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# Students' Strategies and Reasoning in School Mathematics Problems Solving

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**Abstract**

Although there has been many studies carried out considering different teaching and learning approaches on different perspectives and contexts, the aspects regarding influence of the student reasoning and choice of adequate strategies and procedures in solving problems still be influential to their achievement in mathematics tests. Though, based on this consideration, the purpose of this study was to explore Mozambican students' engagement with algebraic symbolism and their achievement in school mathematics. With a stronger focus on what is specific to 'algebraic thinking', as one aspect of thinking and reasoning employed in mathematical work, making generalizations and expressing generality, the aim of the study touch a variety of fields that have been researched in mathematics education. As findings, there were seen cases or situation that learners do not strictly the formal strategies because of some linguistic or scientific constraints as they do not translate correctly the everyday language into symbolic one.

***Keywords:*** *Strategies, problems solving, mathematical language, social context, social class, school algebra, logical reasoning*

## 1. Introduction

Although there has been many studies carried out considering different teaching and learning approaches, different perspectives and contexts, the aspects regarding influence of the student reasoning and choice of adequate strategies and procedures in solving mathematics problems still be influential to their achievement in mathematics tests. Thus, from this context we decided to analyse students understanding and preferred strategies in solving school mathematics tasks, with particular emphasis on algebra problems. Therefore, students' difficulties in learning and solving school mathematics problems has been one of the priority issues of researchers in the last three to four decades. Many of these difficulties are framed in research studies as related to cognitive challenges and obstacles in algebraic thinking, without taking the context in which this thinking is to happen into account.

Sometimes 'algebraic thinking' is described as a very general activity, such as work with a generalized unknown and the search for patterns. For example, [10] adopted such a very broad view, stated as follow: "Algebraic thinking is using mathematical symbols and tools to analyze different situations by *(i) extracting information from the situation... (ii) Representing that information mathematically in words, diagrams, tables, graphs, and equations; and (iii) interpreting and applying mathematical findings, such as solving for unknowns, testing conjectures, and identifying functional relationships*.

During last three decades researchers have studied students' problems with the learning of formal algebra, especially at the beginning of the learning process when learners are introduced into the subject (see [16]). It is not possible to synthesize results, as those studies were conducted from various theoretical perspectives (e.g., [12], [13]), which are not based on common assumptions, such as the following groups of approaches: *(i) Cognitive psychological, which includes Piagetian approaches, embodied cognition, constructivism, and/or those who do not consider any theory; (ii) Sociocultural, which includes Vygotskyan approaches, situated cognition, activity theory, communities in practice, social interactions, socio-semiotic approaches, social psychology and discourse analysis; (iii) Sociological, which includes sociology of education, hermeneutics and critical theory.*

Thus, [10] very much stressed the idea of relevance of algebraic thinking for real life applications. The most common approaches propagated in introducing algebra in school are generalizing, problem solving, modelling and functions [1]. A point commonly made by researchers is that the central feature of the algebra is the symbolic system considered to be the algebraic language, which can be described by the concept of a specific *mathematics register (that is,* within social functional linguistics, a register is a set of meanings that is appropriate to a particular function of language, together with the words and structures that express these meanings. We can refer to a 'mathematics register' in the sense of the meanings

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that belong to the language of mathematics [9]. So, a mathematics register uses specific grammatical constructions to emphasize particular relationships between ideas [9]. Placeholders are the particular (grammatical) participants in algebraic constructions. When undertaking transformations, it is not necessary to specify their field of reference. It can at the end be a surprise that the result still has some meaning in a particular context of a problem.

Considering the characterizations in the paragraphs before, it seems reasonable to assume that algebra is a language that requires and employs a specific mathematics register, even if each curriculum and classroom has its own specific forms of algebra. And, on one hand, as [22] argues, in school algebra and its applications, the concept of variable adopts different meanings depending on the form of a range of mathematically equivalent expressions used in different contexts, creating a different "feel" for the meaning. These are: *(i) variables in formulas (e.g.* $A = l\,w$ *), (ii) unknowns in equations to be solved (e.g.* $6\,x = 30$ *), (iii) an identity (e.g.* $\sin x = \cos x \cdot \tan x$ *), (iv) generalized patterns (e.g. when an arithmetic pattern is generalized and n stands for one instance), and (5) relationships (e.g.* $y = k\,x$ *).* This characterization of variables points out that there is a variety of meanings to placeholders in school algebra. On the other hand, [23] describes algebra as a symbolic system (to describe patterns and relationships without the need for the use of ordinary language), a calculus (among its primary elementary uses is the computation of numerical solutions to problems), and a representational system (tables and graphs from where the needed or presented information can be extracted and interpreted, for use in the mathematisation of situations and experiences).

