IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 2 Issue 4, April 2015.
www.ijiset.com
ISSN 2348 – 7968

ON FUZZY ALMOST P-SPACES

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ABSTRACT

In this paper we discuss several characterizations of fuzzy almost P-spaces and the conditions under which fuzzy topological spaces become fuzzy almost P-spaces, are investigated. Some results concerning functions that preserve fuzzy almost P-spaces in the context of images and preimages are obtained.

KEY WORDS : Fuzzy G_δ-set, fuzzy F_σ -set, fuzzy dense set, fuzzy nowhere dense set , fuzzy submaximal space, fuzzyD-Bairespace,fuzzy strongly irresolvable space , somewhat fuzzy continuous function , somewhat fuzzy open function.

1 .INTRODUCTION

In order to deal with uncertainties, the concept of fuzzy sets and fuzzy set operations were first introduced by **L.A.ZADEH** [23] in his classical paper in the year 1965, describing fuzziness mathematically for the first time. This inspired mathematicians to fuzzify mathematical structures. The concepts of fuzzy topology was defined by **C.L.CHANG** [3] in the year 1968. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed.

A.K.MISHRA [8] introduced the concepts of P-spaces as a generalization of ω_α -additive spaces of SIKORSKI [10] and COHEN, L.W. and C. GOFFMAN [5]. The concept of P-spacesin fuzzy setting was introduced by G. BALASUBRAMANIAN in [13]. Almost P-spaces in classical topology was introduced by A .I. Veksler [21] and wasalso studied further by R.Levy [7]. Chang IL Kim [4] studied several characterizations almost P-spaces.The concept of almost P-spaces in fuzzy setting was introduced by the authors in [16]. In this paper we discuss several characterizations of fuzzy almost P-spaces and the conditions under which fuzzy topological spaces become fuzzy almost P-spaces, are investigated.Some results concerning functions that preservefuzzy almost P-spaces in the context of images and preimages are obtained.

2. PRELIMINARIES

Now we introduce some basic notions and results used in the sequel.

In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to CHANG (1968).

Definition 2.1 : Let λ and μ be any two fuzzy sets in a fuzzy topological space (X,T). Then we define :

(i).$\lambda \vee \mu$: $X \to [0,1]$ as follows : $(\lambda \vee \mu)(x) = \text{Max}\ \{\lambda(x), \mu(x)\}$.

(ii).$\lambda \wedge \mu$: $X \to [0,1]$ as follows : $(\lambda \wedge \mu)(x) = \text{Min}\ \{\lambda(x), \mu(x)\}$.

(iii). $\mu = \lambda^c \Leftrightarrow \mu(x) = 1 - \lambda(x)$.

More generally, for a family $\{ \lambda_i \ / i \in I \}$ of fuzzy sets in (X,T), $\psi = \vee_i (\lambda_i)$ and $\delta = \wedge_i (\lambda_i)$, are defined respectively as $\psi(x) = sup_i \{ \lambda_i(x), x \in X \}$ and $\delta(x) = inf_i \{ \lambda_i(x), x \in X \}$.

Definition 2.2 : Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). We define the interior and the closure of λ respectively as follows :

(i). $int(\lambda) = \vee \{ \mu / \mu \leq \lambda, \mu \in T \}$

(ii). $cl(\lambda) = \wedge \{ \mu / \lambda \leq \mu, 1-\mu \in T \}$.

Lemma 2.1 [1] : For a fuzzy set λ of a fuzzy topological space X,

(i). $1 - int(\lambda) = cl(1 - \lambda)$,

(ii). $1 - cl(\lambda) = int(1 - \lambda)$.

Definition 2.3 [14] : A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy dense if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, $cl(\lambda) = 1$.

Definition 2.4 [14] : A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy nowhere dense if there exists no non - zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, $int\, cl(\lambda) = 0$.

Definition 2.5 [2] : A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy F_σ-set in (X,T) if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$ where $1 - \lambda_i \in T$ for $i \in I$.

Definition 2.6 [2] : A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy G_δ-set in (X,T) if $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$ where $\lambda_i \in T$ for $i \in I$.

Definition 2.7 [1]: A fuzzy set λ in a fuzzy topological space (X,T) is called (i).a fuzzy regular openset in (X,T)ifint cl (λ) = λ ,

(ii).a fuzzy regular closed set in (X,T) if cl int (λ) = λ ,

(ii).a fuzzy semi-open setin (X,T) ifλ ≤cl int (λ) ,

(ii).a fuzzy semi- closed set in (X,T) if int cl (λ) ≤ λ .

