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Expected time to recruitment in single grade manpower system under correlated wastage

J. Sridharan
Department of Mathematics, Government Arts College, Kumbakonam, TamilNadu, India.
email:jayabala_dharan@yahoo.co.in
K. Elangovan
Department of Mathematics, Rajah Serfoji Govt. College, Thanjavur, TamilNadu, India.
email:mkelango@ymail.com
and
A. Srinivasan
Department of Mathematics, Bishop Heber College, Trichy, TamilNadu, India.
email:mathsrinivas@yahoo.com

Abstract

In this paper, a single graded marketing association which is subject to loss on man-hour due to its policy decisions is considered. As the exit of personnel is unpredictable, a new univariate recruitment policy involve mandatory threshold is suggested to enable the organization to plan its decision on recruitment.

Keywords: Manpower planning, correlated wastage, Order statistics, shock model, univariate recruitment policy.
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1 Introduction

The concept of manpower planning is a very important role in Decision making problems. Many mathematical models have been discussed using different kinds of wastages and different type of distributions (see [1], [2]). Since then several authors [7]-[9] contributed to the development of the problem of time to recruitment in a single graded marketing organization involving only one threshold under different conditions. Since the number of exits in every policy decision-making epoch is unpredictable at the time, at which the cumulative loss of manhour crossing a single threshold is probabilistic, the organization has left with no choice except making recruitment immediately upon threshold crossing. Recently, Muthaiyan et. al. [6] have introduced and studied the system characteristics, that is, mean and variance of time to recruitment when the inter-decision times form an order statistics. A sequence of independent and identically distributed exponential random variables and the amount of wastage at each decision epoch forms a sequence of exchangeable and constantly correlated exponential random variables. Recall that, if the recruitment is necessary whenever the cumulative loss of manhour crosses the mandatory threshold. In view of this policy, the organization can plan the decision upon the time for recruitment. This present paper we extend the results in [5] for a single grade manpower system having loss of man-hours are exchangeable and constantly correlated exponential random variables that occur in the grade for the mathematical model which differ from each threshold for the loss of man-hour in the organization. The present paper is organized as follows: The model description is given in the second section. The mean and variance of the time to recruitment are obtained in the next section. Finally, the analytical results are numerically verified by assuming specific distributions, and the relevant conclusion is stated.

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2 Model Description and Analysis

Consider an organization with single grade in which the decisions are taken at random epochs in the interval $[0, \infty)$ and at every policy decision epoch, a random number of persons quit the organization. There is an associated loss of manhour when a person quits. It is assumed that the loss of manhour is linear and cumulative. Let X_i (i = 1, 2, ...) be the loss of manhour due to the i^{th} decision epoch forming a sequence of exchangeable and constantly correlated exponential random variables with distribution $G(\cdot)$. If $U_{(1)}$, $U_{(2)}$, $U_{(3)}$.....$U_{(k)}$ are the order statistics selected from the sample U_1, U_2, U_3,...U_k with the density functions $f_{u(1)}(\cdot)$, $f_{u(2)}(\cdot)$, $f_{u(3)}(\cdot)$....$f_{u(k)}(\cdot)$, respectively. If T is the time to recruitment in the organization with cumulative distribution function $L(\cdot)$, then the probability density function (resp. mean, variance) is denoted by $l(\cdot)$ (resp. $E(T)$, $V(T)$). Let $F_k(\cdot)$ be the k-fold convolution of $F(\cdot)$. Also $l^\star(\cdot)$, $f^\star(\cdot)$, $f^\star_{u(1)}(\cdot)$ and $f^\star_{u(k)}(\cdot)$ denotes the Laplace transform of $l(\cdot)$, $f(\cdot)$, $f_{u(1)}(\cdot)$ and $f_{u(k)}(\cdot)$, respectively. Let $V_k(t)$ be the probability that exactly k decision epochs exist in the interval $(0, t]$. We recall from Renewal theory [4] that, $V_k(t) = F_k(t) - F_{k+1}(t)$ with $F_0(t) = 1$. The univariate recruitment policy employed in this paper is as follows: If the total loss of manhour exceeds the mandatory threshold level Y, the organization must go for recruitment.
Now, we recall some basic results which was used in this paper.
The probability that the threshold level is not reached till t, that is,

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t) P(\sum_{i=1}^{k} X_i < Y).$$

Clearly we have $L(t) = 1 - P(T > t)$ and $l^\star(s) = \frac{d}{dt}L(t)$. The random variables U_1, U_2, U_3,.....U_k can be arranged in an increasing order so that we have a sequence of order statistics, namely, $U(1)$, $U(2)$, $U(3)$,....$U(k)$. Note that the random variables $U_{(1)}$, $U_{(2)}$, $U_{(3)}$,....$U_{(k)}$ are all not independent. The probability density function of $U_{(r)}$ $(r = 1, 2, ...k)$ is given by

$$f_{u(r)}(t) = r \binom{k}{r} (F(t))^{r-1} f(t) (1 - F(t))^{k-r}, \text{ where } r = 1, 2, ...k.$$

The cumulative distribution function of $S_n = X_1 + X_2 + ... + X_k$ is given by [3]

$$(1-\rho) \sum_{i=0}^{\infty} \frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}(k+i-1)!} \int_0^{\frac{y}{b}} e^{-z} z^{k+i-1} dz,$$

where $b = \alpha(1 - \rho)$, α is the mean of X_i and ρ the correlation coefficient between X_i and X_j $(i \neq j)$.

