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IDEAL BITOPOLOGICAL b-T_0 (-T_1, -T_2) SPACES

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Abstract. In this paper we introduce and study (i, j)-b-$\mathcal{I}$-T_0-spaces, (i, j)-b-$\mathcal{I}$-T_1 and (i, j)-b-$\mathcal{I}$-T_2 spaces.

1. Introduction and Preliminaries

The concept of ideal bitopological spaces was first introduced by Kelly [2]. After the introduction of the definition of a ideal bitopological space by Kelly, a large number of topologists have turned their attention to the generalization of different concepts of a single topological space in this space. The concept of ideals in topological spaces has been introduced and studied by Kuratowski [3] and Vaidyanathasamy [4]. An ideal $\mathcal{I}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies (i) $A \in \mathcal{I}$ and $B \subset A$ implies $B \in \mathcal{I}$ and (ii) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$. Given a topological space (X, τ) with an ideal $\mathcal{I}$ on X and if $\mathcal{P}(X)$ is the set of all subsets of X, a set operator $(.)^*$: $\mathcal{P}(X) \to \mathcal{P}(X)$, called the local function [4] of A with respect to τ and $\mathcal{I}$, is defined as follows: for $A \subset X$, $A^*(\tau, \mathcal{I}) = \{x \in X | U \cap A \notin \mathcal{I}$ for every $U \in \tau(x)\}$, where $\tau(x) = \{U \in \tau | x \in U\}$. A Kuratowski closure operator $\mathrm{Cl}^*(.)$ for a topology $\tau^*(\tau, \mathcal{I})$ called the $*$-topology, finer than τ is defined by $\mathrm{Cl}^*(A) = A \cup A^*(\tau, \mathcal{I})$ when there is no chance of confusion, $A^*(\mathcal{I})$ is denoted by A^* and τ_i-$\mathrm{Int}^*(A)$ denotes the interior of A in $\tau_i^*(\mathcal{I})$. If $\mathcal{I}$ is an ideal on X, then ideal bitopological spaces is called an ideal bitopological space. Let A be a subset of a ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$. We denote the closure of A and the interior of A with respect to τ_i by τ_i-$\mathrm{Cl}(A)$ and τ_i-$\mathrm{Int}(A)$, respectively. A subset A is called (i, j)-b-$\mathcal{I}$-open [1] if $A \subset j\,\mathrm{Cl}^*(i\,\mathrm{Int}(A)) \cup i\,\mathrm{Int}(j\,\mathrm{Cl}^*(A))$. The complement of a (i, j)-b-$\mathcal{I}$-open set is called a (i, j)-b-$\mathcal{I}$-closed set. The the intersection of all (i, j)-b-$\mathcal{I}$-closed sets of X containing A is called the (i, j)-b-$\mathcal{I}$-closure of A and is denoted by (i, j)-$b\mathcal{I}\,\mathrm{Cl}(A)$. In this paper we introduce and study (i, j)-b-$\mathcal{I}$-T_0-spaces, (i, j)-b-$\mathcal{I}$-T_1 and (i, j)-b-$\mathcal{I}$-T_2 spaces.

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2. Properties of (i,j)-b-$\mathcal{I}$-T_0 (-T_1, -T_2) spaces

In this section, we introduce and study (i,j)-b-$\mathcal{I}$-T_0-spaces, (i,j)-b-$\mathcal{I}$-T_1 and (i,j)-b-$\mathcal{I}$-T_2 spaces.

