffened short or intermediate lengths, thin-walled cylindrical shell is subjected to external hydrostatic pressure, the tangential compressive stress will be developed in the shell and increases with the increases of the external pressure. When the tangential compressive stress reaches a critical value, that the pipe is no longer able to maintain its initial circular shape, the pipe failure is called elastic buckling failure. This failure is governed by von Mises elastic buckling equation [11] or the simplified formula developed by David Taylor elastic buckling equation [4] .

$$P_{cr} = \frac{E \left( \frac{t}{D} \right)}{\left( n^2 - 1 + 0.5 \left( \frac{\pi D}{L} \right)^2 \right)} * \left( \frac{1}{\left( N^2 \left( \frac{L}{\pi D} \right)^2 + 1 \right)} + \frac{t^2}{12 D^2 (1 - \nu)^2} \right) \left( n^2 - 1 + \left( \frac{\pi D}{L} \right)^2 \right)^2 \dots \dots \dots (3)$$

Windenburg and Trilling's states the buckling equation is given by equation (4) which can be considered a simplification for Von Mises buckling

equation. This formula also known as the David Taylor Model:

$$P_{cr} = \frac{2.42 E \left(\frac{t}{D}\right)^{2.5}}{(1 - \nu^2)^{0.75} \left(\frac{L}{D}\right) - 0.447 \left(\frac{t}{D}\right)^{0.5}} \dots \dots (4)$$

The denominator of Eq.4 consists of two terms the first is function of L/D and the second is function in T/D ratio. The first term is much larger than the second term , Therefore, the term  $0.447(t/d)^{1/2}$  can be neglected and by using  $\mu = 0.3$ , we can simplify Eq (4) to the form [9]:

$$P_{cr} = 2.6 E \frac{\left(\frac{t}{D}\right)^{2.5}}{\left(\frac{L}{D}\right)} \dots \dots (5)$$

This calculated critical pressure using in Eqn.5 depends on two main geometrical parameters, which are the (length to diameter ratio (L/D) & diameter to thickness ratio (D/t)), and material parameter which is (modules of elasticity (E)).

When the pipe length to diameter ratio (L/D) exceeds a certain value for each diameter to thickness ratio (D/t), the length has no effect on the maximum allowable external pressure. This value is called the critical length and it is an important parameter to distinguish whether the cylinder is long or short cylinder. There is a lot of differences between the design codes in defining the critical length [3] [4] [5] [7].

For long cylindrical shells, the buckling behavior is considered as a column buckling case with circular ring cross section [7]. The buckling equation can be derived using the Euler buckling theory, the pipe second section moment of inertia can be calculated by  $I = \frac{\pi^3}{12}$  and the buckling pressure can be

calculated from Eqn.6.

$$P_{cr} = \frac{(n^2 - 1) EI}{r^3} = \frac{(n^2 - 1) E}{12} \left(\frac{t}{r}\right)^3 \dots \dots (6)$$

However, if the pipe is longitudinally restrained, Eq. (6) should be modified by considering the Poisson's effect. The equation of critical pressure is then given by the following, Bresse, Bryan [23] [24]

$$P_{cr} = \frac{(n^2 - 1) E}{12(1 - \nu^2)} \left(\frac{t}{r}\right)^3 \dots \dots (7)$$

As for a long free pipe, since the lowest critical pressure is always produced when the number of wave  $n$  is equal to 2 [25]. Thus the buckling equation of a long pipe can be expressed by Eq. (8) [12].

$$P_{cr} = \frac{E}{4(1 - \nu^2)} \left(\frac{t}{r}\right)^3 \dots \dots (8)$$

For most metallic materials which are commonly used in piping application the Poisson ratio is approximately  $\nu = 0.3$ . Substitution by  $\nu = 0.3$

in equation (8) the Eq. becomes as follows:

$$P_{cr} = 0.275 E \left(\frac{t}{r}\right)^3 = 2.2E \left(\frac{t}{D}\right)^3 \dots \dots (9)$$

The analytical solution from equation (9) is valid as long as the load is normal to the surface.

## 2.2 Geometric Imperfection

Geometric imperfections caused during fabrication and construction are the main cause of the significant differences between critical buckling loads calculated using classical methods and experimental buckling loads. Very small imperfections can cause a substantial drop in the buckling load of the shell.

Geometric imperfections influence is presented in various codes as a reduction factor, despite the reduced imperfection sensitivity. Considerable differences exist between various codes, possibly due to the different theoretical approaches adopted, also some codes had exaggerated in evaluating the impact of Geometric imperfections on critical pressure reduction.