Within the context of pattern generalization, some authors suggests a typology of different forms of algebraic thinking (factual, contextual, and symbolic), based on a definition of algebraic thinking as being about dealing with indeterminacy in analytic ways. For doing this, learners resort also to other semiotic resources than alphanumeric symbolism, such as spoken words or gestures. For instance, [7] observe that the importance of parameters and their relation to unknowns and variables were emphasized through the history of mathematics, giving an elucidative example from Euclid (325 -265 BC) where letters were used to represent quantities (known or unknowns). Nineteen centuries later the French mathematician Viète (1540-1603) initiated the systematic use of letters to denote both the coefficients (parameters) and unknowns. This marked and was the beginning of a new type of algebra expressed in terms of abstract formulas and general rules. From these considerations in [7], and somehow in [21] description in discussing students' interpretation of parameters in algebra, the importance of the concepts of variable, unknown and parameters for modern algebra is evident.

On the other hand, [18], when analyzing test item responses from secondary school students, present a brief overview of students' developing competence in four essential basic algebraic skills, as: *(i) Recognizing which operation relates two quantities and choosing the right operation; (ii) Using algebraic notation to write an expression; (iii) Interpreting an equation in a mathematical context and in the context of a described situation; (iv) Writing an equation (such as generating a formula from a table of values).*

Some of the general problems students face when learning algebra includes the following ([17], [19]): *(i) Students frequently base interpretations of letters and algebraic expressions on intuition and guessing, on analogies with other familiar symbol systems (for example associating m to metro, l to liter, and k to kilo).; (ii) Students' misinterpretations lead to difficulties in making sense of algebra and may persist for several years if not recognized and worked on; (iii) At all year-levels there are some students who seem to be unable to deal with precise distinctions between letters and their referents as necessary for a proper understanding of algebra; (iv) When algebraic concepts and methods are not used in other parts of the mathematics curriculum, students forget them and the notation for expressing them.*

Thus, the notion of intuition is often used in these accounts but remains largely undefined. It could refer to implicit assumptions made on the basis of the frames of reference that the students have to their disposal. Much of the listed "failures" or "misinterpretations" seem to be due to the fact that school algebra largely remains a self-referential activity. While some conclusions for the design of teaching units could be drawn from this kind of research, the quality of the teaching is not an issue for the present study. The school algebra curriculum that the students of the present study experience is what often has been called "traditional" that is, combining a teacher-centered instruction with a formal type of mathematics.

In other studies within the wide range of school algebra approaches, attention has been paid to realistic situations and to the process of mathematisation, and to the development of informal problem solving strategies [4], issues also relevant for the design of the tasks in the present study.

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Though, based on the above considerations, the purpose of the study is, however, to explore Mozambican students' engagement with algebraic symbolism and their achievement in school mathematics under the following research question that was set up for the study:

> "*What are the students' mainly used strategies and logical reasoning when solving mathematics problems in school algebra?*"

With a stronger focus on what is specific to 'algebraic thinking', as one aspect of thinking and reasoning employed in mathematical work, making generalizations and expressing generality, the above research questions posed to the present study touch a variety of fields that have been researched in mathematics education. Consequently, it is not possible to provide a comprehensive review of the literature in all the related subfields, that is, in the teaching and learning of algebra, mathematics learning and teaching processes in relation to achievement in school mathematics. And, within the context of pattern generalization, [14] suggests a typology of different forms of algebraic thinking (factual, contextual, and symbolic), based on a definition of algebraic thinking as being about "dealing with indeterminacy in analytic ways" (ibid., p. 15). For doing this learners resort also to other semiotic resources than alphanumeric symbolism, such as spoken words or gestures.

## 2. Material and Methods

### 2.1 Sampling and Data Gathering

The study adopted a discursive approach for learning mathematics, and then based on this theoretical perspective, a sample of 41learners of a semi-rural public secondary school were selected.

The learners participants were submitted to a written test, composed by two word problems elaborated in mathematics school perspectives, although it was possible to look at those task in everyday perspective.

The content of the test comprised the following items as described (in the original test all the item were given in Portuguese language as the medium of instruction and official language in Mozambique and here translated for the purposes of this research:

**Test item 1**

| | |
|---|---|
| Portuguese Language | Item 1: Pensei num número e achei o seu quádruplo. A este resultado adicionei o quíntuplo do número pensado dividido por ele mesmo. Ao resultado obtido subtrai o dobro do número inicialmente pensado. Como resultado final obtive o número 11. Encontre o número em que pensei? |
| | Q-1: *Apresente o raciocínio e os procedimentos seguidos na resolução do problema* |
| **English Translation** | **Item-1: I thought of a number, and calculated its quadruple. To this result I added the quintuple of the number considered, divided by itself. Then I subtracted twice the number I had thought of. As a final result I got the number 11. What is the number that I thought of?** |
| | **Q-1: *Write down your reasoning and procedures in solving the problem.*** |

This item/question is a typical algebraic word problem with a comparatively simple algebraic structure. However, the language has difficult grammatical structure and the vocabulary contains specialized, institutionalized mathematical terms ("quadruple", "divided by itself", "twice the number"). Thus, this item can be considered to be a form of esoteric domain text. The respondent is expected to translate the sentence into symbolic language and solve the resulting equation. The reader also has to recognize that the "I" who poses this puzzle and is in the possession of the answer, is an imaginative person who is not present. The style of the text is reminiscent of some guessing games one plays with children that start with "I am thinking of…"

Thus it is also of interest here, whether such a "reading across tasks" will be produced. This would allow a solution without contextualizing it as an equation.