Definition 1. *An ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is said to be (i,j)-b-$\mathcal{I}$-T_0 if for every pair of distinct points in X, there exists an (i,j)-b-$\mathcal{I}$-open set of X containing one of the points but not the other.*

Theorem 2.1. *An ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is (i,j)-b-$\mathcal{I}$-T_0 if, and only if for each pair of distinct points x, y of X, (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$ $\neq$ (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$.*

Proof. Let $(X,\tau_1,\tau_2,\mathcal{I})$ be an (i,j)-b-$\mathcal{I}$-T_0 space and x, y any two distinct points of X. Then there exists an (i,j)-b-$\mathcal{I}$-open set G containing x or y, say, x but not y. Then $X\backslash G$ is an (i,j)-b-$\mathcal{I}$-closed set, which does not contain x but contains y. Since (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$ is the smallest (i,j)-b-$\mathcal{I}$-closed set containing y, (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{y\}) \subset X\backslash G$, and so $x \notin (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Consequently, (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \neq (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Conversely, let $x, y \in X$, $x \neq y$ and (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \neq (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Then there exists a point $z \in X$ such that z belongs to one of the two sets, say, (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\})$ but not to (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$. If we suppose that $x \in (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$, then $z \in (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$, which is a contradiction with $z \notin (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$. Hence $x \in X\backslash(i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$, but $X\backslash(i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})$ is an (i,j)-b-$\mathcal{I}$-open set and does not contain y. This shows that $(X,\tau_1,\tau_2,\mathcal{I})$ is (i,j)-b-$\mathcal{I}$-T_0. □

Definition 2. *An ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is said to be (i,j)-b-$\mathcal{I}$-T_1 if for every pair of distinct points x, y of X, there exists a pair of (i,j)-b-$\mathcal{I}$-open sets one containing x but not y and the other containing y but not x.*

Proposition 2.2. *Every (i,j)-b-$\mathcal{I}$-T_1 space is (i,j)-b-$\mathcal{I}$-T_0.*

Proof. The proof is clear. □

The following example shows that the converse of the Proposition 2.2 is not true.

Example 2.3. *Let $X = \{a,b,c\}$, $\tau_1 = \{\emptyset,\{a\},X\}$, $\tau_2 = \{\emptyset,\{a\},X\}$ and $\mathcal{I} = \{\emptyset\}$. Clearly, $(1,2)$-$B\mathcal{I}O(X) = \{\emptyset,\{a\},\{a,b\},\{a,c\},X\}$. Then the ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is $(1,2)$-b-$\mathcal{I}$-T_0 but not $(1,2)$-b-$\mathcal{I}$-T_1.*

Theorem 2.4. *For an ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$, the following statements are equivalent:*

(1) *$(X,\tau_1,\tau_2,\mathcal{I})$ is (i,j)-b-$\mathcal{I}$-T_1.*

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(2) *Each singleton subset of X is (i,j)-b-$\mathcal{I}$-closed in X.*
(3) *Each subset of X is the intersection of all (i,j)-b-$\mathcal{I}$-open sets containing it.*
(4) *The intersection of all (i,j)-b-$\mathcal{I}$-open sets containing the point $x \in X$ is the set $\{x\}$.*

Proof. (1)⇒(2): Let $x \in X$. Then by (1), for any $y \in X$, $y \neq x$, there exists an (i,j)-b-$\mathcal{I}$-open set V_y containing y but not x. Hence $y \in V_y \subset X\backslash\{x\}$. Now varying y over $X\backslash\{x\}$, we get $X\backslash\{x\} = \cup\ \{V_y\colon y \in X\backslash\{x\}\}$. So $X\backslash\{x\}$ being a union of (i,j)-b-$\mathcal{I}$-open sets. Hence $\{x\}$ is (i,j)-b-$\mathcal{I}$-closed.
(2)⇒(3): If $A \subset X$, then for each point $y \notin A$, there exists a set $X\backslash\{y\}$ such that $A \subset X\backslash\{y\}$ and each of these sets $X\backslash\{y\}$ is (i,j)-b-$\mathcal{I}$-open. Hence $A = \cap\ \{\ X\backslash\{y\}\colon \text{y} \in X\backslash A\}$ so that the intersection of all (i,j)-b-$\mathcal{I}$-open sets containing A is the set A itself.
(3)⇒(4): Obvious.
(4)⇒(1): Let $x, y \in X$ and $x \neq y$. Hence there exists an (i,j)-b-$\mathcal{I}$-open set, say, U_x such that $y \notin U_x$. Similarly, there exists an (i,j)-b-$\mathcal{I}$-open set U_y, $x \notin U_y$. Hence $(X, \tau_1, \tau_2, \mathcal{I})$ is (i,j)-b-$\mathcal{I}$-T_1. □