Lemma 2.2 [1] : For a family $\mathcal{A}$ of {λ_{α}} of fuzzy sets of a fuzzy topological space (X,T), ∨ cl (λ_{α})≤ cl (∨λ_{α}). In case $\mathcal{A}$ is a finite set,∨ cl (λ_{α})= cl(∨(λ_{α})). Also ∨ int (λ_{α})≤ int (∨λ_{α}) in (X,T).

Definition 2.9 [14] : A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy first category set if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where where$(\lambda_i)'s$ are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category.

Definition 2.10 [14]: A fuzzy topological space (X,T) is called fuzzy first category if $\bigvee_{i=1}^{\infty}(\lambda_i) = 1_X$, where$(\lambda_i)'s$ are fuzzy nowhere dense sets in (X,T). A topological space which is not of fuzzy first category, is said to be of fuzzy second category.

Definition 2.11 [3] : Let (X ,T) and (Y,S) be any two fuzzy topological spaces. Let f be a function from the fuzzy topological space (X ,T) to the fuzzy topological space (Y,S). Let λ be a fuzzy set in (Y,S). The inverse image of λ under f written as $f^{-1}(\lambda)$ is the fuzzy set in (X ,T) defined by $f^{-1}(\lambda)(x) = \lambda(f(x))$ for all $x \in$ X. Also the image of λ in (X,T) under f written as f (λ) is the fuzzy set in (Y,S) defined by

$$f(\lambda)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \lambda(x) & if \ f^{-1}(y) \text{ is non} - empty; \\ 0 & otherwise. \end{cases} \quad \text{for each } y \in Y.$$

Lemma 2.3[3] : Let $f : (X, T) \to (Y, S)$ be a mapping. For fuzzy sets λ and μ of (X, T) and (Y, S) respectively, the following statements hold :

(1) $ff^{-1}(\mu) \leq \mu$;

(2) $f^{-1}f(\lambda) \geq \lambda$;

(3) $f(1 - \lambda) \geq 1 - f(\lambda)$;

(4) $f^{-1}(1 - \mu) = 1 - f^{-1}(\mu)$;

(5) If f is one - to - one , then $f^{-1}f(\lambda) = \lambda$;

(6) If f is onto , then $ff^{-1}(\mu) = \mu$;

(7) If f is one - to - one and onto, then $f(1 - \lambda) = 1 - f(\lambda)$.

LEMMA 2.4 [1] : Let $f : (X, T) \to (Y, S)$ be a mapping and $\{\lambda_\alpha\}$ be a family of fuzzy sets of Y. Then

(a) $f^{-1}(\cup_\alpha \lambda_j) = \cup_\alpha f^{-1}(\lambda_j)$.

(b) $f^{-1}(\cap_\alpha \lambda_j) = \cap_\alpha f^{-1}(\lambda_j)$.

Lemma 2.5 [6] : Let $f : (X, T) \to (Y, S)$ be a mapping and $\{A_j\}$, $j \in J$ be a family of fuzzy sets in X. Then

(a) $f(\cup_{j \in J} A_j) = \cup_{j \in J} f(A_j)$.

(b) $f(\cap_{j \in J} A_j) \leq \cap_{j \in J} f(A_j)$.

3. FUZZY ALMOST P-SPACES

Almost P-spaces in classical topology was introduced by A .I. Veksler [21] and was also studied further by R.Levy [7]. Chang IL Kim [4] studied several characterizations ofalmost P-

ISSN 2348 – 7968

spaces.Motivated bythese ideas,theconcept of almost P-spaces in fuzzy setting ,isintroduced in [16].

Definition 3.1 [16]: A fuzzy topological space (X,T) is called a fuzzy almostP-space if for every non- zero fuzzyG_δ-set λ in (X,T), int (λ) $\neq 0$ in (X,T).

Proposition 3.1 : If λ is a fuzzy F_σ -set in a fuzzyalmost P- space (X, T), then cl(λ) $\neq 1$.

