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Optimal Riser design by Fuzzy Geometric Programming Technique

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Abstract

This paper represents an approach for solving geometric programming problem under fuzzy environment. A pair of bi-level mathematical program is formulated to calculate the lower and upper bounds of the objective value. The solution is in an interval. A mathematical tool fuzzy geometric programming has made a approach to the problem of riser design. With this technique generalized solutions can be obtained for dimensioning risers.

Key words: Optimization, fuzzy geometric programming, duality theorem, riser design problem, Chvorinov's rule.

1. Introduction

Geometric programming (GP) is a technique to solve the special class of non - linear programming problems subject to linear or non-linear constraints. The original mathematical development of this method used the arithmetic–geometric mean inequality relationship between sums and products of real numbers. In 1967 Duffin, Peterson and Zener put a foundation stone to solve wide range of engineering problems by developing basic theories of geometric programming in the book **Geometric Programming [3]**. Beightler and Phillips gave a full account of entire modern theory of geometric programming and numerous examples of successful applications of geometric programming to real-world problems in their book **Applied Geometric Programming [1]**. GP method has certain advantages. The advantage is that it is easy to solve the dual problem than primal. In 1990 R. k. verma [**4**] has studied fuzzy programming technique to solve geometric programming problems. In this field a paper named Geometric programming problems with fuzzy parameters and its application to crane load sway by S.Yousef, N.Badra and T.G. Abu-El Yazied [5] has been published in world applied sciences journal in 2009.

This paper represents an approach for solving geometric programming problem under uncertainty. A pair of bi-level mathematical program is formulated to calculate the lower and upper bounds of the objective value. The solution is in an interval. One illustrative example is presented to clarify the proposed approach.

2. Fuzzy approach for geometric programming problem

A geometric programming problem can be defined as

Find $x=(x_1, x_2,\ldots\ldots\ldots\ldots,x_n)^T$,so as to

$$\text{Min: } f_{k0}(x) = \sum_{t=1}^{T_{k0}} \widetilde{c_{k0t}} \prod_{j=1}^{n} x_j^{a_{k0tj}} \qquad k=1,2,\ldots\ldots\ldots\ldots,p \qquad \ldots\ldots\ldots(1)$$

$$\text{Such that } f_i(x) = \sum_{t=1}^{T_i} \widetilde{c_{it}} \prod_{j=1}^{n} x_j^{a_{itj}} \leq \widetilde{b_i}\ , \qquad i=1,2,\ldots\ldots,m$$

$$x_j > 0, \qquad j=1,2,\ldots\ldots\ldots\ldots,n$$

Where $c_{k0t} > 0$ for all k and t. a_{itj} ,a_{k0tj} are all real ,for all i, k, t, j .

The objective function coefficients, the constraints coefficients and the right-hand sides of the constraints may be represented in triangular fuzzy number (TFN).

i.e. $\widetilde{c_{k0t}} = [c_{k0t}^L, c_{k0t}^M, c_{k0t}^U]$, $\widetilde{c_{it}} = [c_{it}^L, c_{it}^M, c_{it}^U]$, $\widetilde{b_i} = [b_i^L, b_i^M, b_i^U]$

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Let f_{k0}^{L} and f_{k0}^{U} be the minimum and maximum of f_{k0}. Fuzzy primal geometric programming problem (FPGPP) as follows:

$$f_{k0}^{L} = min \sum_{k=1}^{p} w_k \sum_{t=1}^{T_{k0}} (c_{k0t}^{L} + \alpha(c_{k0t}^{M} - c_{k0t}^{L}) \prod_{j=1}^{n} x_j^{a_{k0tj}}$$

Subject to $\sum_{t=1}^{T_i} (c_{it}^{L} + \alpha(c_{it}^{M} - c_{it}^{L})((b_i^{U} + \alpha(b_i^{U} - b_i^{M})^{-1}) \prod_{j=1}^{n} x_j^{a_{itj}} \leq 1,$

i=1,2,……….m

$x_j > 0,$ j= 1,2,………..,n

$0 \leq \alpha \leq 1$ ………………….(2)

Beightler and Philips [1] and Duffin et al. [3], the model (2) can be transformed to the corresponding dual geometric programming problem (FDGPP) as follows:

f_{k0}^{L} = Maximize

$$\prod_{k=1}^{p} \prod_{t=1}^{T_{k0}} \left(\frac{w_k (c_{k0t}^{L} + \alpha(c_{k0t}^{M} - c_{k0t}^{L})}{w_{kt}} \right)^{w_{kt}}$$

$$\prod_{i=1}^{m} \prod_{t=1}^{T_i} \left(\frac{(c_{it}^{L} + \alpha(c_{it}^{M} - c_{it}^{L})((b_i^{U} + \alpha(b_i^{U} - b_i^{M})^{-1})}{w_{it}} \right)^{w_{it}} \prod_{i=1}^{m} \lambda_i(w)^{\lambda_i(w)}$$

