

# Optimal PMU Placement Using linear integer Programming Technique

G.Sandhya rani,

Sr.Assitant Professor,

Vasavi College of Engineering,  
Hyderabad, Telangana, INDIA

## Abstract

The aim of the article is a study undertaken to determine the optimal locations of phasor measurement units (PMUs) for a given power system. Power systems are rapidly becoming populated by PMUs. PMUs have multiple uses at substations. They provide valuable phasor information for protection and control of power systems during abnormal operation. Under normal operations, they also help in monitoring the system state. This paper focuses on the use of PMU measurements in state estimators. The principle objective was to investigate methods of determining optimal locations for PMUs so that the system state of an entire power system can be observable. Several factors affect how this can be accomplished, such as the available data from existing conventional measurements, the number and location of zero injection buses, the number and location of installed PMUs, and, of course, the system topology. Two new procedures were used to solve the problem of optimally locating PMUs. One is a numerical procedure where the problem is formulated as an integer optimization problem. The solution algorithm was implemented in a computer program. Two case studies were conducted to evaluate the algorithm's performance on IEEE -14 bus, IEEE -30 bus.

**Keywords:** Smart grid, wide area protection, wide area control, power outage, power quality, Phase measurement unit.

## 1. Introduction

Secure operation of power systems requires close monitoring of the system operating conditions. This is traditionally accomplished by the state estimator which resides in the control centre computer and has access to the measurements received from numerous substations in the monitored system. By collecting analog measurements and the status data of the circuit breakers from remotely monitored and controlled substations and feeding them as input into state estimation function, state estimation can provide an estimate for all metered and un-metered electrical quantities and network parameters of the power system, detect and filter out gross errors in the measurement set and detect the topology errors in the network configuration. Until recently, available measurement sets did not contain phase angle measurements due to the technical difficulties associated with the synchronization of measurements at remote locations. Global positioning satellite (GPS) technology alleviated these difficulties and lead to the development of phasor measurement units.

Synchronized Phase Measurement Unit (PMU) is a

monitoring device, which was first introduced in mid-1980s. Phasor measurement units (PMU) are devices, which use synchronization signals from the global positioning system (GPS) satellites and provide the phasors of voltage and currents measured at a given substation. As the PMUs become more and more affordable, their utilization will increase not only for substation applications but also at the control centres for the EMS applications. One of the applications, which will be significantly affected by the introduction of PMUs, is the state estimator.

Phasor Measurement Units (PMUs) become more and more imported and attractive to power engineers because they can provide synchronized measurements of real-time phasors of voltage and currents[1].As the state estimator play an important role in the security of power system to enhance state estimation in a problem needed to be solved .Several algorithms have been published in the literature, it was not possible to measure phase angle of the bus voltage in real time due to the technical difficulties in synchronizing measurements from distant locations,But introducing the PMUs in power system ,possible to measure the real-time phasors of voltages and currents at widely dispersed locations with respect to a global positioning system (GPS) clock [2].

The methodology is needed to determine the optimal location of PMUs in a power system. In addition to its ability to measure voltage and current phasors, a state-of-the-art PMU may include other features such as protective actions. The objective of the present work is to find the minimum number of PMUs to make the system topologically observable,as well as the optimal locations of these PMUs. In recent year, there has been a significant research activity on the problem of finding the minimum number of PMUs and their optimal locations. In [3], a bisecting search method is implemented to find the minimum number of PMUs to make the system observable. The simulated annealing method is used to randomly choose the placement sets to the test for observability at each step of the bisecting search. In [1], the authors use a simulated annealing technique in their graph-theoretic procedure to find the optimal PMU locations.

In [5] and [6] the authors use integer programming to determine the minimum number of PMUs.The method, however may suffer from the problem.