The most effective geometrical imperfection forms which have an effect on the critical pressure are the initial ovality and thickness eccentricity [8], Jong-hyn.B, claims that the effect of the ovality is much higher than eccentricity [10], however thickness eccentricity in piping case replaced by the fabrication thickness tolerance .

Most design codes evaluate the effect of geometric imperfections as a reduction factor by establishing limitation for the excepted initial ovality and eccentricity, for example EN 1993-1-6 code presented three reduction factors 0.75, 0.65, and 0.50 which are selected based on the shell fabrication quality class A, B, or C [4] , respectively , European convention for constructional Steelwork (ECCS) recommends a redaction factor of 0.50 [6] and DIN 18800-4 code recommends a redaction factor of 0.65 [3] .

ASME B&PV code did not present the effect of geometric imperfections as a reduction factor like other codes, but it presents a procedure with certain safety factor and establishing limitations of geometric imperfections as acceptance criteria which limiting its effect .

In the current work the commonly used materials in process piping has been surveyed to specify the

geometrical imperfections limitations, the results of this survey show the following limits [14] :

Initial ovality =  $0.01 D_o$ .

Diameter tolerance =  $\pm 0.01 D_o$

Thickness tolerance =  $\pm 0.125 T$  .

The severest case are expected when the pipe has (thickness tolerance =  $-0.125 t$ , diameter tolerance =  $+ 0.01 D_o$  and initial ovality =  $0.01 D_o$ ) all three case were studied individually and together, although the possibility for all geometric imperfections to be gathered in the same pipe is so rare.

### 3 Methodology

Analytical calculation along with mathematical FE model were performed for different D/t ratios and L/D ratios to obtain the critical pressure of cylindrical shells subjected to external pressure and the critical length of different D/t ratios. The obtained results are interpreted and compared to provide an adopted design proposal for long thin pipes.

This study was carried out on carbon steel pipes with the following characteristics: Young's modulus  $E = 200,000 \text{ N/mm}^2$ , Yield strength  $F_y = 240 \text{ N/mm}^2$  and Poisson ratio  $\nu = 0.3$

Pipes geometrical characteristics are: Pipes thickness  $t=10 \text{ mm}$  D/t ratio is investigated between (20 to 50), the selected D/t range covering all thin walled pipes of the most commonly used sizes in piping industry, the selected L/D range is (0.88 up to 20). This range covers the region in which the critical length  $L_c$  for long pipes is located .

Pipe geometrical imperfections are: Initial ovality =  $0.01 D_o$ , Thickness tolerance =  $- 0.125 t$  and Diameter tolerance =  $+0.01 D_o$ , The imperfections values are selected considering the most conservative pipe geometrical imperfections used in ASME B31.3 piping material specifications.

The investigated cases are summarized as following:

Initial ovality (OV)

Thickness tolerance (Tt)

Diameter tolerance (Dt)

Thickness tolerance +Diameter tolerance ( C1)

Initial ovality + Thickness tolerance ( C2)

Initial ovality + Diameter tolerance ( C3)

Initial ovality + Thickness tolerance + Diameter tolerance ( C4)

### 4 Analysis

#### 4.1 Analytical calculations

There are two different method used for the analytical calculation in the current work. The first one is by using the long pipe theoretical buckling

equation (9) and the second one is by using the code procedure, based on ASME B31.3 process piping code paragraph (304.1.3), which refer to the procedure in the BPV Code, Section VIII, Division 1, paragraph (UG-28 through UG-30 ) and ASME B&PV code case 2286 .

#### 4.2 FE Mathematical model

The current work was investigated using FE method, the software used is (ANSYS WORKBENCH 14), liner buckling module is used to simulate and accurately predict how the structure with certain geometrical imperfection behaves under external pressure load.

A 3D models were created using Solid Work software. These models were then imported to ANSYS. Various models were created to cover the D/t ratios concerned in this study (as discussed before in subsection 2.1.2). The length to diameter ratio of all models is exceeding its relevant critical length ratios to satisfy long pipe conditions.

The left hand side of the model was constrained along all three axes, X, Y and Z (fixed end) and the right hand side was constrained along the X and Y directions only free along the Z axis (simply-supported end) this combination of constraints best represent the most common piping support effect in process industry as shown in Figure 2 .