Where $\lambda_i(w) = \sum_{t=1}^{T_i} w_{it},$ i= 1,2,………..m

Subject to $\sum_{k=1}^{p} \sum_{t=1}^{T_{k0}} w_{kt} = 1$

$\sum_{k=1}^{p} \sum_{t=1}^{T_{k0}} a_{k0tj} w_{kt} + \sum_{i=1}^{m} \sum_{t=1}^{T_i} a_{itj} w_{it} = 0$, j =1,2,…………,n

$W_{kt} \geq 0$, k= 1,2,…………,p

t=1,2,…………, T_{k0}

$w_{it} \geq 0$ i= 1,2,…………,m.

…………………(3)

also for the upper bound of the objective value we have the following FPGPP.

$$f_{k0}^{U} = min \sum_{k=1}^{p} w_k \sum_{t=1}^{T_{k0}} (c_{k0t}^{U} - \alpha(c_{k0t}^{U} - c_{k0t}^{M}) \prod_{j=1}^{n} x_j^{a_{k0tj}}$$

Subject to $\sum_{t=1}^{T_i} (c_{it}^{U} - \alpha(c_{it}^{U} - c_{it}^{M})((b_i^{L} + \alpha(b_i^{M} - b_i^{L})^{-1}) \prod_{j=1}^{n} x_j^{a_{itj}} \leq 1,$

I=1,2,……….m

$x_j > 0,$ $\quad j = 1,2,\ldots\ldots\ldots,n$

$0 \le \alpha \le 1$ $\quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$

Beightler and Philips [1] and Duffin et al.[3], the model (2)can be transformed to the corresponding dual geometric programming problem(FDGPP)as follows:

$f_{k0}{}^{U}$ = Maximize

$$\prod_{k=1}^{p}\prod_{t=1}^{T_{k0}}\left(\frac{w_k\,(c_{k0t}{}^{U}-\alpha(c_{k0t}{}^{U}-c_{k0t}{}^{M})}{w_{kt}}\right)^{w_{kt}}$$

$$\prod_{i=1}^{m}\prod_{t=1}^{T_i}\left(\frac{(c_{it}{}^{U}-\alpha(c_{it}{}^{U}-c_{it}{}^{M})((b_i{}^{L}+\alpha(b_i{}^{M}-b_i{}^{L})^{-1})}{w_{it}}\right)^{w_{it}}\prod_{i=1}^{m}\lambda_i(w)^{\lambda_i(w)}$$

Where $\lambda_i(w) = \sum_{t=1}^{T_i} w_{it},$ $\quad i = 1,2,\ldots\ldots\ldots m$

Subject to $\sum_{k=1}^{p}\sum_{t=1}^{T_{k0}} w_{kt} = 1$

$$\sum_{k=1}^{p}\sum_{t=1}^{T_{k0}} a_{k0tj}\, w_{kt} + \sum_{i=1}^{m}\sum_{t=1}^{T_i} a_{itj}\, w_{it} = 0\,, \quad j = 1,2,\ldots\ldots\ldots,n$$

$W_{kt} \ge 0$, $\quad k = 1,2,\ldots\ldots\ldots,p$

$\quad t = 1,2,\ldots\ldots\ldots, T_{k0}$

$w_{it} \ge 0$, $\quad i = 1,2,\ldots\ldots\ldots,m.\ \ldots\ldots(5)$

3. Riser design problem

A cylindrical side riser which consists of a cylinder of height H and diameter D. The theoritical basis for riser design is Chvorinov's rule, which is $t = k\,(V/SA)^2$.

Where t = solidification time (minutes/seconds)

K = solidification constant for molding material (minutes/in^2 or seconds/cm^2)

V = riser volume (in^3 or cm^3)

SA = cooling surface area of the riser.

The objective is to design the smallest riser such that $t_R \ge t_C$

Where t_R = solidification time of the riser.

t_C = solidification time of the casting.

$K_R (V_R/SA_R)^2 \geq K_C (V_C/SA_C)^2$

The riser and the casting are assumed to be molded in the same material, so that K_R and K_C are equal. So $(V_R/SA_R) \geq (V_C/SA_C)$.