**Test item 2**

| | |
|---|---|
| Portuguese Language | Item 2: Um cinema móvel vende bilhete para criança a metade do preço do bilhete para adulto. Sabendo que cinco (5) bilhetes para adulto e oito (8) bilhetes para criança custam 180.00 MT, quanto custa o bilhete para adulto? |
| | Q-1: *Apresente de forma clara o seu raciocínio e os passos seguidos* |
| **English Translation** | **Item 2: A moving cinema sells children's tickets for half the adult price. Knowing that 5 adult tickets and 8 child tickets cost a total of 180.00 MT (Mozambican currency) how much does the adult ticket cost?** |
| | **Q-2: *Write down your reasoning and the procedures in solving the problem.*** |

This item 2 looks like a version of a 'standard' type of algebraic problems encountered at school. But as it is a simple whole-half relation and the numerical values come out easily, it can be solved in an elementary way. It can of course also be solved by means of school algebra. The item can be considered as public domain text: It is a question in realistic context, and the student must re-contextualize it in mathematical school language. One can expect that this item will be interpreted differently, depending whether it is posed in a mathematics classroom/ or seen as a mathematics task or not.

## 2.2 Language Difficulties of the Items

On Item 1**:**

In the previous section when discussing this item 1, it was explicitly referred to as a simple algebraic word problem in esoteric domain text, though it includes some complexity in the grammatical structure. There, regarding the first sentence, the phrase "*Pensei num número e achei o seu quádruplo*" {'I thought of a number, and calculated its quadruple'} might be considered containing a level 2 complexity in the following way:

[*Pensei num número* [*e achei o seu quádruplo*]]
[*Número pensado* [*achar o quádruplo do número*]]
[Number thought of [calculating the quadruple of the number thought of]].

In Portuguese Language, when the sentence is written in first person - Singular or Plural - the pronoun, '**I**' or '**we**', is usually omitted, since the concordance is made by the termination of the verb.

The coherence and lexical cohesion is assured by reference to the use of the term 'seu' meaning '**its**' referring to the number previously thought of. Otherwise the text sentence would be longer and repetitive in writing: "*Pensei num número e achei o quádruplo do número. Em que número pensei*?" [I thought of a number and I calculated the quadruple of it. Which is the number I thought of?].

Following, the second sentence, "*A este resultado adicionei o quíntuplo do número pensado dividido por ele mesmo*". This sentence might be decomposed in the following way:

[*Adicionar* [*número pensado* [*seu quíntuplo* [*dividir este produto pelo número pensado*] (*dito dividido por ele mesmo*)]]].

That is,

[Adding [the number thought of [quintuple (i.e. multiplying by five) [dividing the result quintuple of the number thought of by the number thought of] (as is said divided by itself)]]].

The expression "*dividido por ele mesmo*", meaning "divided by itself", refers to the number thought of not including the quintuple of the number. Yet the possibility of an incorrect interpretation is not ruled out and respondents might divide the result of the multiplication by five. The recursive depth is high.

The next sentence, the third sentence of item 1: "*Ao resultado obtido subtrai o dobro do número inicialmente pensado*" explicitly takes into account the previous one and continues with a sequential explanation which might be decomposed in this way:

[*Ao resultado obtido* [*subtrair* [*dobro* [*número inicialmente pensado*]]]].

That is,

[To the obtained result [subtract [twice [the number initially thought of]]]].

The high recursive depth of this sentence comes from the fact that it considers the result of the sequential events up to now and the subtraction of double the number thought of at the beginning. The "number thought of" is repeated to provide coherence.

The next sentence (the fourth): "*Como resultado final obtive o número 11*", i.e. "as final result I got the number 11", is clearly of level 1 in terms of grammar complexity but the "final result" has to be understood to relate to the whole previous process, only said by referring to it as "final".

Finally the sentence (the fifth and the last) "*Encontre o número em que pensei*", "Find out the number I thought of", specifies the task to be done by the respondent. The grammar complexity might be considered to be of level 1 as the sentence does not require any decomposition.

In terms of the specificity of terminology the term "*encontre*" (find out) may not be precise enough to denote a mathematical activity. It could also mean one can guess the number. In fact, there are such games, for example "I'm seeing something beautiful that some of you would like to play with. What is it?"

On Item 2:

In discussing the construction of this item 2, it was said that it looks like a typical school algebra task and that it can be solved in an elementary standard way.

The first sentence: "*Um cinema móvel vende bilhete para criança a metade do preço do bilhete para adulto*", i.e. "A moving cinema sells children's tickets for half the adult price", can be decomposed into two levels of complexity:

> [*Preço do bilhete de adulto* (*incógnita*) [*Preço do bilhete de criança*) (*metade da incógnita de adulto*)]].

That is,

> [Adult ticket price (an unknown) [child ticket price (half of the adult unknown)]].

