

Analysis of Photonic Band Structure in 1-D Photonic Crystal using PWE and FDTD Method

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Abstract

A comparative study is performed for calculating the photonic band gap in photonic crystal by using two numerical method, namely Plane Wave Expansion (PWE) and Finite Difference Time Domain Method (FDTD). A photonic crystal (PC) is said to be an artificially periodic layered structure that is known to possess photonic band gaps (PBGs). Photonic band gap is the range of frequency where the light can not propagate through the structure. In order to obtain most general idea of Phc's characteristics as well as to effectively design it, it is convenient to use band structures which gives full information about radiation behavior when propagating within the specific direction inside the photonic crystal using band structure.[1]

Keywords: *Forbidden gap, PWE, FDTD, Eigen state, photonic band gap.*

1.Introduction

Photonic crystals are periodic material structures that affect the motion of photons propagating through them. They affect the photons in the same way as semiconductor crystal affects the motion of electrons. The crystals consist of periodic dielectric structures that affect electromagnetic wave propagation. A dielectric is a substance that resists electric current, meaning it acts as an electrical insulator. In some instances, a lack of substance or vacancy can act as a dielectric (e.g., air). The photonic crystals affect propagation by allowing electromagnetic waves of certain wavelengths to pass while blocking others. A range of blocked wavelengths is called a photonic band gap. There are some naturally occurring photonic crystals such as the gemstone opal and the substance that comprises butterfly wings [2].

Diffraction is a basic principle behind the photonic crystal function. Diffraction refers to different wave propagation characteristics such as bending, spreading, and interference. Diffraction effects occur most when the wavelength and affecting medium structure are on the same scale. The visible spectrum occurs on the 400- to 700-nm wavelength scale. As a result, the feature periodicity of the photonic crystals must also exist on this scale. The most important property which determines practical significance of the PhC is the presence of the photonic band gap. The photonic band gap (PBG) refers to the energy or frequency range where the light propagation is prohibited inside the PhC. When the radiation with frequency inside the PBG incidents the structure, it appears to be completely reflected. However, if one introduces the defect to the strictly periodic structure, the effect of such a defect is the same as the defect introduction to the crystalline structure of a semiconductor. This means that new eigen-state appears inside the PBG with energy corresponding to the eigen-frequency of the defect. Thus, the radiation within the defect frequency will propagate inside the structure or, in case of multiple defects radiation will be guided like in waveguide. Thus, there exist quite strong analogy between PhC physics and solid-state physics both from the physical and mathematical points of view[3]. Nature already makes photonic crystals, in the sparkling gem opal, and in the colors of butterfly wings. These have photonic band structures though not full photonic bandgaps.[4]

2. Basic Equations

In the band structure computation of a PhC by PWE method, Dispersion relation is calculated. In order to obtain the dispersion relation, it is necessary to solve the eigen- problem formulated for the Helmholtz equation inside infinite periodic structure which is given below:

$$\hat{\theta}H = \left(\frac{\omega^2}{c^2}\right)H$$

Where

$$\hat{\theta} = -\left(\frac{\partial}{\partial x}\right)\left(\frac{1}{\epsilon(x)}\right)\left(\frac{\partial}{\partial x}\right)$$

Where H is the eigen function, $\epsilon(x)$ is the periodic dielectric function, ω is eigen frequency.

Band structure computation of 1-D photonic crystal is carry out by using the equation of magnetic field component given below;

$$\frac{\partial}{\partial x} \frac{1}{\epsilon(x)} \frac{\partial}{\partial x} H(x) + \frac{\omega^2}{c^2} H(x) = 0$$

Where H is the magnetic field component, ω is the eigen frequency, c is the speed of light.

In the band structure computation by FDTD method, Bloch periodic boundary conditions are using as given by:

$$E(x + a, y + b, z + c) = E(x, y, z). e^{-i.k_x.a - j.k_y.b - k.k_z.c}$$

$$H(x + a, y + b, z + c) = H(x, y, z). e^{-i.k_x.a - j.k_y.b - k.k_z.c}$$

3. Techniques Involved

Two techniques are used :

- Plane Wave Expansion (PWE) method.
- Finite Difference Time Domain (FDTD) method

The PWE method is widely used for the computation of the band structures. It allows to solve the eigen- problem formulated for the specific periodic structure

and requires mathematical description of the PhC lattice and the Bloch theorem is used for representation of the eigen-functions in the general form. In contrast to PWE method, the FDTD provides possibility of the refractive index variation during the computation process , which allows to take into account losses and nonlinearity when computing the band structure.