The casting has a specified volume and surface area, so $V_C/SA_C = Y$ = constant, which is called the casting modulus.

$(V_R/SA_R) \geq Y$, $V_R = \pi D^2H/4$, $SA_R = \pi DH + 2\pi D^2/4$

$(\pi D^2H/4)/(\pi DH + 2\pi D^2/4) = (DH)/(4H+2D) \geq Y$

Or, $4YD^{-1} + 2YH^{-1} \leq 1$

Primal side cylindrical riser design problem can be stated as:

Minimize $V = \pi D^2H/4$

Subject to $4YD^{-1} + 2YH^{-1} \leq 1$.

4. Illustrative example

Min $\pi D^2H/4$

Subject to $4YD^{-1} + 2YH^{-1} \leq 1$.

Here Y is a fuzzy number, $Y = (0.75, 1, 1.2)$

Min $z = \frac{\pi D^2 H}{4}$

Subject to $4(0.75, 1, 1.2)\, D^{-1} + 2(0.75, 1, 1.2)\, H^{-1} \leq 1$

$D, H > 0$

Here degrees of difficulty = 3-(2+1) = 0

Dual geometric programming problem is

$$Z^L = \text{Max} \left(\frac{\pi}{4w_{01}}\right)^{w_{01}} \left(\frac{4(0.75+0.25\alpha)}{w_{02}}\right)^{w_{02}} \left(\frac{2(0.75+0.25\alpha)}{w_{03}}\right)^{w_{03}} (w_{02}+w_{03})^{w_{02}+w_{03}}$$

Subject to $w_{01} = 1$

$2w_{01} - w_{02} = 0$

$W_{01} - w_{03} = 0$

So $w_{01} = 1, w_{02} = 2, w_{03} = 1.$

Primal-dual variable relations are

$\frac{\pi D^2 H}{4} = w_{01}\, d(w)$

$4(0.75+0.25\alpha)\, D^{-1} = \frac{w_{02}}{w_{02}+w_{03}}$

$2(0.75+0.25\alpha)\, H^{-1} = \frac{w_{03}}{w_{02}+w_{03}}$

$D = H = 6(0.75+0.25\alpha)$

$Z^L = \frac{297}{14}(1.5+0.50\alpha)^3$

Similarly

$$Z^U = \text{Max}\left(\frac{\pi}{4w_{01}}\right)^{w_{01}}\left(\frac{4(1.2-0.2\alpha)}{w_{02}}\right)^{w_{02}}\left(\frac{2(1.2-0.2\alpha)}{w_{03}}\right)^{w_{03}}(w_{02}+w_{03})^{w_{02}+w_{03}}$$

Subject to $w_{01} = 1$

$2w_{01} - w_{02} = 0$

$W_{01} - w_{03} = 0$

So $w_{01} = 1, w_{02} = 2, w_{03} = 1.$

Primal-dual variable relations are

$\frac{\pi D^2 H}{4} = w_{01}\, d(w)$

$4(1.2-0.2\alpha)\, D^{-1} = \frac{w_{02}}{w_{02}+w_{03}}$

$2(1.2-0.2\alpha)\, H^{-1} = \frac{w_{03}}{w_{02}+w_{03}}$

$D = H = 6(1.2-0.2\alpha)$

$Z^U = \frac{297}{14}(2.4-0.4\alpha)^3$

α	D=H	Z^L
0	4.50	71.59
0.1	4.65	78.99
0.2	4.80	86.89
0.3	4.95	95.29
0.4	5.10	104.22
0.5	5.25	113.69
0.6	5.40	123.72
0.7	5.55	134.32
0.8	5.70	145.50
0.9	5.85	157.30
1	6.00	169.71

Table 1: Results of fuzzy optimization for the lower bound objective function

α	D=H	Z^U
0	7.20	293.26
0.1	7.08	278.84
0.2	6.96	264.90
0.3	6.84	251.43
0.4	6.72	238.43
0.5	6.60	225.88
0.6	6.48	213.79
0.7	6.36	202.13
0.8	6.24	190.90
0.9	6.12	180.10

1	6.00	169.71

Table 2: Results of fuzzy optimization for the upper bound objective function.

5. Conclusion

Here lower and upper bounds of objective functions are calculated by a pair of bi-level mathematical program using fuzzy geometric programming technique. The solution is obtained in an interval. The optimal solution together with optimal decision variables and optimal objective functions for different values of the parameter α in [0, 1] are obtained for lower and upper bounds. This proposed approach can also be solved by intuitionistic fuzzy geometric programming technique.

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