The second and last sentence, "*Sabendo que cinco (5) bilhetes para adulto e oito (8) bilhetes para criança custam 180.00 MT, quanto custa o bilhete para adulto?*", i.e. "Knowing that five (5) adult tickets and eight (8) child tickets cost a total of 180.00 MT, how much does the adult ticket cost?", is connected by the words "adult ticket" and "child ticket".

This sentence may be decomposed recursively in two levels:

> [*Preço do bilhete de adulto* [5 *bilhetes de adulto por preço, 8 bilhetes de criança por metade do preço, montante gasto*]].

That is,

> [Adult ticket price [5 adult tickets times' unknown price, 8 child tickets times' unknown half price, spent amount]].

## 3. Results and Discussion

The students' results and solutions strategies were analysed by clusters. Thus, knowing school mathematics, e.g. algebra has its rules and procedures to write, translate into symbols and solve the school mathematic tasks we opted to consider these rules as formal strategies. There are cases or situation that learners do not strictly the formal strategies i.e. because of some linguistic or scientific constraints they do not translate correctly the everyday language into symbolic one. Then they mix strategies using in part the formal and an informal one. The third case is when the learners do not interpret the problem as school mathematics task, solving it empirically; in this case we decide to call informal strategy. To facilitate the task interpretation the strategies were coded as follow:

Table 1: Strategy Codes for Students Responses

| *Nr.* | *Strategy* | *Code* |
|---|---|---|
| ***01*** | Formal | 3 |
| ***02*** | Mixed | 2 |
| ***03*** | Informal | 1 |

In the first item the student has a possibility to use different strategies, i.e. a formal one according to rules and procedures translating the given sentences into symbolic language, writing the equation and find the solution, or using an informal strategy such as guessing the solution and trying to verify the result, or finally the mixed strategy, i.e. the both formal and informal. And, the second item looks like a version of a 'standard' type of algebraic problems encountered at school. But it can be solved in an elementary way or can of course also by means of school algebra. The codes for item 2 responses were given in a similar way as item 1 codes.

### 3.1 First Cluster Analysis on Item 1

Here half of the students from the low status cluster (seven students) achieved the correct solution and seven did not. The more interesting outcome lays in their strategies with which they attempted to solve it. All students attempted a solution. As the text deals with an unknown number and in a quite complex grammatical structure proceeds to explain operations done with that number, one would expect difficulties. When looking at their strategies, it is possible to distinguish between formal mathematical, informal, and mixed strategies. The students used formal or informal strategy no one recurred to the mixed one. The following Table 2 depicts the choices.

Table 2: First cluster: Results and Strategies used in Solving Item 1

| ***Solution*** | | | | |
|---|---|---|---|---|
| ***Count & %*** | | ***Correct*** | ***Incorrect*** | ***No attempt*** |
| ***Female*** | 8 | 2 | 6 | 0 |
| ***Male*** | 6 | 5 | 1 | 0 |
| ***Total*** | 14 | 7 | 7 | 0 |
| ***%*** | 28 | 50% | 50% | - |
| ***Strategy*** | | | | |
| ***Count & %*** | | ***Formal*** | ***Mixed*** | ***Informal*** |
| ***Female*** | 8 | 4 | 0 | 4 |
| ***Male*** | 6 | 5 | 0 | 1 |
| ***Total*** | 14 | 9 | 0 | 5 |
| ***%*** | 28 | 64% | - | 36% |

In total five students used an informal strategy of whom four are girls, and only one boy obtained the right solution via this strategy. All those who did not succeed in translating the word problem are main Local Language speakers, but there were some Local Language speakers who did it correctly. An expected aspect causing difficulties is the students' constraints in translating the word problem into symbolic language. Some of them did not understand the meaning of the words: quadruple, twice the number, five times the number, and quintuple. An example for this misunderstanding can be given by presenting Hevito's solution attempt:

- *The "quadruple of the number thought of": $x^4$ he knows that the word 'quadruple' is related to the number 4 but he thinks this number operates as exponent.*
- *"To this result I added the quintuple of the number thought of divided by itself": $x^4 + \frac{x^5}{x^5}$.*

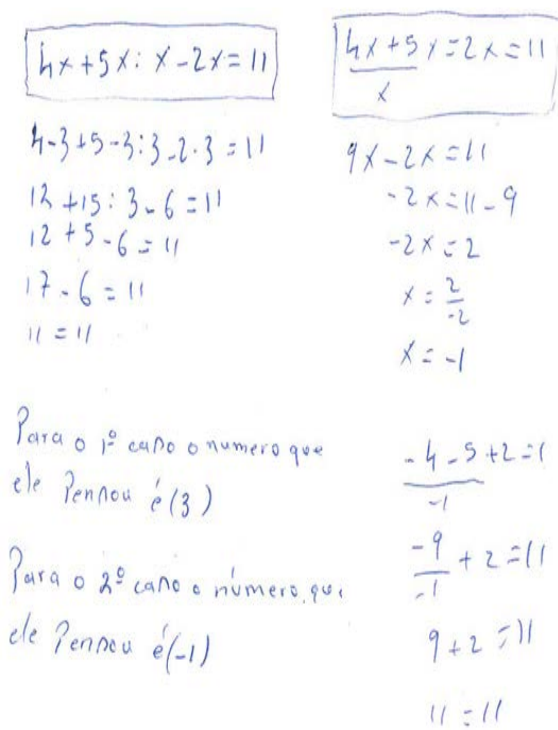
  *He understands "divided by itself" referring to the whole last phrase (to what is added) and not only to the number thought of, and in "quintuple" the number 5 is also interpreted as an exponent. Hevito has positive marks in all subjects; in mathematics he has a ten (out of 20).*