Computation of the band structure is more convenient to carry out using the equation for magnetic field component. The equation in this case takes the following form:

$$\frac{\partial}{\partial x} \frac{1}{\epsilon(x)} \frac{\partial}{\partial x} H(x) + \frac{\omega^2}{c^2} H(x) = 0 \quad (3.1)$$

In order to solve the eigen-value equation, some approximate methods should be used which employ the periodicity of the permittivity distribution. For instance, we know that the eigen-function of an infinite periodic structure will also be infinite and periodic. That's why the Bloch theorem should be used for representation of the eigen-function of the PhC. The Bloch theorem says that eigen-function of an infinite periodic structure can be represented in the form of plane wave multiplied by some kind of periodic function with periodicity of lattice. Such a function has the following form [5]:

$$H(x) = h_{k,n}(x). \exp(j. k. x) \quad (3.2)$$

Where $h_{k,n}(x)$ is the periodic function in representation of wave vectors k and state number n. expand the above equation to fourier series by reciprocal lattice vectors

$$H(x) = \sum_G h_{k,n}(G). \exp(j. (k + G). x) \quad (3.3)$$

Where $h_{k,n}(G)$ is the previously defined periodic function. It is also the fourier expansion coefficient.

However for convenience of further operation, we will expand the fourier series not permittivity but inverted dielectric functions as it is written down

$$\frac{1}{\epsilon(x)} = \sum_{G'' \in G} \chi(G''). \exp(j. G''. x) \quad (3.4)$$

Where $\chi(G)$ are fourier expansion coefficients of the inverted dielectric function. After the expansion of all the infinite function we put them in equation (3.1)

$$\sum_{G''} \frac{\partial}{\partial(x)} \chi(G) \cdot \exp(j \cdot G'' \cdot x) \frac{\partial}{\partial(x)} \sum_{G'} h_{k,n}(G') \cdot \exp(j \cdot (k + G') \cdot x) + \frac{\omega^2}{c^2} \sum_G h_{k,n}(G) \cdot \exp(j \cdot (k + G) \cdot x) = 0 \quad (3.5)$$

Taking into account that $G=G''+G'$ we reduce the first summand of the expression to the common summation index

$$\sum_G \sum_{G'} \frac{\partial}{\partial(x)} \chi(G - G') \cdot \exp(j \cdot (G - G') \cdot x) \cdot h_{k,n}(G') \frac{\partial}{\partial(x)} \exp(j \cdot (k + G') \cdot x) + \frac{\omega^2}{c^2} \sum_G h_{k,n}(G) \cdot \exp(j \cdot (k + G) \cdot x) = 0 \quad (3.6)$$

Then taking the derivatives and combining the exponents. We obtain:

$$\sum_G \sum_{G'} \chi(G - G') h_{k,n}(G') \exp(j \cdot (k + G) \cdot x) \times (j \cdot (k + G') \cdot j \cdot (k + G)) + \frac{\omega^2}{c^2} \sum_G h_{k,n}(G) \cdot \exp(j \cdot (k + G) \cdot x) = 0 \quad (3.7)$$

The projection to the basic $\exp(j \cdot (k + G) \cdot x)$ gives the equation:

$$-\sum_{G'} \chi(G - G') ((k + G') \cdot (k + G)) \cdot h_{k,n}(G') + \frac{\omega^2}{c^2} h_{k,n}(G) = 0 \quad (3.8)$$

This equation is known as “Master equation” for 1-D Phc. A set of solutions of the equations system (3.8) can be found as eigen- values of matrix differential operator

FDTD technique which is able to show transient evolution of the interactions between electromagnetic wave and physical objects, not only has the advantage in dispersive and nonlinear material simulations, but also has the ability to model circuit elements including quiet a few semiconductor devices. We incorporated FDTD after the analysis by PWE method for the band structure computation. Latest techniques involved in the FDTD simulations of metamaterials.