Proof: Let λ be a fuzzy F_σ -set in a fuzzy almost P- space (X,T). Then, ($1 - \lambda$) is a fuzzyG_δ-set in (X,T) .Since (X,T) is a fuzzy almostP-space, for the fuzzy G_δ-set ($1 - \lambda$), we have int ($1 - \lambda$) $\neq 0$. This implies that $1 -$ cl (λ) $\neq$ 0 and hence we have cl (λ) $\neq 1$.

Remark 3.1: In view of the proposition 3.1 , we have the following result : " If λ is a fuzzy G_δ-set in a fuzzy almost P- space (X, T), then cl($1 - \lambda$) $\neq$ 1 ."

The following propositions give the conditions for a fuzzy topological space to be a fuzzy almost P-space.

Proposition 3.2 : If eachnon-zero fuzzy G_δ-set is a fuzzy regular closed set in a fuzzy topological space (X,T), then (X, T) is a fuzzy almost P- space.

Proof: Let λ be a non-zero fuzzy G_δ-set in (X,T) such that cl int (λ) $= \lambda$.We claim that int (λ) $\neq 0$.Assume the contrary. Then int (λ) $=0$, will imply that cl int (λ) $=$ cl (0) $=$ 0and hence we will have $\lambda = 0$, a contradiction to λ beinga non-zero fuzzy G_δ-set in (X,T). Hence we must haveint (λ) $\neq 0$,for a fuzzy G_δ-set λin (X,T)and therefore(X, T) is a fuzzy almost P - space.

ISSN 2348 – 7968

Proposition 3.3 : If eachnon-zero fuzzy G_δ-set is a fuzzy semi-open set in a fuzzy topological space (X,T), then (X, T) is a fuzzy almost P - space.

Proof: Let λ be a non-zero fuzzy G_δ-set in (X,T) such that $\lambda \leq cl[\,int(\lambda)]$. We claim that $int(\lambda) \neq 0$. Assume the contrary. Then $int(\lambda) = 0$, will imply that $cl\, int(\lambda) = cl\,(0) = 0$ and hence we will have $\lambda = 0$, a contradiction to λ beinga non-zero fuzzy G_δ-set in (X,T). Hence we must have $int(\lambda) \neq 0$, for a fuzzy G_δ-set λin (X,T) and therefore (X, T) is a fuzzy almost P - space.

The following proposition shows that a fuzzy first category set, is not a fuzzy dense set in a fuzzy almost P-space.

Proposition 3.4 : If $\lambda$is a fuzzyfirst category set in a fuzzy almost P – space (X,T), then $cl(\lambda) \neq 1$, in (X,T).

Proof : Let λ be a fuzzy first category set in (X,T). Then,$\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'$sare fuzzy nowhere dense sets in (X,T).Now $\lambda_i \leq cl(\lambda_i)$, implies that $\vee_{i=1}^{\infty}(\lambda_i) \leq \vee_{i=1}^{\infty} cl(\lambda_i)$ and hence we have $cl(\vee_{i=1}^{\infty}(\lambda_i)) \leq cl(\vee_{i=1}^{\infty} cl(\lambda_i))$. Thencl $(\lambda) \leq cl(\vee_{i=1}^{\infty} cl(\lambda_i))$ ….(1). Now $\vee_{i=1}^{\infty} cl(\lambda_i)$ is a fuzzy F_σ -set in (X,T). Since (X, T) is a fuzzy almost P-space, by proposition 3.1 , $cl(\vee_{i=1}^{\infty} cl(\lambda_i)) \neq 1$. ….(2). Now we claim that$\lambda$is not a fuzzy dense set in (X,T).Assume the contrary. Suppose that $\lambda$is afuzzy dense set , then $cl(\lambda) = 1$, implies from (1) , that $1 \leq cl(\vee_{i=1}^{\infty} cl(\lambda_i))$. That is, $cl(\vee_{i=1}^{\infty} cl(\lambda_i)) = 1$, a contradiction to (2). Hence we must have $cl(\lambda) \neq 1$, in (X,T).

Proposition 3.5 :If eachnon-zero fuzzy first category set is a fuzzy dense set in a fuzzy topological space (X,T), then (X, T) is not a fuzzy almost P -space.