Others students presented a chain of equalities, with an equal sign between all operations according to number words found in the given word problem. In this way each sentence forms a new equation amounting to a "new" variable denoted by the same symbol x. The example below from Irene, Sofia and Sara shows this procedure. While Irene and Sofia have positive marks in mathematics (a ten), Sara has a nine: $x-4 = x+\frac{5}{5} = x-2x$

- *One can also see here that for these students quadruple of the number thought of is indeed four times x. Correct, but they consider that this results again is x as shown after equal sign.*
- $x+\frac{5}{5}$. *Here the students added $\frac{5}{5}$ considering this as five times the number thought of divided by itself which is mistake from misreading the text by leaving out the "number thought of" as a factor. For them this results again in x when they write $x+\frac{5}{5}=x$. Still at least they could interpret the phrase "divided by itself".*
- $x-2x$ *(To the found result x they subtracted twice the number thought of).*

However, there were other students who used to translate the sentence in Portuguese language to their own mother thong (as, Jongo, Mirigu and Roda) for better understanding the question and then getting then solution. For instance, Jongo's reasoning and procedures are displayed below. He seems to first have guessed the number, after writing down the right equation, and then tried to solve the equation.

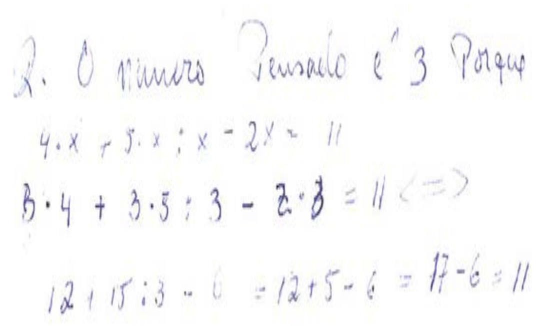
***Jongo's reasoning and procedures***



The sentence means: *"For the 1st case the number thought of is (3). For the 2nd case the number thought of is (-1)".*

Mirigu's reasoning displayed below also shows a correct equation without a solution. In the numerical version below the equation some of the factors are revised in order, so it is not clear whether this is related or whether the similarity with item 1 was discovered.

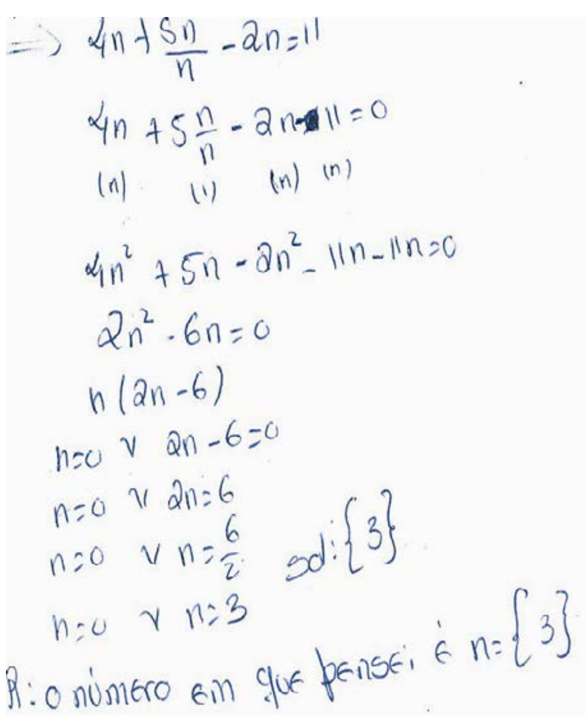
***Mirigu's reasoning and procedures***



The sentence means: "*The number thought of is 3, because …*"

Roda's reasoning and procedures below are different. She chose the letter n and not x denoting the variable and solved the equation formerly including some writing errors. The result is written as a set.