In [7]-[8], the OPP optimization problem is solved using PSAT, a MATLAB based toolbox, and depth first search (DeFS) method is compared with other methods. Another depth first search (DeFS) method is proposed in [9]. The DeFS algorithm is computationally faster, but the solution is not optimum, because the optimization criterion is stiff. A modified depth first approach is the minimum spanning tree (MST) method [9]. The MST algorithm improves the DeFS approach, which also has fast computing characteristics, and improves DeFS's complex and weak convergence. A novel topological method based on the augment incidence matrix and Tabu Search(TS) algorithm, is proposed in [10]. The solution of the combinatorial OPP problem requires less computation and is highly robust. The method is faster and more convenient than conventional observability analysis methods using complicated matrix analysis, because it manipulates integer numbers. A TS method on meter placement to maximize topological observability is presented in [10]. The GA method suggested in [11] solves the OPP problem using different PMU placement criteria, such as the absence of critical measurements and critical sets from the system, maximum quantity of measurements received as compared to the initial one, maximum accuracy of estimates, minimum cost of PMU placement, and transformation of the network graph into tree. The immune algorithm (IA) is a search strategy based on genetic algorithm principles and inspired by protection mechanisms of living organisms against bacteria and viruses. In reference [12], the application of the immune genetic algorithm (IGA) method to the OPP problem is presented.

The Algorithm is developed in Matlab using an external optimization program (TomLab) which is used to implement the integer programming solution. The results can be extended to investigate the benefits of adding a small number of PMUs at strategic locations to improve bad data detection and identification capability in the system. Similarly, it may be worthwhile to investigate the placement of a few PMUs for purposes of eliminating the possibility of unobservable states during expected topology changes or contingencies.

## 2. Problem Formation

PMUs provide two types of measurements: bus voltage phasors and branch current phasors. Depending on the type of PMUs used the number of channels used for measuring voltage and current phasors will vary. In this report, it is assumed that each PMU has enough channels to record the bus voltage phasor at its associated bus and current phasors along all branches that are incident to this bus. The objective of the PMU placement problem is to render an observable system by using a minimum number of PMUs. An example of an optimally placed set of PMUs in a 14-bus system is shown below in Figure 1.

In this system, there are three PMUs placed at buses 2, 6 and 9 respectively. Bus 7 is the only zero injection bus.

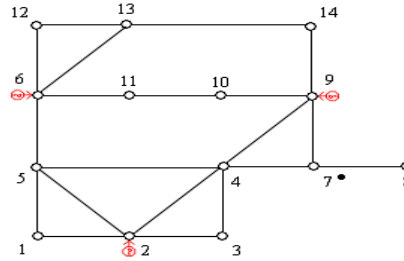


Figure 1. Optimal PMU placement for a 14-bus test system

The PMU at bus 2 can not only measure the voltage phasor of bus 2, but also the current phasors of branches 2-1, 2-3, 2-4 and 2-5. Using Ohm's law, the voltage phasors at buses 1, 3, 4 and 5 can be obtained from the branch currents and the voltage at bus 2. Having determined voltage phasors at buses 1, 2, 3, 4, and 5, the current phasors of branches 1-5, 3-4 and 4-5 can be calculated. Following the same logic, PMU at bus 6 can measure the voltage phasor at bus 6 and the current phasors of branches 6-5, 6-11, 6-12 and 6-13, thus allowing the calculation of the voltage phasors at buses 5, 11, 12, 13 and the current phasor of branch 12-13. PMU at bus 9 can measure the voltage phasor at bus 9 and the current phasors of branches 9-4, 9-7, 9-10, 9-14 and allow the calculation of the voltage phasors at buses 4, 7, 10, 14, and the current phasors of branches 4-7. As voltage phasors of buses 10, 11, 13, 14 are known, current phasors of branches 10-11 and 13-14 can now also be calculated. Using the known current phasors of branches 4-7 and 9-7, and the zero injection at bus 7, the current phasor of branch 7-8 can be derived using the Kirchhoff's Current Law. The only remaining unknown voltage phasor at bus 8 can now be calculated by using the voltage phasor at bus 7 and the current phasor of branch 7-8. Thus the entire system becomes observable by placing only three PMUs at buses 2, 6, 9 and by considering the zero injection at bus 7.