In general, PBG computation using FDTD should be carried out as follows:

1. Determine the computation area.
2. Set up the periodic boundary conditions.
3. Define the radiation excitation function. The radiation spectrum should be wide enough to cover whole investigated frequency range.
4. Carry out the spectral analysis of time-dependent response of the structure on the probe pulse by searching all of local maxima and plotting them over frequency axis.
5. Repeat steps from 2 to 4 at different values of the phase shift in periodic boundary conditions corresponding to all selected points within the PhC Brillouin zone the band structure is computed for solution.

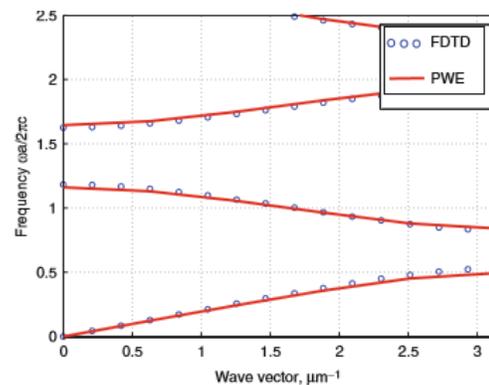


Fig.1. Comparison of 1- D Phc Band structure between PWE and FDTD.

4.Numerical Results and Discussions

We analysed the program for photonic band structure computation of 1D photonic crystal. Unit cell consist of the two layers. Structure is defined by the thickness of layers within the unit cell as well as their refractive index. Input parameters: thickness of layers and refractive index of each layer inside the unit cell. Output data: dependence of normalized frequency on wave vector (photonic band structure).

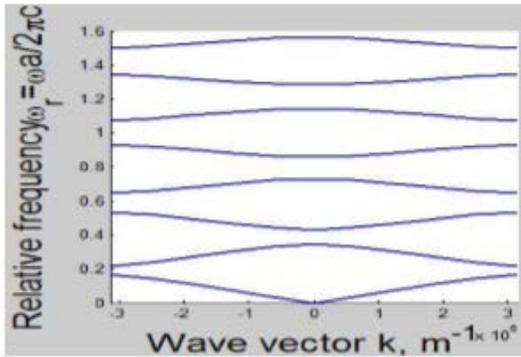


Fig.2. Band structure computation of 1D Phc for eps1=-1; eps2=9

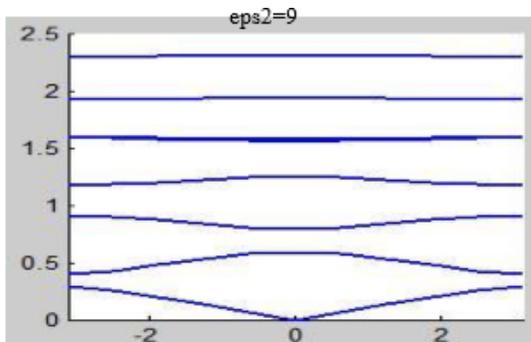


Fig.2(a). Band structure computation of 1D Phc for eps1=-1; eps2=3

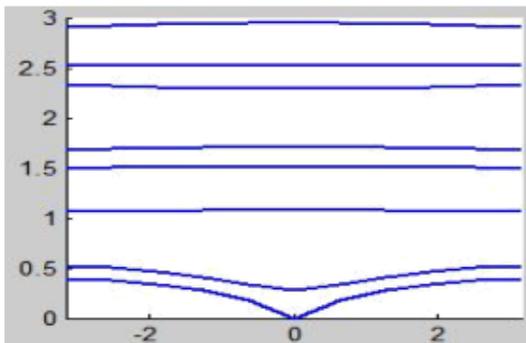


Fig.2(b) Band structure computation of 1D Phc for eps1=-5; eps2=1

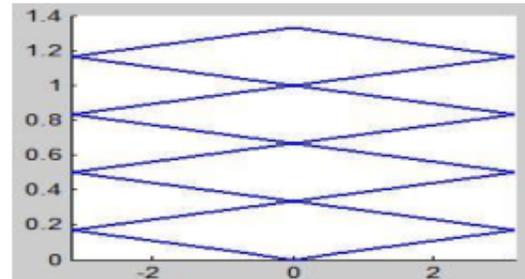


Fig.2(c) Band structure computation of 1D Phc for eps1=9; eps2=9

We have analysed the program for 1D Phc band structure computation by means of FDTD method. Input data: 1D Phc band structure. Where the variable a is lattice constant $a=1e-6$ and c is light speed in vacuum $c=3e8$