***Roda's reasoning and procedures***



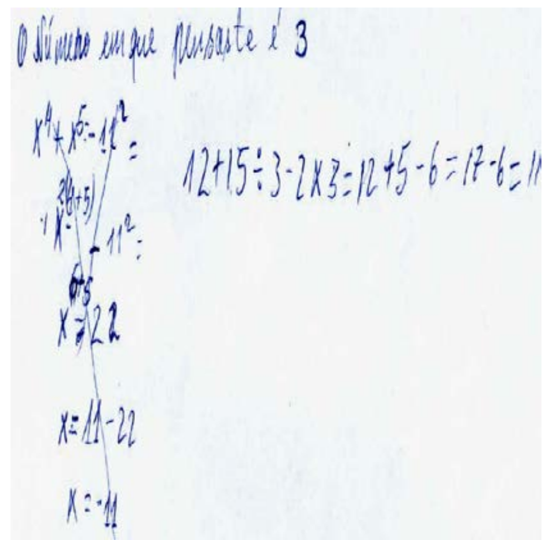
The sentence means: "*The number I taught of is,*"

Mofino was the only student establishing a relationship between the first and second items, just writing

"*The number you thought of is the 3*",

before an unsuccessful attempt of setting up an equation that included the interpretation of quadruple and quintuple as exponents. He also wrote the procedure of "student 1" from the item 1:

***Mofino's reasoning and procedures***



The sentence means: "*The number I taught of, is 3*"

As to the strategies, the students in this cluster either started out with writing an equation (be it correct or incorrect), or without using variables. No one used a mixed strategy.

### 3.2 First Cluster Analysis on Item 2

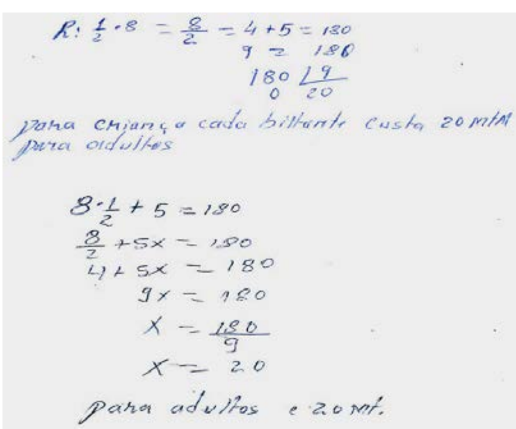
This was a typical word problem about ticket prices. It can be solved by an informal strategy, as is the case with most of such tasks, but also by setting up an equation. The results presented in Table 3 show that seven students (50%) achieved the correct solution and only one of them, Jongo, contextualized the item to mathematics classroom procedures using a formal strategy. He has positive marks in all subjects and an eleven in mathematics (out of 20). Two students, Lumba and Mirigu, used mixed formal-informal strategies. Mirigu has insufficient marks in mathematics (a seven) and two other areas, but Lumba has positive marks in all subjects (a ten in mathematics). These students had in their mind that there is a fixed unknown x (the adult or children ticket price), and after they had written a calculation or an "equation" without a variable, they included the unknown in one of the additional steps of their procedures. Ten students (72%) used informal strategies, eight girls and two boys. One participant, Énica, did not try to solve the item. As in the algebra word problem in item 2, the girls tended to resort to an informal strategy more than the boys.

Table 3: First Cluster: Results and Strategies used in Solving Item 2

| | | | | |
|---|---|---|---|---|
| | | ***Solution*** | | |
| ***Count & %*** | | ***Correct*** | ***Incorrect*** | ***No attempt*** |
| ***Female*** | 8 | 2 | 5 | 1 |
| ***Male*** | 6 | 5 | 1 | 0 |
| ***Total*** | 14 | 7 | 6 | 1 |
| ***%*** | 28 | 50% | 43% | 7% |
| | | ***Strategy*** | | |
| ***Count & %*** | | ***Formal*** | ***Mixed*** | ***Informal*** |
| ***Female*** | 8 | 0 | 0 | 7 |
| ***Male*** | 6 | 1 | 2 | 3 |
| ***Total*** | 14 | 1 | 2 | 10 |
| ***%*** | 28 | 7% | 14% | 72% |

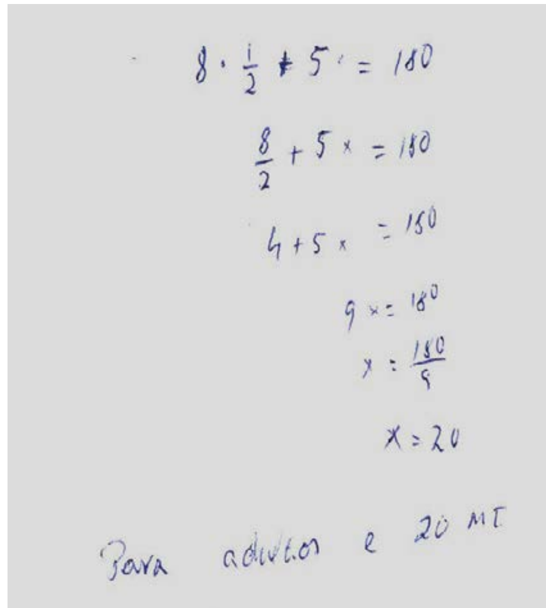
The high number of students attempting to solve the task by using an informal strategy could perhaps reflect a tendency to approach it from an everyday perspective, which would reflect outcomes of earlier research about strategies in relation to socio-economic status. While normally in a school context the approach is not valued or not successful, here it was possible. Lumba's procedures (below) that he tried to adapt his solution to the form of an equation after he had worked it out.