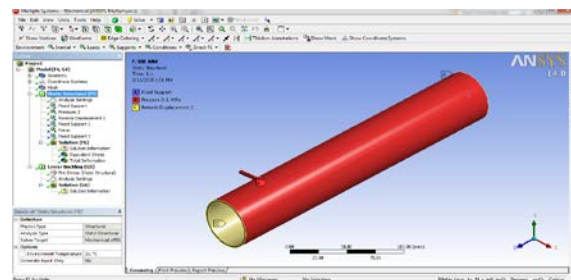
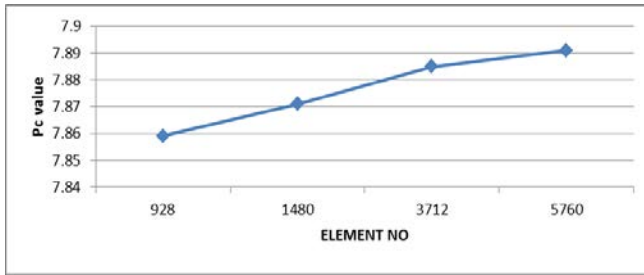


Figure. 2. FE model (simply supported pipe).

By using liner buckling module in ANSYS to solve the model after applying boundary condition of the pipe edges, a unit load (1MPa) was applied on the outer surface of the pipe. The results were directly obtained the critical pressure of the model pipe from the load multiplayer result summary. Based on the type of the simulated geometric imperfection in the model the critical pressure was obtained reflecting the effect of the imperfection.

The effect of mesh size on the accuracy of the FE model results was investigated, to insure the precision of the-analysis as shown in Fig 3.

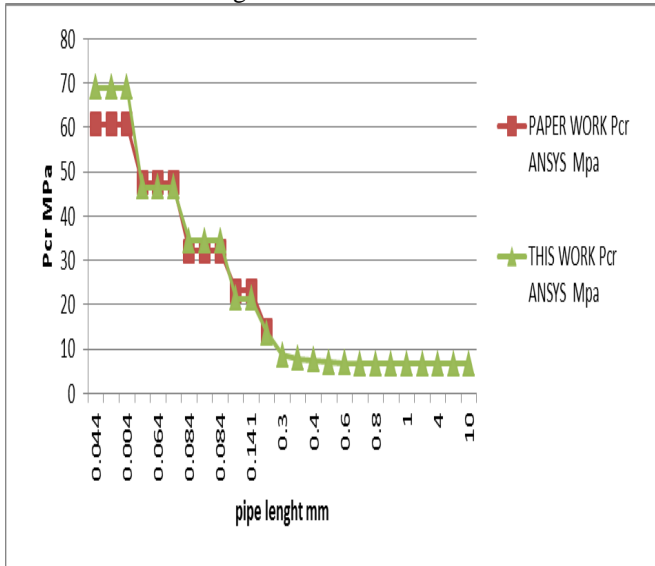


**Figure 3 FE Model elements number**

By comparing the predicted pressure values with the auto-meshed predicted pressure (average elements number not less than 3000), the error percentage was found less than 1%. Thus, it was decided that all models could be auto-meshed.

**4.2.1 ANSYS WORKBENCH verification**

The results of the current model has been verified by comparing it with previous work done in the same field which study the effect of external pressure on cylindrical shell along different lengths [9], current model show a good convergence with the previous model as shown in Fig 4.



**Figure 4 FE Model verification**

**5 Results and discussion**

**5.1 Critical Length Ratio**

The effect of changing L/D ratio on critical buckling pressure at different D/t ratio was investigated as shown in Table 1.

**5.1.1 Mathematical FE model**

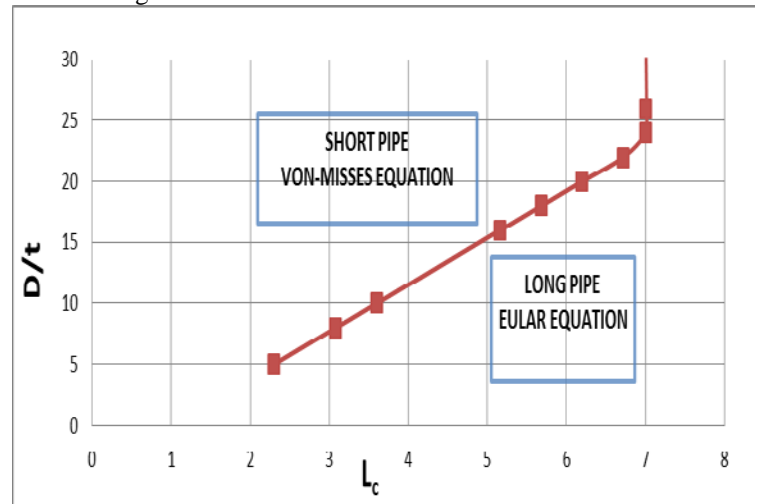
FE model was performed to demined the critical ratio (L/D)<sub>c</sub> at D/t ranges from 5 to 50 the following table was obtained

The results of Table 1 show that for each D/t ratio after certain value of L/D the critical pressure remain constant. The L/D value after which the critical pressure stabilizes is called the critical length L<sub>c</sub>. The critical length to diameter ratio at various values of D/t are selected and presented in Table 2.