Proof : Let λ be a fuzzy first category set in (X,T) such that $cl(\lambda) = 1$. Then $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Now $1 - cl(\lambda_i)$ is a fuzzy open set in (X,T). Let $\mu = \wedge_{i=1}^{\infty} [1 - cl(\lambda_i)]$. Then μ is a fuzzy G_δ-set in (X, T). Now $\mu = \wedge_{i=1}^{\infty} [1 - cl(\lambda_i)] = 1 - [\vee_{i=1}^{\infty} cl(\lambda_i)] \leq 1 - [\vee_{i=1}^{\infty} (\lambda_i)] = 1 - \lambda$. That is, $\mu \leq 1 - \lambda$. Then $int(\mu) \leq int(1 - \lambda)$ and hence $int(\mu) \leq 1 - cl(\lambda) = 1 - 1 = 0$. That is, $int(\mu) = 0$. Hence, for the fuzzy G_δ-set μ in (X, T), $int(\mu) = 0$. Therefore (X, T) is not a fuzzy almost P-space.

Proposition 3.6 : If γ is a fuzzy residual set in a fuzzy almost P - space (X,T), then $int(\gamma) \neq 0$, in (X,T).

Proof : Let γ be a fuzzy residual set in (X,T). Then, $(1 - \gamma)$ is a fuzzy first category set in (X,T) and hence by proposition 3.4, $cl(1 - \gamma) \neq 1$ in (X,T). Therefore $1 - int(\gamma) = cl(1 - \gamma) \neq 1$. Therefore $int(\gamma) \neq 0$, in (X,T).

Proposition 3.7 : If γ is a fuzzy residual set in a fuzzy almost P - space (X,T), then there exists a fuzzy G_δ-set μ in (X, T) such that $\mu \leq \gamma$ in (X,T).

Proof : Let γ be a fuzzy residual set in (X,T). Then, $(1 - \gamma)$ is a fuzzy first category set in (X,T). Let $\lambda = 1 - \gamma$. Then $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Now $[1 - cl(\lambda_i)]$ is a fuzzy open set in (X,T). Let $\mu = \wedge_{i=1}^{\infty} [1 - cl(\lambda_i)]$. Then μ is a fuzzy G_δ-set in (X, T). Since (X, T) is a fuzzy almost P-space, $int(\mu) \neq 0$, in (X,T). Let $int(\mu) = \delta$, where $\delta \in T$. Now $\mu = \wedge_{i=1}^{\infty} [1 - cl(\lambda_i)] = 1 - [\vee_{i=1}^{\infty} cl(\lambda_i)] \leq 1 - [\vee_{i=1}^{\infty} (\lambda_i)] = 1 - \lambda$. That is, $\mu \leq \gamma$. Thus, if γ is a fuzzy residual set in a fuzzy almost P - space (X,T), then there exists a fuzzy G_δ-set μ in (X, T) such that $\mu \leq \gamma$ in (X,T).

IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 2 Issue 4, April 2015.
www.ijiset.com
ISSN 2348 – 7968

Proposition 3.8 : If μ is a fuzzy residual set in a fuzzy almost P - space (X,T), then μ is not a fuzzy nowhere dense set in (X,T).

Proof : Let μ be a fuzzy residual set in a fuzzy almost P – space(X,T).Then, by proposition 3.6, int (μ) $\neq$ 0, in (X,T).We claim that int cl (μ) $\neq$0. Assume the contrary. Then int cl(μ) =0 and int (μ) $\leq$ int cl(μ), will imply that int (μ) = 0,a contradiction. Hence μ is not a fuzzy nowhere dense set in (X,T).

Remark 3.1: If λ is a fuzzy semi-open set in a fuzzy topological space (X,T), then cl(λ) is a fuzzy regular closed set in (X,T).

For, if λ is a fuzzy semi-open set in (X,T), then $\lambda \leq$ cl [int (λ)] and hence cl (λ) $\leq$cl(cl[int (λ)]) = cl [int (λ)] $\leq$ cl [int cl(λ)] ……(1).Also we have cl [int cl(λ)] $\leq$ cl [cl(λ)] = cl(λ) …..(2).From (1) and (2), cl [int (cl(λ))] = cl(λ) and hence cl(λ) is a fuzzy regular closed set in (X,T).

Proposition 3.9 : If (λ_i)'s are fuzzy regular closed sets in a fuzzy almost P - space(X,T), then cl ($\vee_{i=1}^{\infty}(\lambda_i)$) $\neq$ 1, in (X,T).