In this article, a procedure based on Integer Linear Programming using MATLAB software to find out the minimum number of PMUs and their location so that the entire power system network will be observable. The procedure is explained with a IEEE 14-bus system clearly.

### 2.1 Integer Programming Based Procedure

In this section, a numerical method based on Integer Programming will be presented to solve the optimal PMU placement problem. The formulation of problem is shown as below.

For an n-bus system, the PMU placement problem can be formulated as follows:

$$\min \sum w_i \cdot x_i$$

$$s.t. f(X) \geq 1$$

where

$$X_i = \begin{cases} 1 & \text{if a PMU is installed at bus } i \\ 0 & \text{otherwise} \end{cases}$$

$w_i$  is the cost of the PMU installed at bus  $i$ .

$f(X)$  is a vector function, whose entries are non-zero if the corresponding bus voltage is solvable using the given measurement set and zero otherwise.

$\hat{1}$  is a vector whose entries are all ones. Inner product of the binary decision variable vector and the cost vector represents the total installation costs of the selected PMUs. Constraint functions ensure full network observability while minimizing the total installation cost of the PMUs.

The procedure for building the constraint equations will be described for three possible cases where there are (1) no conventional measurement or zero injections, (2) flow measurements or (3) flow measurements as well as injection measurements (they may be zero injections or measured injections). Description of the procedure for each case will be given using IEEE 14-bus system example for clarification. However, the entire procedure is actually programmed and successfully tested on different size systems with diverse measurement configurations.

Consider the IEEE 14-bus system and its measurement configuration shown in Figure 2. The black dot near bus 7 represents that bus 7 is a zero injection bus (or has a injection measurement installed) while the black box on line 5-6 represents a paired flow measurement on line 5-6.

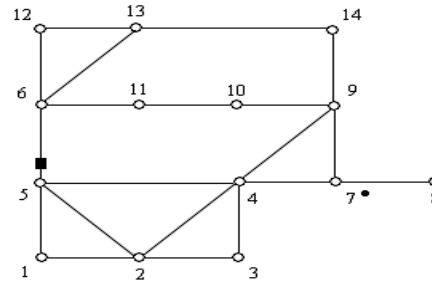


Figure 2. IEEE 14-bus system with conventional measurements

Here, we have two cases:

**Case 1.** A system with no conventional measurements and/or zero injections

**Case 2:** A system with some flow measurements

But, in this we deal only with a system considering no conventional measurements and/or zero injections

**Case: A system with no conventional measurements and/or zero injections**

In this case, the flow measurement and the zero injection are ignored. In order to form the constraint set, the binary connectivity matrix  $A$ , whose entries are defined below, will be formed first:

$$A_{k,m} = \begin{cases} 1 & \text{if } k = m \text{ or } k \text{ and } m \text{ are connected} \\ 0 & \text{if otherwise} \end{cases}$$

Matrix  $A$  can be directly obtained from the bus admittance matrix by transforming its entries into binary form. Building the  $A$  matrix for the 14-bus system yields:

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

The constraints for this case can be formed as:

$$f(X) = A \cdot X = \begin{cases} f_1 = x_1 + x_2 + x_5 & \geq 1 \\ f_2 = x_1 + x_2 + x_3 + x_4 + x_5 & \geq 1 \\ f_3 = x_2 + x_3 + x_4 & \geq 1 \\ f_4 = x_2 + x_3 + x_4 + x_5 + x_6 + x_7 & \geq 1 \\ f_5 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 & \geq 1 \\ f_6 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 & \geq 1 \\ f_7 = x_1 + x_2 + x_3 + x_4 + x_5 & \geq 1 \\ f_8 = x_1 + x_2 & \geq 1 \\ f_9 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 & \geq 1 \\ f_{10} = x_9 + x_{10} + x_{11} & \geq 1 \\ f_{11} = x_9 + x_{10} + x_{11} & \geq 1 \\ f_{12} = x_9 + x_{12} + x_{13} & \geq 1 \\ f_{13} = x_9 + x_{12} + x_{13} + x_{14} & \geq 1 \\ f_{14} = x_9 + x_{13} + x_{14} & \geq 1 \end{cases}$$