**Table 1 FEA results of critical length values for each D/T ratios**

D/t	5	8	10	16	18	20	25	30	40	50
L <sub>c</sub>	1.7	2.8	4.2	6	6	6	7	7	7	7

Figure 5 shows that increasing the D/t ratio results in increase of the pipe critical length , the figure also show that after a significant value L<sub>c</sub> remain constant with the increase of the D/t ratio this value is the margin limits when the pipe is considered short or long.



**Figure 5 Design chart of long and short pipes margin limits**

**5.1.2 Analytical procedures**

The calculation of L/D ratio based on the analytical procedure of ASME B&PV code is evaluated using two methods which are the chart procedure and the code case procedure as showing in tables 3 & 4 :

**Table 2 FE Model Results**

L/D	D/t 5	D/t 8	D/t 10	D/t 16	D/t 18	D/t 20	D/t 25	D/t 30	D/t 40	D/t 50
	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)
0.88	7540	2142	1256	403	284	217	117	76.6	36.2	22
1.28	6322	1512	796	235	180	148	94	58.6	24.5	13.5
1.68	5981	1337	590	170	124	97	57	41	21.6	11.3
2.82	5838	1222	582	132	92	69	36	22.5	9.8	5.6
4.22	5838	1222	572	126	87	64	32	18.8	8	4
6	5838	1222	572	125	85	63	31.8	18.2	7.5	3.8
7	5838	1222	572	124	85	63	31.6	19	7.46	3.7
8	5838	1222	572	124	85	63	31.6	19	7.46	3.7
10	5838	1222	572	124	85	63	31.6	19	7.46	3.7

**Table 3 The value of L/D versus D/t ratio using chart procedure results**

L/D	D/t 5	D/t 8	D/t 10	D/t 16	D/t 18	D/t 20	D/t 25	D/t 30	D/t 40	D/t 50
	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)
0.88	42.0	23.0	16.3	10.2	9.1	8.100	6.500	5.300	4.000	3.150
1.28	42.0	23.0	16.3	10.0	8.8	8.000	6.250	5.100	3.790	3.000
1.68	42.0	23.0	16.3	9.8	8.5	7.650	6.000	5.000	3.620	2.840
2.82	42.0	23.0	16.3	9.7	8.4	7.350	5.500	4.450	3.220	2.500
4.22	42.0	23.0	16.3	9.7	8.4	7.250	5.380	4.200	2.830	2.170
6	42.0	23.0	16.3	9.7	8.4	7.250	5.380	4.000	2.310	1.650
7	42.0	23.0	16.3	9.7	8.4	7.250	5.380	4.000	2.300	1.410
8	42.0	23.0	16.3	9.7	8.4	7.250	5.380	4.000	2.300	1.240
10	42.0	23.0	16.3	9.7	8.4	7.250	5.380	4.000	2.300	1.170

**Table 4 The value of L/D versus D/t ratio using code case procedure results**

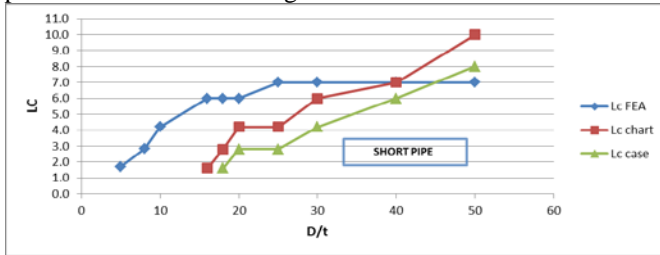
L/D	D/t 5	D/t 8	D/t 10	D/t 16	D/t 18	D/t 20	D/t 25	D/t 30	D/t 40	D/t 50
	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)	Pcr (MPa)
0.88	58.0	36.2	29.0	18.0	16.1	14.500	11.600	9.650	7.240	6.000
1.28	58.0	36.2	29.0	18.0	16.0	14.480	11.600	9.650	6.870	5.170
1.68	58.0	36.2	29.0	18.0	15.4	14.000	10.600	8.410	5.850	4.400
2.82	58.0	36.2	29.0	18.0	15.4	12.300	7.850	6.250	4.390	3.330
4.22	58.0	36.2	29.0	18.0	15.4	12.300	7.850	5.360	3.550	2.700
6	58.0	36.2	29.0	18.0	15.4	12.300	7.850	5.360	2.750	1.880
7	58.0	36.2	29.0	18.0	15.4	12.300	7.850	5.360	2.750	1.600
8	58.0	36.2	29.0	18.0	15.4	12.300	7.850	5.360	2.750	1.410
10	58.0	36.2	29.0	18.0	15.4	12.300	7.850	5.360	2.750	1.410