Proof: Let (λ_i)'s be fuzzy regular closed sets in (X,T). Then, ($1-\lambda_i$)'s are fuzzy regular open sets in (X,T). Since the fuzzy regular open sets are fuzzy open sets in a fuzzy topological space ,($1-\lambda_i$)'s are fuzzy open sets in (X,T) and hence $\wedge_{i=1}^{\infty}(1-\lambda_i)$ is a fuzzy G_δ-set in (X,T). Since (X, T) is a fuzzy almost P-space, int ($\wedge_{i=1}^{\infty}(1-\lambda_i)$) $\neq$0 in (X,T). Then we have int ($1-\vee_{i=1}^{\infty}(\lambda_i)$) $\neq$0 and therefore cl ($\vee_{i=1}^{\infty}(\lambda_i)$) $\neq$ 1, in (X,T).

Proposition 3.10 : If (λ_i)'s are fuzzy semi-open sets in a fuzzy almost P – space(X,T), then cl ($\vee_{i=1}^{\infty}$[cl (λ_i)]) $\neq$ 1, in (X,T).

ISSN 2348 – 7968

Proof: Let (λ_i)'s be fuzzy semi-open sets in (X,T). Then , by remark 3.1, cl (λ_i)'s are fuzzy regular closed sets in(X,T) and hence,by proposition 3.8, cl ($\vee_{i=1}^{\infty}$ [cl(λ_i)]) $\neq$ 1, in (X,T).

4. FUZZY ALMOST P-SPACESand OTHER FUZZY TOPOLOGICAL SPACES

Definition 4.1 [13] : A fuzzy topological space (X,T) is called a fuzzy P-space if countable intersection of fuzzy open sets in (X,T) is fuzzy open. That is , every non-zero fuzzy G_δ-set in (X,T), is fuzzy open in (X,T).

Proposition 4.1 : Ifa fuzzy topological space (X, T) is a fuzzy almost P -space, then (X, T) is a fuzzy second category space.

Proof : Let the fuzzy topological space (X, T) be a fuzzy almost P –space.Suppose that(X, T) is a fuzzy first category space. Then$\vee_{i=1}^{\infty}(\lambda_i) = 1$, where $(\lambda_i)'$sare fuzzy nowhere dense sets in (X,T).Now $\lambda_i \leq$ cl (λ_i), implies that $\vee_{i=1}^{\infty}(\lambda_i) \leq \vee_{i=1}^{\infty} \mathrm{cl}\,(\lambda_i) \ldots.(1)$. Also $\vee_{i=1}^{\infty} \mathrm{cl}\,(\lambda_i)$ is a fuzzy F_σ -set in (X,T). Since (X, T) is a fuzzy almost P-space, by proposition 3.1 , cl($\vee_{i=1}^{\infty}$ cl (λ_i)) $\neq$ 1 .....(2). From (1), we havecl ($\vee_{i=1}^{\infty}(\lambda_i)$) $\leq$ cl($\vee_{i=1}^{\infty}$ cl (λ_i)) . This implies that cl(1) $\leq$ cl($\vee_{i=1}^{\infty}$ cl (λ_i))and hence we have 1$\leq$ cl($\vee_{i=1}^{\infty}$ cl (λ_i)). That is, cl($\vee_{i=1}^{\infty}$ cl (λ_i)) =1, a contradiction to (2). Hence we must have $\vee_{i=1}^{\infty}(\lambda_i)$ $\neq$ 1, in (X,T) and therefore (X, T) is a fuzzy second category space.

Definition 4.2 [20] : A fuzzy topological space (X,T) is said to be a fuzzy strongly irresolvable space if cl int (λ) = 1 , for each fuzzy dense set λ in (X,T).

Proposition 4.2 : Ifeach non-zero fuzzy G_δ -set is a fuzzy dense set in a fuzzy strongly irresolvable space, then (X, T) is a fuzzy almost P –space.

Proof : Let λbe a non-zero fuzzy G_δ -set in (X,T). Then, by hypothesis, λ is a fuzzy dense set in (X,T).Since (X, T) is a fuzzy strongly irresolvable space, we have

cl (int (λ)) = 1. Then, we have int (□) ≠ 0,[Otherwise int (□) = 0, will imply cl (int (□)) = cl (0) = 0, a contradiction]. Hence , for every non- zero fuzzyG_δ-set λ in (X,T), we have int (λ) ≠0 in (X,T). Therefore (X, T) is a fuzzy almost P - space.