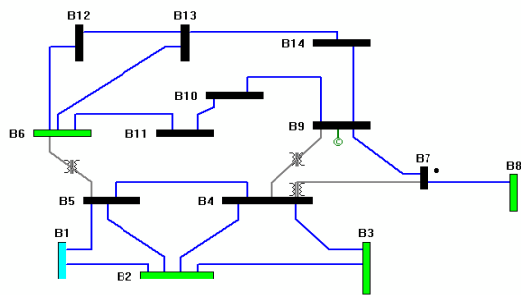
The operator ‘+’ serves as the logical ‘OR’ and the use of 1 in the right hand side of the inequality ensures that at least one of the variables appearing in the sum will be non zero. For example, consider the constraints associated with bus 1 and 2 as given below,

$$f_1 = x_1 + x_2 + x_5 \geq 1$$

$$f_2 = x_1 + x_2 + x_3 + x_4 + x_5 \geq 1$$

The first constraint  $f_1 \geq 1$  implies that at least one PMU must be placed at either one of buses 1, 2 or 5 in order to make bus 1 observable. Similarly, the second constraint  $f_2 \geq 1$  indicates that atleast one PMU should be installed.

**i) IEEE 14-bus system**



IEEE 14-bus

Figure 3. IEEE 14-bus system

The Information of the IEEE 14 system and simulation

results are given in the Table 1.

Table 1 System information of IEEE 14-bus system

System	# branches of	# of PMUs	Location of PMUs at buses
IEEE 14-bus	20	4	2,6,7,9

**ii) IEEE 30-bus system**

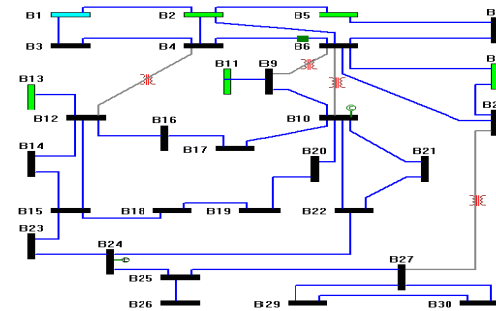


Figure 4. IEEE 30-bus system

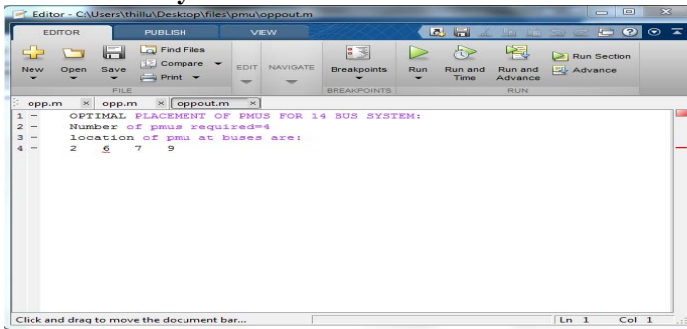
The Information of the system and simulation results are given in the Table 2.

Table 2 System information of IEEE 30-bus system

System	# branches of	# of PMUs	Location of PMUs at buses
IEEE 30-bus	41	10	2, 4, 6, 9, 10, 12, 15, 18, 25, 27