Lemma 2.5. *If every finite subset of an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ is (i,j)-b-$\mathcal{I}$-closed, then it is (i,j)-b-$\mathcal{I}$-T_1.*

Proof. Let $x, y \in X$ such that $x \neq y$. Then by hypothesis, $\{x\}$ and $\{y\}$ are (i,j)-b-$\mathcal{I}$-closed sets in X. Hence $X\backslash\{x\}$ and $X\backslash\{y\}$ are (i,j)-b-$\mathcal{I}$-open subsets of X such that $x \in X\backslash\{y\}$ and $y \in X\backslash\{x\}$. Therefore, $(X, \tau_1, \tau_2, \mathcal{I})$ is (i,j)-b-$\mathcal{I}$-T_1. □

Theorem 2.6. *If an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ is (i,j)-b-$\mathcal{I}$-T_0, then (i,j)-$b\mathcal{I}\,\mathrm{Int}((i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\}))\cap(i,j)$-$b\mathcal{I}\,\mathrm{Int}((i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})) = \emptyset$ for every pair of distinct points x and y in X.*

Proof. Let $(X, \tau_1, \tau_2, \mathcal{I})$ be (i,j)-b-$\mathcal{I}$-T_0 and $x, y \in X$ such that $x \neq y$. Then there exists an (i,j)-b-$\mathcal{I}$-open set G containing one of the point, say, x but not the other. This implies that $x \in G$ and $y \notin G$, then $y \in X\backslash G$ and $X\backslash G$ is (i,j)-b-$\mathcal{I}$-closed. Now (i,j)-$b\mathcal{I}\,\mathrm{Int}(\{y\}) \subset (i,j)$-$b\mathcal{I}\,\mathrm{Int}((i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})) \subset X\backslash G$. Hence $G \subset X\backslash(i,j)$-$b\mathcal{I}\,\mathrm{Int}((i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\}))$. But $x \in G \subset X\backslash(i,j)$-$b\mathcal{I}\,\mathrm{Int}((i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\}))$. Then (i,j)-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset X\backslash((i,j)$-$b\mathcal{I}\,\mathrm{Int}((i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})))$. This implies that (i,j)-$b\mathcal{I}\,\mathrm{Int}((i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\})) \subset (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset X\backslash(i,j)$-$b\mathcal{I}\,\mathrm{Int}((i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\}))$. Therefore, (i,j)-$b\mathcal{I}\,\mathrm{Int}((i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{x\}))\cap(i,j)$-$b\mathcal{I}\,\mathrm{Int}((i,j)$-$b\mathcal{I}\,\mathrm{Cl}(\{y\})) = \emptyset$. □

Definition 3. *An ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ is said to be (i,j)-b-$\mathcal{I}$-T_2 if for every pair of distinct points x, y of X, there exists a pair of disjoint (i,j)-b-$\mathcal{I}$-open sets, one containing x and the other containing y.*

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Proposition 2.7. *Every* (i, j)*-*b*-*$\mathcal{I}$*-*T_2 *space is* (i, j)*-*b*-*$\mathcal{I}$*-*T_1*.*

Proof. The proof is clear. □

The following example shows that the converse of the Proposition 2.7 is not true.

Example 2.8. *Let* $X = \mathbb{R}$*,* τ_1 *be the cofinite topology on* $\mathbb{R}$ *and* τ_2 *the discrete topology on* $\mathbb{R}$ *and* $\mathcal{I} = \{\emptyset\}$*. Then the ideal bitopological space* $(X, \tau_1, \tau_2, \mathcal{I})$ *is* $(1, 2)$*-*b*-*$\mathcal{I}$*-*T_1 *but not* $(1, 2)$*-*b*-*$\mathcal{I}$*-*T_2*.*

Theorem 2.9. *For a ideal bitopological space* $(X, \tau_1, \tau_2, \mathcal{I})$*, the following statements are equivalent:*