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3 Mean and variance of time to recruitment

Suppose the mandatory threshold follows an exponential distribution (called model I). Then we have

$$P(T > t) = (1-\rho)\sum_{k=0}^{\infty}\frac{(F_k(t)-F_{k+1}(t))}{(1+b\theta)^{k-1}((1-\rho+k\rho)(1+b\theta)-k\rho)}.$$

It is clear that

$$l^{\star}(s) = \frac{-(1-\rho)\sum\limits_{k=0}^{\infty}(f^{\star}(s))^k-(f^{\star}(s))^{k+1}}{(1+b\theta)^{k-1}((1-\rho+k\rho)(1+b\theta)-k\rho)}.$$

Now we shall discuss the following two cases:
Case(i): The probability density function $f(t)$ take it as a first order statistics. Then we have $f_{u(1)}(t) = kf(t)(1-F(t))^{k-1}$. Since $f(t) = \lambda e^{-\lambda t}$, the probability density function of the first order statistics is given by $f^{\star}_{u(1)}(s) = \frac{k\lambda}{s+k\lambda}$. Clearly we have

$$E(T) = -(\frac{d}{ds}l^{\star}(s))_{s=0} = \frac{(1-\rho)}{\lambda}\sum_{k=0}^{\infty}\frac{1}{k(1+b\theta)^{k-1}[(1-\rho+k\rho)(1+b\theta)-k\rho]}$$

and

$$E(T^2) = (\frac{d^2}{ds^2}l^{\star}(s))_{s=0} = \frac{2(1-\rho)}{\lambda^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2(1+b\theta)^{k-1}[(1-\rho+k\rho)(1+b\theta)-k\rho]}.$$

The variance of time to recruitment can be obtained in an usual technique and hence omitted.

Case(ii): The probability density function $f(t)$ take it as a k^{th} order statistics. Then we have $f_{u(k)}(t) = kf(t)(F(t))^{k-1}$. Since $f(t) = \lambda e^{-\lambda t}$, the probability density function of the k^{th} order statistics is given by $f^{\star}_{u(k)}(s) = \frac{k!\,\lambda^k}{(s+\lambda)(s+2\lambda)....(s+k\lambda)}$. Clearly we have

$$E(T) = \frac{(1-\rho)}{\lambda}\sum_{k=0}^{\infty}(\sum_{n=1}^{k}\frac{1}{n})\frac{1}{(1+b\theta)^{k-1}[(1-\rho+k\rho)(1+b\theta)-k\rho]}$$ and

$$E(T^2) = \frac{(1-\rho)}{\lambda^2}\sum_{k=0}^{\infty}\frac{[(2k+1)(\sum\limits_{n=1}^{k}\frac{1}{n})^2+\sum\limits_{n=1}^{k}\frac{1}{n^2}]}{(1+b\theta)^{k-1}[(1-\rho+k\rho)(1+b\theta)-k\rho]}.$$

The variance of time to recruitment can be obtained in an usual technique and hence omitted.

Suppose the mandatory threshold follows an extended exponential distribution with shape parameter 2 (called model II). Then we have

$$P(T > t) = \frac{(1-\rho)}{\theta}\sum_{k=0}^{\infty}(F_k(t)-F_{k+1}(t))(2A_k-B_k),$$

where

$$A_k = \frac{1}{(1+b\theta)^{k-1}[(1-\rho+k\rho)(1+b\theta)-k\rho]}$$

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and

$$B_k = \frac{1}{(1+b2\theta)^{k-1}[(1-\rho+k\rho)(1+b2\theta)-k\rho]}.$$

Clearly we have

$$l^\star(s) = -\frac{(1-\rho)}{\theta}\sum_{k=0}^{\infty}[(f^*(s))^k - (f^*(s))^{k+1}][2A_k - B_k],$$

Case(i): The probability density function $f(t)$ take it as a first order statistics. Clearly we have

$E(T) = \frac{(1-\rho)}{\lambda\theta}\sum_{k=0}^{\infty}\frac{1}{k}[2A_k - B_k]$ and $E(T^2) = \frac{2(1-\rho)}{\theta\lambda^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2}[2A_k - B_k]$.

The variance of time to recruitment can be obtained in an usual technique and hence omitted.

Case(ii): The probability density function $f(t)$ take it as a k^{th} order statistics.

It is clear that

$E(T) = \frac{(1-\rho)}{\lambda\theta}\sum_{k=0}^{\infty}(\sum_{n=1}^{k}\frac{1}{n})(2A_k - B_k)$ and $E(T^2) = \frac{(1-\rho)}{\theta\lambda^2}\sum_{k=0}^{\infty}[(2k+1)(\sum_{n=1}^{k}\frac{1}{n})^2 + \sum_{n=1}^{k}\frac{1}{n^2}](2A_k - B_k)$.