***<u>Lumba's reasoning and procedures for item 2</u>***




Mirigu's (below) introduces the "unknown" at the end of his calculation.

***<u>Mirigu's reasoning and proceduresfor item 2</u>***



## 3.3 Second Cluster Analysis on Item 1

As mentioned above, the problem allowed for formal (school mathematical) or informal strategies. The table below provides an overview.

Table 4: Second Cluster: Results and Strategies used in Solving Item 1

| | | | | |
|---|---|---|---|---|
| | | ***Solution*** | | |
| ***Count & %*** | | ***Correct*** | ***Incorrect*** | ***No attempt*** |
| ***Female*** | 11 | 4 | 7 | 0 |
| ***Male*** | 6 | 5 | 1 | 0 |
| ***Total*** | 17 | 9 | 8 | 0 |
| ***%*** | 34 | 53% | 47% | - |
| | | ***Strategy*** | | |
| ***Count & %*** | | ***Formal*** | ***Mixed*** | ***Informal*** |
| ***Female*** | 11 | 5 | 1 | 5 |
| ***Male*** | 6 | 3 | 3 | 0 |
| ***Total*** | 17 | 8 | 4 | 5 |
| ***%*** | 34 | 47% | 24% | 29% |

Nine students (53%) of this middle status cluster achieved the correct solution. Three of them (Isa), Mapezo and João) used a formal strategy through writing and solving a linear equation. Four students (Cecil, Emidio, Emilio and Frago) used a mixed strategy. Isa and Mapezo have an insufficient mark in mathematics (an eight out of 20), and also insufficient marks in other subjects (notably Isa in Portuguese). João achieved sufficient marks in all areas. The students who used a formal strategy all have Portuguese as their first language, but one of them (Isa) states she mostly communicates in a Local Language outside school. Three of the ones who adopted a mixed strategy have a Local Language as their first language and two also mainly communicate in this medium. One (Frago) communicates mostly in a Local Language as well, but Portuguese is his first language. It was considered a mixed strategy when after setting up the equation the student did not solve it following the formal procedures but instead tentatively or by guessing. In this particular case the students wrote the correct linear equation and found a number verifying the equality as follow:

$$4\times x+5\times x\div x-2\times x=11 \quad (1)$$
$$4\times 3+5\times 3\div 3-2\times 3=11 \quad (2)$$
$$12+15\div 3-6=11 \quad (3)$$
$$17-6=11 \quad (4)$$
$$11=11 \quad (5)$$

After writing the equation (1) they looked for the x verifying the equality (5). Two students, Tácia and Comay, both main Portuguese speakers, used an informal strategy without showing any reasoning and procedure only stating: *"The number thought of is 3"*. Tácia has an eight (out of 20) in mathematics, but otherwise sufficient marks, while Comay has a ten in mathematics (sufficient) and an insufficient mark in another subject.

## 3.4 Second Cluster Analysis on Item 2

In this item 2 that dealt with the cinema tickets fourteen students (82%) achieved the correct solution, while one participant did not touch the question and two failed.

Table 5: Second Cluster: Results and Strategies used in Solving Item 2

| | | | | |
|---|---|---|---|---|
| | | ***Solution*** | | |
| ***Count & %*** | | ***Correct*** | ***Incorrect*** | ***No attempt*** |
| ***Female*** | 11 | 8 | 2 | 1 |
| ***Male*** | 6 | 6 | 0 | 0 |
| ***Total*** | 17 | 14 | 2 | 1 |
| ***%*** | 34 | 82% | 12% | 6% |
| | | ***Strategy*** | | |
| ***Count & %*** | | ***Formal*** | ***Mixed*** | ***Informal*** |
| ***Female*** | 11 | 2 | 0 | 8 |
| ***Male*** | 6 | 4 | 1 | 1 |
| ***Total*** | 17 | 6 | 1 | 9 |
| ***%*** | 34 | 35% | 6% | 53% |

In contrast to the students from the low status cluster, more students from this group (35%) used a formal school mathematics strategy by setting up an equation. In the first cluster there was only one boy doing so. Nine students (53%) used an informal strategy in solving this item, it means they interpreted it in the context of everyday life and did not re-contextualize it to a school mathematics activity. Here, girls showed a high propensity to use an informal strategy as eight of them (72%) recurred to this strategy. For example, Isa a girl, first Portuguese speaker but often communicating in a Local Language, wrote the following strategy solution:

$$20\times 5=100$$
$$100+80=180$$
$$10\times 8=80$$

*Adult ticket costs 20 MT,*
*Child ticket costs 10 MT.*

An example of a mixed strategy comes from Comay, a girl for whom Portuguese is the first and mostly used language she used mixed strategy in doing the following sequence:

$$8\times\frac{1}{2}+5=180\,,$$
$$\frac{8}{2}+5x=180\,,$$
$$4+5x=180\,,$$
$$9x=180\,,$$
$$x=\frac{180}{9}\,,$$
$$x=20\,,$$

*For adult costs 20 MT*

## 3.5 Third Cluster Analysis on Item 1

In this group, girls also showed a tendency to use informal strategies in this item as all students recurring to this strategy were girls (representing 45% of the middle status cluster's female students). For the item 1, only students from this cluster (24%) recurred to a mixed strategy.