The value of D/t after which the critical pressure remains constant with the increase of L/D ratio using both procedure where arranged in the following table : the minimum value  $L_c$  after which the critical pressure remains constant is the critical length ( $L_c$ ).

**Table 5 Analytical results of critical length values for each D/t ratios**

D/t	5	8	10	16	18	20	25	30	40	50
$L_c$ chart	-	-	-	1.6	2.8	4.2	4.2	6	7	10
$L_c$ code case	-	-	-	-	1.6	2.8	2.8	4.2	6	8

The critical length ( $L_c$ ) calculated using the FE (Table 2), ASME code (Table.5) are plotted in figure in order to compare the difference between these methods which shows significant difference in specifying the critical length between FEA and code procedures as shown in Figure 6:



**Figure 6 comparison between mathematical & analytical results of long and short pipes margin limits**

The results of the figure show :

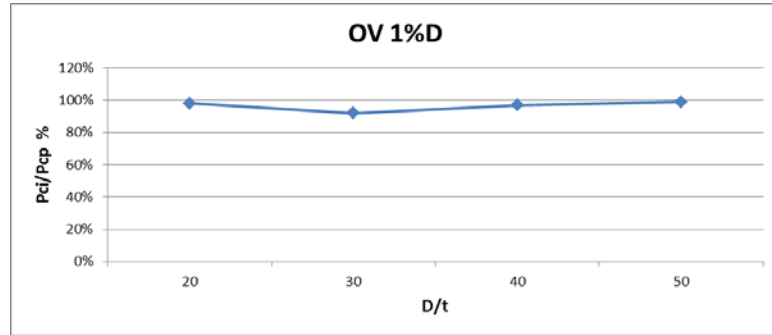
For FEA  $L_c = (L/D)_c = 0.26 D/t + 1$  where  $D/t \leq 20$  and  $L_c = 7$  where  $D/t > 20$ . The chart procedure show that for all D/T ratios  $L_c = 50$  [1]. The code case procedure show that for all D/t ratios  $L_c = 25$  [1]. That demonstrate the conservatively of the code design procedures comparing to FE result in determining the critical length ratio for each D/t ratio.

### 5.2 Geometrical imperfections analysis

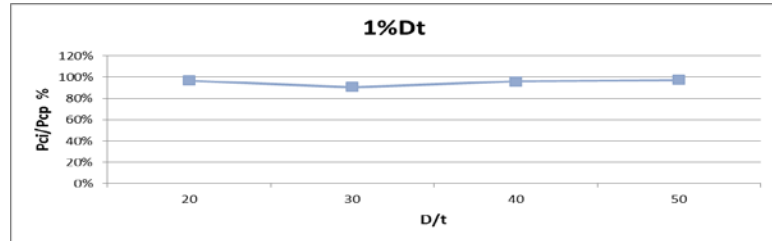
#### 5.2.1 Mathematical FE model

##### *pipes geometrical imperfections cases*

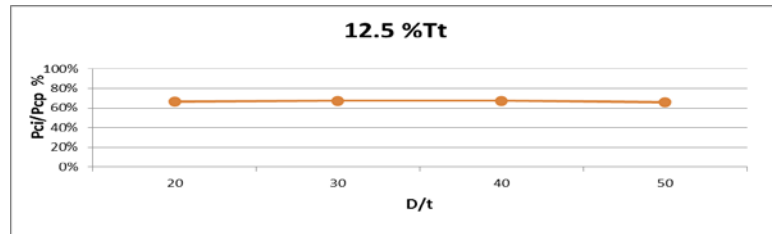
Based on geometrical imperfections cases previously mentioned, mathematical model results are evaluated and compared with perfect pipe to demonstrate the effect of each geometrical imperfections parameter were plotted individually as shown in Fig (7,8 &9).