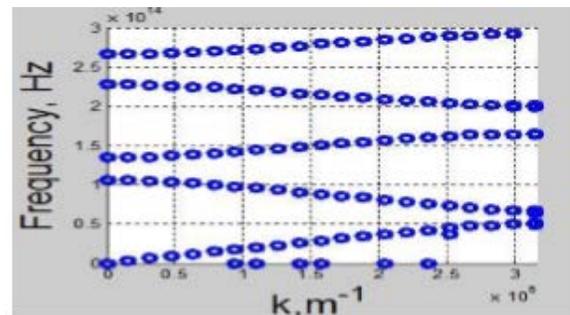


Fig.3 Band structure computation of 1D Phc for eps1=1; eps2=9

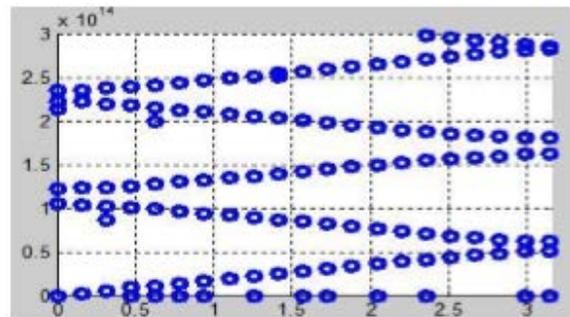


Fig.3(a) Band structure computation of 1D Phc for eps1=3; eps2=9

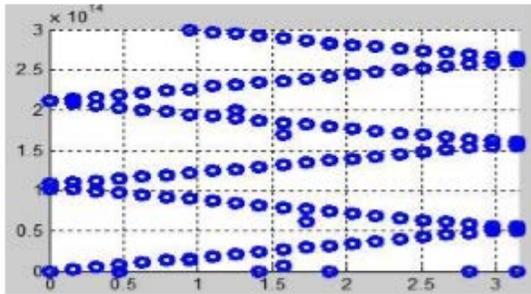


Fig.3(b) Band structure computation of 1D Phc for eps1=6; eps2=9

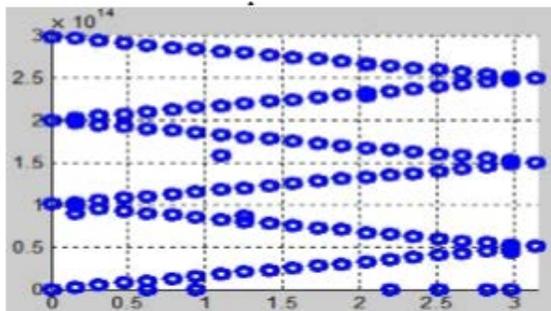


Fig.3(c) Band structure computation of 1D Phc for eps1=9; eps2=9

5. Summary and Future research

Future work includes Finite difference time domain combined with SPICE (Simulation Program With integrated circuit emphasis) and GPU (graphics processing units). Message passing interface (MPI) could be used to make parallel computing for FDTD and SPICE respectively. To overcome the GPU memory limitation of large scale structure simulation with FDTD, divided computational region can be utilized. Now this is well promising area will be useful especially for the ever-growing high frequency of the scientific and commercial applications. Due to such exuberant and wide range of properties exhibited by the photonic crystals that are widely used in nano-manipulation techniques, holographic techniques, quantum computing and communications, semiconductor process techniques, self organization techniques etc.

6. Conclusion

To increase the band gap so that reflectance should occur. Ideally it should act as a mirror. So in both the methods either by PWE or FDTD our aim was always to increase the forbidden gap so that the

maximum reflections should occur inside the material itself. Since the main use of photonic crystals is as low-loss mirrors characterizing high reflectivity and low absorption loss per single reflection thus, for maximum reflectivity we should choose a crystal geometry that has the maximum photonic bandgap(PBG).

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