The variance of time to recruitment can be obtained in an usual technique and hence omitted.

Suppose that the mandatory threshold has Setting Clock Back to Zero (SCBZ) property (called Model III).
Then we have

$$P(T > t) = (1-\rho)\sum_{k=0}^{\infty}(F_k(t) - F_{k+1}(t))(pC_k + qD_k),$$

where $C_k = \frac{1}{(1+b(\theta_1+\lambda))^{k-1}[(1-\rho+k\rho)(1+b(\theta_1+\lambda))-k\rho]}$,
$D_k = \frac{1}{(1+b\theta_2)^{k-1}[(1-\rho+k\rho)(1+b\theta_2)-k\rho]}$, $p = \frac{\theta_1-\theta_2}{\lambda+\theta_1-\theta_2}$ and $q = \frac{\lambda}{\lambda+\theta_1-\theta_2}$ with $p+q = 1$.

It is clear that

$$l^\star(s) = -(1-\rho)\sum_{k=0}^{\infty}[(f^*(s))^k - (f^*(s))^{k+1}](pC_k + qD_k).$$

Case(i): The probability density function $f(t)$ take it as a first order statistics. Then we have

$$E(T) = \frac{(1-\rho)}{\lambda}\sum_{k=0}^{\infty}\frac{1}{k}(pC_k + qD_k)$$

and

$$E(T^2) = \frac{2(1-\rho)}{\lambda^2} \sum_{k=0}^{\infty} \frac{(k+1)}{k^2}(pC_k + qD_k).$$

The variance of time to recruitment can be obtained in an usual technique and hence omitted.

Case(ii): The probability density function $f(t)$ take it as a k^{th} order statistics.
It is clear that

$$E(T) = \frac{(1-\rho)}{\lambda} \sum_{k=0}^{\infty} (\sum_{n=1}^{k} \frac{1}{n})(pC_k + qD_k)$$

and

$$E(T^2) = \frac{(1-\rho)}{\lambda^2} \sum_{k=0}^{\infty} [(2k+1)(\sum_{n=1}^{k} \frac{1}{n})^2 + \sum_{n=1}^{k} \frac{1}{n^2}](pC_k + qD_k).$$

The variance of time to recruitment can be obtained in an usual technique and hence omitted.

The following tables give the mean and variance of the time to recruitment for all models discussed in this paper by keeping $\theta = \theta_1 = 0.4$, $\theta_2 = 0.2$, $\lambda = 1$, $k = 2$ and $b = 2$ and varying ρ.

Table 1: Effect of ρ on the performance measures E(T) and V(T).

model	order	ρ	-0.6	-0.3	0	0.3	0.6
I	1	E(T)	0.23148	0.19414	0.15432	0.11175	0.06614
I	1	V(T)	0.64085	0.54474	0.43914	0.32276	0.19403
I	k	E(T)	0.69444	0.58243	0.46296	0.33243	0.19841
I	k	V(T)	5.30478	4.51441	3.64368	2.68323	1.61406
II	1	E(T)	0.814	0.71246	0.5867	0.43768	0.26572
II	1	V(T)	1.7794	1.62979	1.41588	1.12148	0.72655
II	k	E(T)	2.442	2.13739	1.7601	1.31305	0.79716
II	k	V(T)	14.38663	13.24318	11.56954	9.21798	6.00753
III	1	E(T)	0.28975	0.25362	0.21835	0.17430	0.11626
III	1	V(T)	0.78529	0.69653	0.60738	0.49252	0.33527
III	k	E(T)	0.86925	0.76085	0.65506	0.52290	0.34879
III	k	V(T)	6.48820	5.76153	5.02976	4.08407	2.78499

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Table 2: Effect of ρ on the performance measures E(T) and V(T).

model	order	$\rho = 0.9$	k = 3	k = 4	k = 5
I	1	E(T)	0.00439	0.00140	0.00050
I	1	V(T)	0.01170	0.00349	0.00119
I	k	E(T)	0.02418	0.01167	0.00575
I	k	V(T)	0.32768	0.22672	0.14817
II	1	E(T)	0.01929	0.00641	0.00237
II	1	V(T)	0.05106	0.01598	0.00568
II	k	E(T)	0.10610	0.05631	0.02709
II	k	V(T)	1.42918	1.09126	0.69722
III	1	E(T)	0.01166	0.00481	0.00224
III	1	V(T)	0.03096	0.01201	0.00539
III	k	E(T)	0.06415	0.04010	0.02554
III	k	V(T)	0.86684	0.77775	0.65729

Findings
Influence of nodal parameters on the performance measures (mean and variance) of the time to recruitment for all the models discussed in this paper are stated below

1. When the correlation coefficient increases, the mean and variance of time to recruitment decreases for both first and k^{th} order statistics.

2. If k, the number of decision epochs in $(0, t]$ increases, the mean and variance of the time to recruitment decreases for both first and k^{th} order statistics..

Conclusion: From the above numerical illustrations, we conclude that the performance measures are fully depends on the values of k and ρ. Moreover, the results show that the k^{th} order statistics is preferable.

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