(1) $(X, \tau_1, \tau_2, \mathcal{I})$ *is* (i, j)*-*b*-*$\mathcal{I}$*-*T_2*.*
(2) *Let* $x \in X$*. For each* $y \neq x$*, there exists* $U \in (i, j)\text{-}B\mathcal{I}O(X, x)$ *and* $y \notin (i, j)\text{-}b\mathcal{I}\,\mathrm{Cl}(U)$*.*
(3) *For each* $x \in X$*,* $\cap\{(i, j)\text{-}b\mathcal{I}\,\mathrm{Cl}(U_x) : U_x$ *is an* (i, j)*-*b*-*$\mathcal{I}$*-open set containing* $x\} = \{x\}$*.*

Proof. (1)⇒(2): Let $x \in X$ and $y \neq x$. Then there exist disjoint (i, j)-b-$\mathcal{I}$-open sets U and V such that $x \in U$ and $y \in V$. Clearly, $X \backslash V$ is (i, j)-b-$\mathcal{I}$-closed. Then $(i, j)\text{-}b\mathcal{I}\,\mathrm{Cl}(U) \subset X \backslash V$ and hence $y \notin (i, j)\text{-}b\mathcal{I}\,\mathrm{Cl}(U)$.
(2)⇒(3): If $y \neq x$, then there exists $U \in (i, j)\text{-}B\mathcal{I}O(X, x)$ and $y \notin (i, j)\text{-}b\mathcal{I}\,\mathrm{Cl}(U)$. Hence $y \notin \cap\{(i, j)\text{-}b\mathcal{I}\,\mathrm{Cl}(U) : U \in (i, j)\text{-}B\mathcal{I}O(X, x)\}$.
(3)⇒(1): Let $x, y \in X$ such that $x \neq y$. Then $y \notin \{x\} = \cap\{(i, j)\text{-}b\mathcal{I}\,\mathrm{Cl}(U) : U \in (i, j)\text{-}B\mathcal{I}O(X, x)\}$. Hence, $y \notin (i, j)\text{-}b\mathcal{I}\,\mathrm{Cl}(U)$ for some (i, j)-b-$\mathcal{I}$-open set U containing x. Clearly, U and $X \backslash (i, j)\text{-}b\mathcal{I}\,\mathrm{Cl}(U)$ are the required (i, j)-b-$\mathcal{I}$-open sets containing x and y, respectively. □

Theorem 2.10. *An ideal bitopological space* $(X, \tau_1, \tau_2, \mathcal{I})$ *is* (i, j)*-*b*-*$\mathcal{I}$*-*T_2 *if, and only if for each pair of distinct points* $x, y \in X$*, there exists an* (i, j)*-*b*-*$\mathcal{I}$*-clopen set* U *containing one of them but not the other.*

Proof. Let $(X, \tau_1, \tau_2, \mathcal{I})$ be an (i, j)-b-$\mathcal{I}$-T_2 space and $x, y \in X$ such that $x \neq y$ implies that there exist two disjoint (i, j)-b-$\mathcal{I}$-open sets U and V such that $x \in U$ and $y \in V$. Since $U \cap V = \emptyset$ and V is an (i, j)-b-$\mathcal{I}$-open set, $x \in U \subset X \backslash V$ and $X \backslash V$ is (i, j)-b-$\mathcal{I}$-closed. Since $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-T_2, for each $x \in X \backslash V$ there exists an (i, j)-b-$\mathcal{I}$-open set U_x such that $x \in U_x \subset X \backslash V$. Then $X \backslash V$ is (i, j)-b-$\mathcal{I}$-open. Therefore, $X \backslash V$ is an (i, j)-b-$\mathcal{I}$-clopen set. Conversely, suppose for every pair of distinct points $x, y \in X$, there exists an (i, j)-b-$\mathcal{I}$-clopen set U containing x but not y. Then $X \backslash U$ is an (i, j)-b-$\mathcal{I}$-open set and $y \in X \backslash U$. Since $U \cap (X \backslash U) = \emptyset$, $(X, \tau_1, \tau_2, \mathcal{I})$ is an (i, j)-b-$\mathcal{I}$-T_2 space. □