Definition 4.3 [15]: A fuzzy topological space (X,T) is called a fuzzy resolvable space if there exists a fuzzy dense set λ in (X,T) such that cl (1 –λ) = 1. Otherwise (X,T) is called a fuzzy irresolvable space.

Theorem 4.1 [11] : If λ is a fuzzy dense and fuzzy G_δ -set in a fuzzy topological space (X,T), then (1 – λ) is a fuzzy first category set in (X,T).

Proposition 4.3 : Ifeachnon-zero fuzzy dense set is a fuzzyG_δ -set in a fuzzy almost P -space, then(X, T) is a fuzzy irresolvable space.

Proof : Let λbe a non-zero fuzzy dense set in(X,T). Then, by hypothesis , λ is a fuzzy G_δ -set and hence λ is a fuzzydense andfuzzyG_δ –set in (X,T). Then , by theorem 4.1, 1 – λ is a fuzzy first category set in (X,T).Since(X, T) is a fuzzy almost P-space, by proposition 3.4, cl (1– □) ≠ 1, in (X,T). Thus , for each fuzzy dense set λ in (X,T) , we have cl (1–□) ≠ 1, in (X,T). Hence (X, T) is not a fuzzy resolvable space and therefore (X, T) is a fuzzyirresolvable space.

Definition 4.4 [2] : A fuzzy topological space (X,T) is called a fuzzy submaximal space if for each fuzzy set □ in (X,T) such that cl (□) = 1 , then λ ∈ T in(X,T).

Proposition 4.4: Ifeach non-zero fuzzy G_δ -set is a fuzzy dense set in a fuzzy submaximal space , then (X,T) is a fuzzy almost P -space.

Proof : Let λbe a non-zero fuzzyG_δ -set in (X,T) such that cl(□) = *1*.Since(X,T) is a fuzzy submaximal space, the fuzzy dense set□ in (X,T), is a fuzzy open set in (X,T) and hence □is a fuzzysemi-open set in (X ,T). Thus, each non-zero fuzzy G_δ-set is a fuzzy semi-open set in (X,T).Then , by proposition 3.3,(X, T) is a fuzzy almost P -space.

Definition4.5 [12] : A fuzzy topological space (X, T) is called a fuzzy D- Baire space if every fuzzy first category set in (X,T), is a fuzzy nowhere dense set in (X,T).

The following propositiongivesacondition for a fuzzy almost P -space to be a fuzzy D-Baire space.

Proposition 4.5 : If int(μ) = *0* , for each fuzzy closed set μ in a fuzzy almost P -space, then (X, T) is a fuzzy D-Baire space.

Proof : Let λ be a fuzzy first category set in (X,T). Since (X, T) is a fuzzy almost P-space, by proposition 3.4, cl (□) ≠ 1, in (X,T). Let cl (□) = μ, where 1−□ ∈ T. Then int [cl (□)] = int [μ]= 0 (by hypothesis). Thus λ is a fuzzy nowhere dense set in (X,T). Hence each fuzzy first category set in (X,T), is a fuzzy nowhere dense set in (X,T). Therefore (X, T) is a fuzzy D-Baire space.

Definition 4.6[18]: A fuzzy topologicalspace (X,T) is called a fuzzyGID-space if for eachfuzzy dense and fuzzy G_δ-set □in (X,T), cl int (□) = *1*,in (X,T).

Theorem 4.2 [18] : If each fuzzyG_δ-set is fuzzy densein afuzzy GID space , then (X,T) is a fuzzy almost P -space .

Definition 4. 6 [9] : A fuzzy topological space X is said to be fuzzy hyperconnected if every non- null fuzzy open subset of X is fuzzy dense in X. That is , a fuzzy topological space (X,T) is fuzzy hyperconnected if cl $(\mu_i) = 1$, for all $\mu_i \in T$.

Proposition 4.7 : If a fuzzy almost P space (X,T) is a fuzzy hyperconnected space, then(X,T) is a fuzzy GID space.

Proof : Let □ be a fuzzy dense and fuzzy G_δ-set in (X,T).Since (X, T) is a fuzzy almost P – space, for the fuzzy G_δ-set □ in (X,T), we have int $(\lambda) \neq 0$ in (X,T). Also, since (X, T) is a fuzzy hyperconnected,for the fuzzy open set int (λ) in (X,T), we have cl (int (λ)) = 1. Hence for a fuzzy dense and fuzzy G_δ-set □ in (X,T), we have cl int (□) = *1*, in (X,T). Therefore (X,T) is a fuzzy GID space.