**Figure 7 1%D ovality effect on critical pressure**



**Figure 8 Diameter tolerance effect on critical pressure**



**Figure 9 Thickness tolerance effect on critical pressure**

As shown in Figs.7 & 8 the effect of ovality and diameter tolerance on critical pressure is almost negligible. However, as shown in Fig.9 the effect of thickness tolerance on critical pressure is severe .

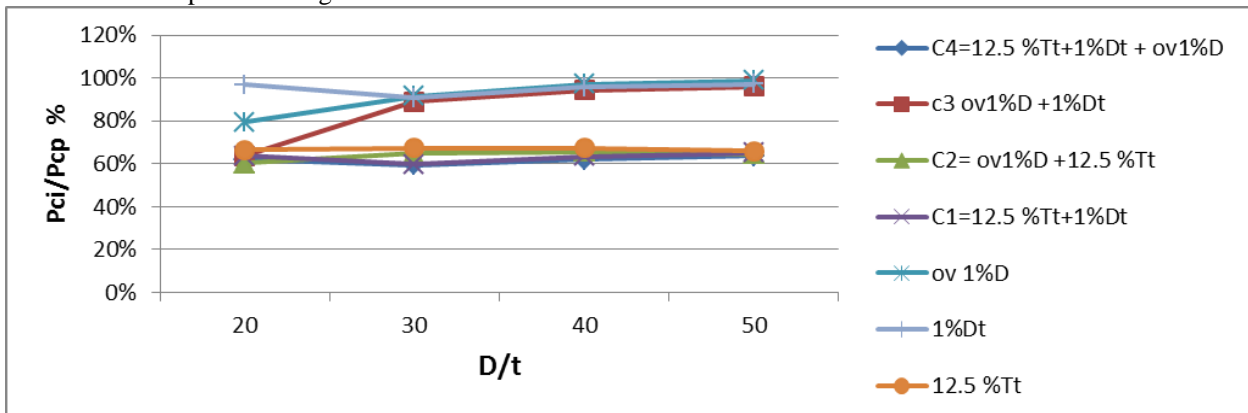
The results of the current analysis shows that the change in the thickness tolerance is the most critical geometric imperfection effect on critical pressure as it reduces the D/t ratio which severely reduce the critical pressure (refer to equation 9). However, there are a lot of ASTM material specification allow less or no thickness tolerance such as ASTM materials A672 and A358 allow only 0.3 mm less than the nominal thickness and A312 doesn't allow any thickness tolerance [14].

The effect of each case previously mentioned imperfection cases were arranged in the following table:

**Table 6 FE Model Results of geometrical imperfections cases**

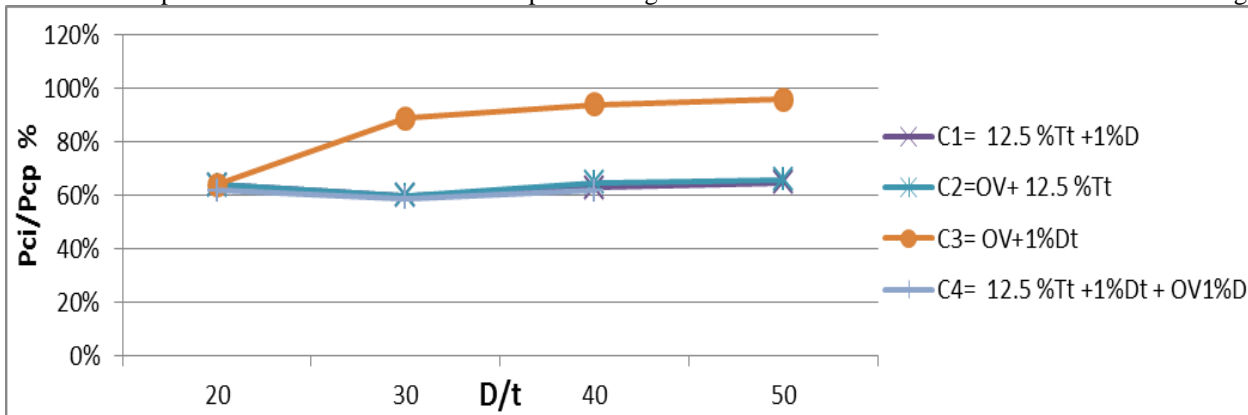
IMPERFECTION TYPE	D/t 20		D/t 30		D/t 40		D/t 50	
	P <sub>ci</sub>	P <sub>ci</sub> /P <sub>cp</sub> %	P <sub>ci</sub>	P <sub>ci</sub> /P <sub>cp</sub> %	P <sub>ci</sub>	P <sub>ci</sub> /P <sub>cp</sub> %	P <sub>ci</sub>	P <sub>ci</sub> /P <sub>cp</sub> %
OV 1%D	61.8	98%	17.5	92%	7.2	97%	3.66	99%
12.5 %Tt	41.5	66%	12.7	66%	5	67%	2.49	67%
1%Dt	61	97%	17.3	91%	7.1	96%	3.6	97%
C1= 12.5 %Tt +1%D	40.2	64%	11.4	60%	4.7	63%	2.41	65%
C2=OV+ 12.5 %Tt	40.6	64%	11.5	60%	4.8	65%	2.43	66%
C3= OV+1%Dt	41.1	64%	17	89%	7	94%	3.55	96%
C4= 12.5 %Tt +1%Dt + OV1%D	39.2	62%	11.3	59%	4.6	62%	2.36	64%