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Theorem 2.11. *For an ideal bitoplogical space $(X, \tau_1, \tau_2, \mathcal{I})$, the following statements are equivalent:*

(1) $(X, \tau_1, \tau_2, \mathcal{I})$ *is* (i, j)-b-$\mathcal{I}$-T_2.
(2) *The intersection of all* (i, j)-b-$\mathcal{I}$-*clopen sets of each point in* X *is singleton.*
(3) *For a finite number of distinct points* x_i $(1 \leq i \leq n)$, *there exists an* (i, j)-b-$\mathcal{I}$-*open set* G_i *such that* G_i $(1 \leq i \leq n)$ *are pairwise disjoint.*

Proof. (1)$\Rightarrow$ (2): Let $(X, \tau_1, \tau_2, \mathcal{I})$ be an (i, j)-b-$\mathcal{I}$-T_2 and $x \in X$. Suppose $\cap\{G : G$ is (i, j)-b-$\mathcal{I}$-clopen and $x \in G\} = \{x, y\}$ where $x \neq y$. Since $(X, \tau_1, \tau_2, \mathcal{I})$ is (i, j)-b-$\mathcal{I}$-T_2, there exist two disjoint (i, j)-b-$\mathcal{I}$-open sets U and V such that $x \in U$ and $y \in V$. Then $x \in U \subset X \backslash V$; hence $X \backslash V$ is (i, j)-b-$\mathcal{I}$-open set and also it is (i, j)-b-$\mathcal{I}$-closed. Hence $X \backslash V$ is (i, j)-b-$\mathcal{I}$-clopen set containing x but not y, which is a contradiction. Thus the intersection of all (i, j)-b-$\mathcal{I}$-clopen sets containing x is $\{x\}$.
(2)$\Rightarrow$ (3): Let $\{x_1, x_2, x_3, ..., x_n\}$ be a finite number of distinct points of X. Then by (2), $\{x_i\} = \cap\{F : F$ is (i, j)-b-$\mathcal{I}$-clopen set and $x_i \in F\}$ for $i = 1, 2, 3..., n$. Since $x_j \in \{x_i\}$, for $i, j = 1, 2...., n$ and $i \neq j$, there exists an (i, j)-b-$\mathcal{I}$-clopen set F_0 such that $x_i \in F_0$ and $x_j \notin F_0$ for $i \neq j$ $(1 \leq i, j \leq n)$. Then $x_i \in X \backslash F_0$, where $X \backslash F_0$ is an (i, j)-b-$\mathcal{I}$-clopen set and $F_0 \cap (X \backslash F_0) = \emptyset$. Hence, $X \backslash F_0$ is an (i, j)-b-$\mathcal{I}$-open set containing x_i. Therefore, for each i, there exist pairwise disjoint (i, j)-b-$\mathcal{I}$-open sets G_i for x_i $(1 \leq i \leq n)$.
(3)$\Rightarrow$(1): Obvious. $\square$

Theorem 2.12. *If A is an (i, j)-b-$\mathcal{I}$-compact subset of an (i, j)-b-$\mathcal{I}$-T_2 space in $(X, \tau_1, \tau_2, \mathcal{I})$ and $x \in X \backslash A$, then there is an (i, j)-b-$\mathcal{I}$-open set B such that $A \subset B$ (or there is a set G such that $x \in G \subset X \backslash B$).*

Proof. Suppose A is an (i, j)-b-$\mathcal{I}$-compact subset of X and $x \in X \backslash A$. Then for each $a \in A$, there exist disjoint (i, j)-b-$\mathcal{I}$-open sets U_x and V_a such that $x \in U_x$ and $a \in V_a$. Then the collection $\{V_a : a \in A\}$ is an (i, j)-b-$\mathcal{I}$-open covering of A. Since A is (i, j)-b-$\mathcal{I}$-compact, there is a finite subcollection, say, $\{V_{a_1}, V_{a_2}, ..., V_{a_m}\}$ of (i, j)-b-$\mathcal{I}$-open sets covering A. Let $B = \overset{m}{\underset{i=1}{\cup}} V_{a_i}$. Then clearly B is (i, j)-b-$\mathcal{I}$-open and $A \subset B$. $\square$