5 . FUZZY ALMOST P-SPACESandFUNCTIONS.

Theorem 5.1 [22] : Letf : (X,T) → (Y,S) be a fuzzy open function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S). Then, for every fuzzy set β in (Y,S), f^{-1} (cl (β)) $\leq$ cl $f^{-1}(\beta)$.

Theorem 5.2 [19] : Letf : (X,T) → (Y,S) be a fuzzy open function from a fuzzy topological space (X,T) into a fuzzy topological space (Y,S). Then, for every fuzzy set δ in (Y,S), int$(f^{-1}(\delta)) \leq f^{-1}$ (int (δ)).

Definition 5.1 [14]: A function f : (X,T) → (Y,S) from a fuzzy topological space (X,T) into another fuzzy topological space (Y, S) is called somewhat fuzzy continuous if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$ implies that there exist a fuzzy open set δ in (X ,T) such that $\delta \neq 0$ and $\delta \leq f^{-1}(\lambda)$.

Definition 5.2 [14] : A function $f : (X,T) \to (Y,S)$ from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is called somewhat fuzzy open if $\lambda \in T$ and $\lambda \neq 0$ implies that there exists a fuzzy open set η in (Y,S) such that $\eta \neq 0$ and $\eta \leq f(\lambda)$.

Let f be a function from the fuzzy topological space (X ,T) to the fuzzy topological space (Y,S).Under what conditions on " f " may we assert that if (X,T) is a fuzzy almost P- space, then (Y,S) is a fuzzy almost P- space? .The following propositions establish the desired conditions.

Proposition 5.1 : Ifa function $f : (X,T) \to (Y,S)$ from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is fuzzy continuous function and ifλ is a fuzzy G_δ-set in (Y,S), then $f^{-1}(\lambda)$ is a fuzzy G_δ-set in (X,T).

Proof: Let λ be a fuzzy G_δ-set in (Y,S).Then, $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where$(\lambda_i)'$sare fuzzy open sets in (Y,S). Now $f^{-1}(\lambda) = f^{-1}(\wedge_{i=1}^{\infty}(\lambda_i)) = \wedge_{i=1}^{\infty} [f^{-1}(\lambda_i)]$. Since the function f is fuzzy continuous and $(\lambda_i)'$s are fuzzy open sets in (Y,S), $[f^{-1}(\lambda_i)]$'s are fuzzy open sets in (X,T). Hence$f^{-1}(\lambda)$ is a fuzzy G_δ- set in (X,T).

Proposition 5.2 : Ifafunction $f : (X,T) \to (Y,S)$ from a fuzzy topological space (X,T) onto another fuzzy topological space (Y,S) is fuzzy continuous, and fuzzy open function and if (X,T) is a fuzzy almost P- space , then (Y,S) is a fuzzy almost P- space.

Proof: Let □ be a fuzzy G_δ-set in (Y,S). Since f is a fuzzy continuousfunction, by proposition 5.1, f^{-1} (□)is a fuzzy G_δ-set in (X,T). Since (X,T) is a fuzzy almost P space, int $[f^{-1}$ (□)] ≠ 0, in (X,T). Since f is a fuzzy open function from (X,T)onto (Y,S),for the fuzzy set □in (Y,S), by theorem 5.2, int(f^{-1} (□)) ≤ f^{-1} (int (□)).Let μ =int $[f^{-1}$ (□)] . Then, μ is a fuzzy open set in (X,T). Now μ ≤ f^{-1} (int (□)) implies that f (μ) ≤ f $[f^{-1}$ (int (□))]. Sincethe function f is onto , f $[f^{-1}$ (int (□))] = int (□) and hence f (μ) ≤int (□). Since f is a fuzzy open function from (X,T) onto (Y,S),for the fuzzy open set μ in (X,T), f (μ) is a fuzzy open set in (Y,S). Hence we haveint (□) ≠ *0,* for the fuzzyG_δ -set □in (Y,S). Therefore(Y,S) is a fuzzy almost P- space.

Proposition 5.3 : If the function f : (X,T) → (Y,S) from a fuzzy topological space (X,T) onto another fuzzy topological space (Y,S) is a fuzzy continuous and somewhat fuzzy open function and if (X,T) is a fuzzy almost P-space , then (Y,S) is a fuzzy almost P-space.