The results were plotted in Figure 10:



**Figure 10 different geometrical imperfections cases effect on critical pressure**

The four imperfection combinations were plotted together to note the differences as shown in Figure 11



**Figure 11 main geometrical imperfections cases effect on critical pressure**

The severest geometrical imperfections case on critical pressure is case C4 (12.5 %Tt+1%Dt + ov1%D) with about 40% reduction factor as shown in (Table 7 & FIG 7 & 8). This case C4 represent a

combinations of all severest imperfections and this condition is rarely accrued.



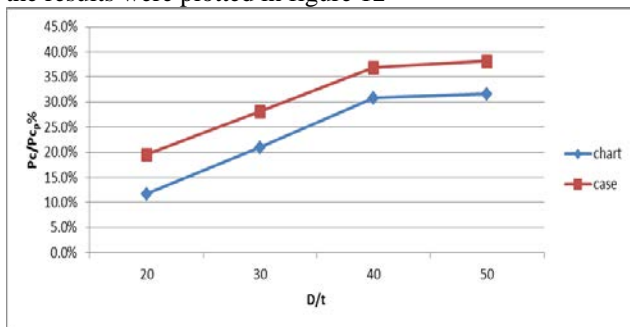
### 4.2.2 Analytical procedures

By solving the previous imperfection cases using both code procedures (chart and code case), the obtained results are presented in Table 7:

**Table 7 Analytical results of geometrical imperfect pipe**

D/T	chart		case		chart/case
	Pct	Pct/Pcp%	Pcs	Pcs/Pcp%	Pcs/Pct%
20	7.350	11.7%	12.290	19.5%	167.2%
30	4.000	20.9%	5.360	28.1%	134.0%
40	2.300	30.8%	2.750	36.9%	119.6%
50	1.170	31.6%	1.410	38.1%	120.5%

the results were plotted in figure 12



**Figure 2 Difference between chart procedure and code case procedures in critical pressure value**

The reduction factor used by code depend on D/t ratio which increase with the increase of D/t ratio and become constant at D/t = 40 with about 35% reduction factor. The difference between the results in chart procedure and code case decreases with the decrease of D/t ratio and becomes constant at D/t = 40 with about 20% critical pressure value in code case higher than chart as shown in ( Table 8 & FIG 12)

**Figure 3 Difference between code procedures and FEA in critical pressure value**

Figure 13 show that the effect of the geometrical imperfection reduction factor calculated by FEA is about 40%. However, the geometrical imperfection reduction factor calculated by ASME B&PV code is about 60% to 70%.

### 5.2.3 Comparison between the mathematical FE model and the analytical procedures

By comparing the results of the mathematical FE model and analytical procedures as shown in ( Table 8 & FIG 13), the difference in the critical pressure was found as following:

In chart procedure at D/T ≥ 40 was about 30% and increased to 50% at D/T < 40

In code case procedure at D/T ≥ 40 was 25% and increased to 40% at D/T < 40 .