Lemma 2.13. *Let $(X, \tau_1, \tau_2, \mathcal{I})$ be an (i, j)-b-$\mathcal{I}$-compact (i, j)-b-$\mathcal{I}$-T_2 space. If every point of X is an (i, j)-b-$\mathcal{I}$-limit point of X and, given x in X and an (i, j)-b-$\mathcal{I}$-open subset A of X, then there exists a nonempty set B contained in A such that (i, j)-$b\mathcal{I}\,\mathrm{Cl}(B)$ does not contain x.*

Proof. Suppose $x \in A$. Since x is an (i, j)-b-$\mathcal{I}$-limit point of X, $A \cap (X \backslash \{x\}) \neq \emptyset$. So we can choose a point $y \in A$. Suppose $x \notin A$, then also we can choose a point $y \in A$, since A is nonempty, that is, $x, y \in X$, $x \neq y$ and $y \in A$. Since the space $(X, \tau_1, \tau_2, \mathcal{I})$ is (i,j)-b-$\mathcal{I}$-T_2, there exist disjoint (i,j)-b-$\mathcal{I}$-open sets U_x and U_y such that $x \in U_x$ and $y \in U_y$. Let $B = A \cap U_y$. Then $B \subset A$ and (i,j)-$b\mathcal{I}\,\mathrm{Cl}(B)$ is the intersection of all (i,j)-b-$\mathcal{I}$-closed set containing B. Clearly, the (i,j)-b-$\mathcal{I}$-closed set (i,j)-$b\mathcal{I}\,\mathrm{Cl}(B)$ does not contain x. □

Theorem 2.14. *Suppose $(X, \tau_1, \tau_2, \mathcal{I})$ is an (i,j)-b-$\mathcal{I}$-compact (i,j)-b-$\mathcal{I}$-T_2 space and every point of X is an (i,j)-b-$\mathcal{I}$-limit point of X. If $\{A_\alpha\}_{\alpha \in \Delta}$ is a nonempty nested collection of (i,j)-b-$\mathcal{I}$-open sets, then X is uncountable.*

Proof. We will prove that the function $f : \mathbb{N} \to X$ defined by $f(x) = x_n$ is not onto. Without loss of generality assume that $A_0 \supset A_1 \supset A_2 \supset .....$ Take $A_0 = X$. Since A_0 is a nonempty (i,j)-b-$\mathcal{I}$-open set and $x_1 \in X$, then by Lemma 2.13, there exists a set $V_1 \subset A_0$ such that (i,j)-$b\mathcal{I}\,\mathrm{Cl}(V_1)$ does not contain x_1. In general, given a nonempty (i,j)-b-$\mathcal{I}$-open set A_{n-1}, there exists a set $V_n \subset A_{n-1}$ such that (i,j)-$b\mathcal{I}\,\mathrm{Cl}(V_n)$ does not contain x_n. Now the collection $\{(i,j)$-$b\mathcal{I}\,\mathrm{Cl}(V_1), (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(V_2), ...\}$ is a nested collection of (i,j)-b-$\mathcal{I}$-closed sets satisfying the finite intersection condition. Since $(X, \tau_1, \tau_2, \mathcal{I})$ is (i,j)-b-$\mathcal{I}$-compact, $\cap(i,j)$-$b\mathcal{I}\,\mathrm{Cl}(V_i) \neq \emptyset$. Let $x \in \cap(i,j)$-$b\mathcal{I}\,\mathrm{Cl}(V_i)$. Then $x \in (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(V_i)$ for every i, then this $x \neq x_n$ for any n. Since $x \in (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(V_i)$ for all i, but $x_n \notin (i,j)$-$b\mathcal{I}\,\mathrm{Cl}(V_n)$ for any n. Hence X is uncountable. □

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