**3. Simulation Results**

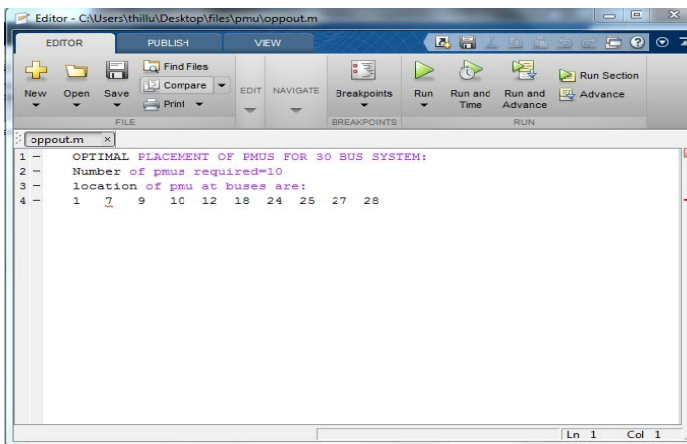
### IEEE 14-bus system simulation results



```

1 - OPTIMAL PLACEMENT OF PMUS FOR 14 BUS SYSTEM:
2 - Number of pmu required=4
3 - location of pmu at buses are:
4 - 2 8 7 9
  
```

### IEEE 30-bus system simulation results



```

1 - OPTIMAL PLACEMENT OF PMUS FOR 30 BUS SYSTEM:
2 - Number of pmu required=10
3 - location of pmu at buses are:
4 - 1 7 9 10 12 18 24 25 27 28
  
```

## 4. CONCLUSION:

Program is developed in MATLAB software using binary linear integer programming for determining optimum no. of PMU's and their location considering 2 cases for 2 different IEEE 14bus ,30 bus power systems. The results are completely observable.

## References

1. R.F. Nuqui and A.G.Phadke ,”Phasor measurement unit placement techniques for complete and incomplete observability, ” IEEE Trans. Power Del.,vol. 20 ,no. 4 ,pp. 2381-2388, Oct. 2005.
2. A.G. Phadke ,”Synchronized phasor measurements in power system,” IEEE Comput . Appl. Power ,vol. 6 ,no. 2 ,pp. 10- 15, April 1993.
3. T.L. Baldwin ,L. Mili ,M.B. Boisen ,and R.Adapa ,”Power

- system observability with minimal phasor measurement placement ,”IEEE Trans. Power System , vol 8 ,no. 2, pp. 707-715 , May 1993.
4. T. W. Haynes et al, “Domination in Graphs Applied to Electric Power Networks”, SIAM J. Discrete Math., Vol. 15,No. 4, 2002.
5. B. Xu and A. Abur, “Observability analysis and measurement placement for systems with PMUs,” in proc. IEEE Power Eng. Soc. Power Systems Conf. Expo., Oct. 2004, pp. 943– 946.
6. M. Farsadi, H. Golahmadi, and H. Shojaei, "Phasor measurement unit (PMU) allocation in power system with different algorithms", in 2009 Int. Conf. on Electrical and Electronics Engineering, pp. 396-400.
7. G. Venugopal, R. Veilumuthu, and P. Avila Theresa, "Optimal PMU placement and observability of power system using PSAT," in 2010 Int. Joint Journal Conf. on Engineering and Technology, pp.67-71.
8. T.-T. Cai and Q. Ai, "Research of PMU optimal placement in power Systems," in 2005 World Scientific and Engineering Academy and Society Int. Conf., pp. 38-43.
9. A. Z.Z. Gamm, I. N. Kolosok, A. M. Glazunova, and E. S. Korkina, PMU placement criteria for EPS state estimation," in 2008 Int. Conf. on Electric Utility Deregulation and Restructuring and Power Technologies,pp. 645-649.
10. Optimal Placement of Phasor Measurement Units for Power System Observability Without Considering Zero Injection Buses International Journal of Smart Sensors and Ad Hoc Networks (IJSSAN) ISSN No. 2248 - 9738 Volume - 1, Issue - 4, 2012 122 .
11. J. Peng, Y. Sun, and H. F. Wang, "Optimal PMU placement for full Network observability using Tabu search algorithm," International Journal of Electrical Power & Energy Systems, vol. 28, no. 4, pp. 223-231, May 2006.
12. M. Hajian, A. M. Ranjbar, T. Amraee, and A. R. Shirani, "Optimal placement of phasor measurement units particle swarm optimization approach," in 2007 Int. Conf. on Intelligent Systems Applications to Power Systems, pp. 1-6.