**Table 8 comparasion between code results and FEA results**

D/T	20	30	40	50
Pc <sub>i</sub> /Pc <sub>p</sub>	62%	59%	62%	64%
Pc <sub>t</sub> /Pc <sub>p</sub> %	11.7%	20.9%	30.8%	31.6%
Pc <sub>s</sub> /Pc <sub>p</sub> %	19.5%	28.1%	36.9%	38.1%
Δ P (Pc <sub>t</sub> - Pc <sub>p</sub> ) %	50.6%	38.5%	31.2%	32.2%
ΔP (Pc <sub>s</sub> - Pc <sub>p</sub> ) %	42.7%	31.4%	25.2%	25.7%

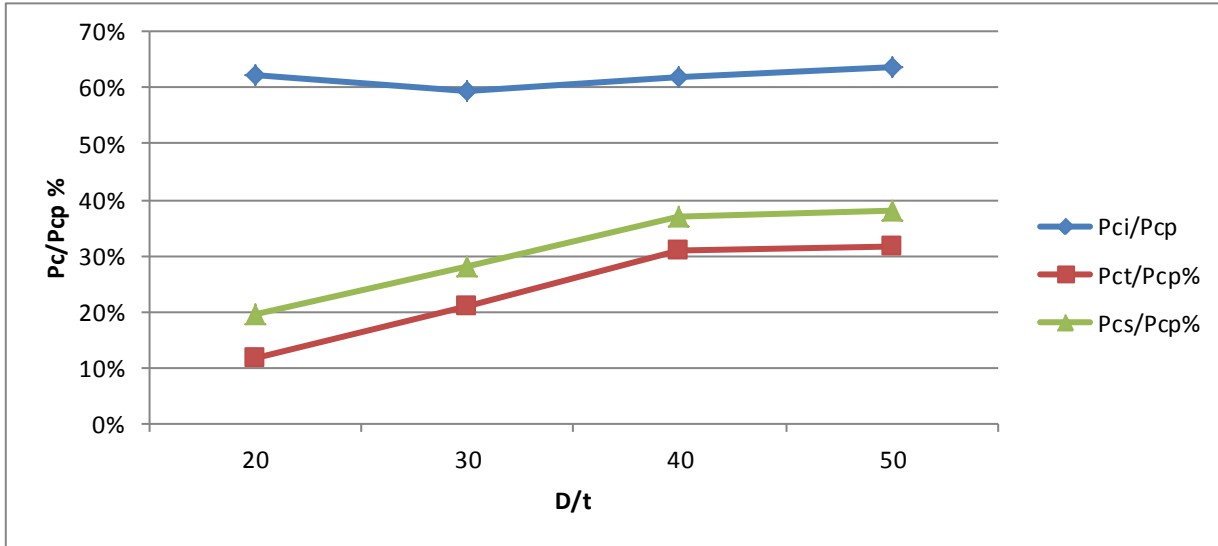


Figure 4 Difference between code procedures and FEA in critical pressure value

Figure 13 show that the effect of the geometrical imperfection reduction factor calculated by FEA is about 40%. However, the geometrical imperfection reduction factor calculated by ASME B&PV code is about 60% to 70%.

Based on previous findings from Figures 7, 8 & 9 where the geometrical imperfection does not change with the change of D/T ratio, and from equation (9), an equation is developed to calculate the maximum allowable external pressure for long thin walled pipes (equation 10) considering the 40% as geometrical imperfection reduction factor for the most severe cases mentioned in ASTM code .

$$P_{cr} = 1.3E \left( \frac{t}{D} \right)^3 \dots \dots \text{ where } \frac{D}{t} \geq 20 \dots \dots (10)$$

This equation is only valid for thin walled pipes where  $D/T \geq 20$  since for lower D/T ratio values where the pipe becomes thicker and the possibility of the plastic failure to occur becomes much higher than the geometric instability failure .

### 6. Conclusion and recommendations

The discrepancies in design codes for very long Pipes subjected to external pressure was highlighted in a comparison between the mathematical FE model and the ASME B&PV code analytical procedures. The severest geometrical imperfection case was determined and compared with code procedures results emphasize the differences between the critical pressure predictions.

This comparison revealed that the code is over exaggerating in evaluating the influence of geometric imperfection and also in determining of the critical length margin. This proves the unsuitability of the design procedures stated in ASME B&PV code to be applied on ASME B31.3 process piping code in calculating the maximum allowable external pressure for long thin walled pipes.

Based on the obtained results an equation was developed to calculate the maximum allowable external pressure for long thin walled pipes (equation 10), the proposed equation is considering the geometrical imperfection reduction factor for the severest case mentioned in ASTM code .

Despite equation 10 can be considered suitable alternative, it is conservative since the geometrical imperfection combination (C4) is rare to happen and the 12.5% thickness tolerance is not stated in the majority of ASTM material specifications.

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