This algebraic word problem was not solved more successfully by students from this group. All but three students chose a school mathematical approach. In this group the ones choosing in informal strategy were boys.

Table 6: Third Cluster: Results and Strategies used in Solving Item 1

| | | *Solution* | | |
|---|---|---|---|---|
| *Count & %* | | *Correct* | *Incorrect* | *No attempt* |
| *Female* | 2 | 1 | 1 | 0 |
| *Male* | 8 | 4 | 4 | 0 |
| *Total* | 10 | 5 | 5 | 0 |
| *%* | 20 | 50% | 50% | - |
| | | *Strategy* | | |
| *Count & %* | | *Formal* | *Mixed* | *Informal* |
| *Female* | 2 | 2 | 0 | 0 |
| *Male* | 8 | 5 | 0 | 3 |
| *Total* | 10 | 7 | 0 | 3 |
| *%* | 20 | 70% | - | 30% |

Of the five students (50%) who succeeded in achieving the right solution four communicate mostly in Portuguese. The three students who did not write down any reasoning or procedures are Jaime, Tiago and Carlos. All three gave the same (wrong) answer in stating: "*The number thought of is 4*". From these students, only Tiago has an insufficient mark in mathematics (a nine out of 20). When excluding these three students, all others contextualized the item as a school mathematics question and only two of the seven did not achieve the right solution as consequence of misinterpretation. Relina, a main Portuguese speaker, who has a sufficient mark (a ten out of 20) in mathematics and an insufficient mark only in physics, translated the word problem into symbolic language in terms of,

$$4 \cdot x^2 + 5 \cdot x - 2 \cdot x - 2 \cdot x \ .$$

It seems she was not sure of the meaning of '*quadruple*' because she multiplies the number thought of by four and she squared it. Also she did not understand that the number multiplied by five was divided by itself neither that the final result is 11.

There is an increase the use of a formal strategy compared with the approaches chosen by students from the low and middle status clusters. Notably, none from this cluster used a mixed strategy for solving this item, a phenomenon also observed in the first cluster. The reasons might be similar (see above).

### 3.6 Third Cluster Analysis on Item 2

Here, the item is also about the tickets, the students of the high status group did not produce more correct solutions. Their strategies were similar in quality to the ones from the other groups, except that nobody operated with a mixed formal-informal strategy. Quite many used an informal approach, but did not do so successfully, except one student.

Table 7: Third Cluster: Results and Strategies used in Solving Item 2

| | | *Solution* | | |
|---|---|---|---|---|
| *Count & %* | | *Correct* | *Incorrect* | *No attempt* |
| *Female* | 2 | 2 | 0 | 0 |
| *Male* | 8 | 3 | 5 | 0 |
| *Total* | 10 | 5 | 5 | 0 |
| *%* | 20 | 50% | 50% | - |
| | | *Strategy* | | |
| *Count & %* | | *Formal* | *Mixed* | *Informal* |
| *Female* | 2 | 1 | 0 | 1 |
| *Male* | 8 | 3 | 0 | 5 |
| *Total* | 10 | 4 | 0 | 6 |
| *%* | 20 | 40% | - | 60% |

From the five students (50%) who achieved the correct solution four used a formal strategy i.e. they re-contextualized the item to school mathematics, writing an equation. All students that used this strategy achieved the right solution. Students using an informal strategy only Relina got the correct solution. Relina has a ten in mathematics and her first language is a Local Language and communicates in Portuguese.

## 4. Conclusions

In the presentation above, attention has been paid to the students' achievement in terms of producing correct solutions and choice of strategies in relation to the tasks and answers requiring high language proficiency and also to the students' performance to contextualize a question given in a school mathematical perspective (item 1) or in re-contextualizing a question presented as public domain text (item 2).

In the first item the students' achievement centred around 50% in all status groups. There were some common aspects between the first and third cluster. In both groups the students did not use a mixed formal-informal strategy, the formal strategy being more common (64% and 70% chose a formal approach, and 36% and 30% recurred to an informal strategy). Less students from the middle status cluster recurred to a formal strategy, and the others used a mixed (24%) and informal (29%) approach. The low achievement of students in this item can be assumed to be partly due to the students' low proficiency in the medium of instruction.

However, there were in fact differences in the choices between the statuses groups, independently of their mathematics achievement expressed in their school marks. The students from the low status cluster more often choose an informal approach (72%), but there were also quite many in the high status group (60%), but only around half of the students from the middle status group did so. It was only in this middle group, where students also used mixed strategies. In general, the girls tended to prefer to approach the task from an informal perspective and not to use school mathematics. For the first cluster the high percentage of students using an informal strategy can reflect the tendency of low social status groups found in other investigations, as does the girls' preference.

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