Proof : Let □ be a fuzzy G_δ-set in a fuzzy topological space (Y,S). Then, □ = $\wedge_{\square=1}^{\infty}(\square_{\square})$, where $(\lambda_i)'$sare fuzzy open sets in (Y,S).Now f^{-1} (□)= $f^{-1}(\wedge_{\square=1}^{\infty}(\lambda_i)) = \wedge_{\square=1}^{\infty} f^{-1}(\lambda_i)$. Since f is a fuzzy continuous function from (X,T) onto (Y,S) and (λ_i)’s are fuzzy open sets in (Y,S), $f^{-1}(\lambda_i)'$s are fuzzy open sets in (X,T). Thenf^{-1} (□) is a fuzzy G_δ-set in (X,T). Since(X,T) is a fuzzy almost P-space,int $[f^{-1}$ (□]≠ *0.* Then there exists a fuzzy open set μ in (X,T) such that μ ≤ f^{-1} (□). Then, f (μ) ≤ f $[f^{-1}$ (□)]. Since the function f is onto, by lemma 2.3,f [$f^{-1}(\lambda)$] = □. Hence we have f (μ) ≤□ . Since f is a somewhat fuzzy openfunction from (X,T) onto (Y,S),for the fuzzy open set μ in (X,T),there exists a fuzzy open set η in (Y,S) such that □ ≠

0 and $\eta \leq f(\mu)$. Thus we have $\eta \leq f(\mu) \leq$and hence$\eta \leq$□.Hence we have int (□) $\neq 0$,for the fuzzyG_δ -set □in (Y,S). Therefore (Y,S) is a fuzzy almost P-space.

Proposition 5.4 : If the function $f : (X,T) \rightarrow (Y,S)$ from a fuzzy topological space (X,T) onto another fuzzy topological space (Y,S) is a somewhat fuzzy continuous, one-to-one and fuzzy open function and if (Y,S) is a fuzzy almost P-space , then (X,T) is a fuzzy almost P- space.

Proof : Let □ be fuzzy G_δ-set inthe fuzzy topological space (X,T). Then □ = $\wedge_{□=1}^{\infty}$(□$_□$), where (λ_i)'sare fuzzy open sets in (X,T) . Then 1 − □ = $1 -$ $\wedge_{□=1}^{\infty}$(□$_□$) = $\vee_{□=1}^{\infty}$($1 -$ □$_□$). Since f is one - to - one and onto, by lemma 2.3, $f(1-\lambda) = 1 - f(\lambda)$ and hence $1 - f(\lambda) = f$ [$\vee_{□=1}^{\infty}$($1 -$ □$_□$)] = $\vee_{□=1}^{\infty}$ f ($1 -$ □$_□$) = $\vee_{□=1}^{\infty}$ [$1 -$ f(□$_□$)] = $1 - \wedge_{i=1}^{\infty} f(\lambda_i)$. Then , we have $f(\lambda) = \wedge_{i=1}^{\infty} f(\lambda_i)$.Since f is a fuzzy open function from (X,T) onto (Y,S), for the fuzzy open sets (λ_i)'s in (X,T), we have $(f(\lambda_i))$'s are fuzzy open sets in (Y,S) and hence $\wedge_{i=1}^{\infty} f(\lambda_i)$, is a fuzzy G_δ-set in (Y,S). Then, $f(\lambda)$ is a fuzzy G_δ-set in(Y,S). Since (Y,S) is a fuzzy almost P- space, int $[f(\lambda)] \neq 0$. Then , there exists a fuzzy open set μ in (Y,S) such that $\mu \leq f(\lambda)$. This implies that$f^{-1}(\mu) \leq f^{-1}f(\lambda)=$□ (since f is one - to - one , $f^{-1} f(\lambda) = \lambda$). Hence we have,$f^{-1}(\mu) \leq$□ . Since f is a somewhat fuzzy continuous function , for the fuzzy open set μ in (Y,S) ,there exist a fuzzy open set δ in (X ,T) such that $\delta \neq 0$ and $\delta \leq f^{-1}(\mu)$. Then$\delta \leq f^{-1}(\mu) \leq$□ and hence int (□) $\neq 0$ in (X,T).Therefore (X, T)is a fuzzy almost P-space.

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