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Robust Tuning of Controller for SISO and MIMO Systems Using Coefficient Diagram Method

Vasu Puligundla[1] and Jayanta Pal[2]

[1]Adama Science and Technology University, Ethiopia, [2]Indian Institute of Technology, Kharagpur, India

***Abstract* – In this paper, robust tuning of controller for SISO and MIMO systems is considered. Firstly Power System Stabilizer(PSS) is taken as an example for SISO System. The first objective is to tune PID-PSS so that closed-loop response of a single machine connected to an infinite bus system (SMIB) is made stable over a wider range of operating points while maintaining the desired damping. The Coefficient Diagram Method (CDM) is used for choosing the coefficients of the target characteristic polynomial of the closed loop system based on performance criteria; such as equivalent time constant, stability indices and stability limits. Standard Manabe form is used for choosing the stability indices. The parametric uncertainties are handled by adding a pre-filter that increases the degree of the CDM based controller (PID-PSS) by one. Genetic Algorithm and pole coloring technique are then used for tuning the pre-filter by minimizing the shift in the closed-loop poles due to perturbations. The robustness of the designed feedback controller for SMIB is verified by using the Kharitonov Theorem and the Zero-exclusion condition. ; Secondly Controller designed for MIMO systems taking some examples which include stable and unstable systems. Pressurized flow-box, Four-input four-output gas fired furnace problem, Mueller's two-shaft aircraft gas turbine are considered as an examples for MIMO Systems. The second objective is to tune a Diagonal controllers for these MIMO systems, such that each output can be controlled independent of other outputs, by varying particular input only. Standard Manabe form and CDM are used for choosing the coefficients of closed loop characteristic equation.**

***Index Terms*-- Power System Stabilizers, Power system dynamic stability, Coefficient Diagram Method, Genetic Algorithm, Robust control, MIMO systems, Zero Frequency Decoupler.**

I. INTRODUCTION

There has been considerable effort for solving the problem of low frequency oscillations leading to instability of power systems. These modes of oscillations are characterized by low mechanical natural frequencies in the range of 0.3-2.0 Hz. To damp out the oscillations, power system stabilizers (PSS) are used to inject a supplementary signal at the voltage reference input of the automatic voltage regulator (AVR). Conventionally a single-input single-output feedback controller is used as PSS. As an input signal to a conventional PSS, anyone of the three signals i.e., machine shaft speed, ac bus frequency or accelerating power can be used. Most commonly used input signal is the machine shaft speed [10].

Many research papers have been published in this area [2, 8, 9]. The PSS design normally uses classical control theory and is based on a model of the power system linearized at some operating point. Properly tuned, a PSS can considerably enhance the dynamic performance of a power system.

Some work in the area of designing self-tuning, adaptive and robust PSS [2, 8] has been reported for achieving better control over wide range of load variations. However, the complexity and/or real-time computational requirement of such controller preclude their use in actual power plants.

Changes in transmission networks, generation and load patterns results in changes in operating conditions of power systems. Thus, the small signal dynamic behavior of a power system is varied, which can be expressed as a parametric uncertainty in the small signal linearized model of the system. In this work, we design a robust PSS so that adequate damping can be provided over a wide range of operating conditions. This work was motivated by some papers [2, 8, 9], where the quantitative feedback theory (QFT) [8], LMI technique [2] and Optimization Techniques [9] have been used for designing a robust PSS.

In this paper coefficient diagram method [3], an algebraic design approach (or polynomial method), is used. The time-domain performance of a system is closely related with its poles or characteristic polynomial. The characteristic polynomial can be defined from stability and response specification, but it is very difficult to choose it with guarantee of robustness. The CDM standard form [3] is used for choosing the target closed loop characteristic polynomial. Although the CDM results in pretty robust controllers, if there are large uncertainties in the system CDM itself may not be enough to satisfy robust stability and performance requirements. The CDM design method is extended to handle all possible parametric uncertainties with satisfactory performance by increasing the degree of the controller offered by the CDM by one. A pole-zero pair is introduced to create extra design freedoms and then a pole-coloring technique [4] to guarantee robust pole assignment. The pole-zero pair is tuned using Genetic Algorithm by minimizing the shift in the closed-loop poles due to perturbations.

II. METHOD

A. *Concept of CDM:*

In CDM, the controllers are designed based on the stability index known as γ_i and the equivalent time constant known as τ which are synthesized from the characteristic polynomial of the closed-loop transfer function.

$$P(s) = a_n s^n + \cdots + a_1 s + a_0 = \sum_{i=0}^{n} a_i s^i ----(3)$$

From the characteristic polynomial *P(s)* given in eq. (3), the stability index γ_i and the equivalent time constant τ are respectively described in general term as the following equations [3]

$$\gamma_i = \frac{a_1^2}{a_{i+1}a_{i-1}}, i = 1 \sim n-1 \; ----(4)$$

$$\tau = \frac{a_1}{a_0} ----(5)$$

In order to meet the specifications, the equivalent time constant τ and the stability index γ_i are normally chosen as

$$\tau = \frac{t_s}{2.5} \sim \frac{t_s}{3}$$

$$\gamma_i > 1.5\gamma_i^*$$

Where t_s is the specified settling time and γ_i^* is the stability limit defined as

$$\gamma_i^* = \frac{1}{\gamma_{i+1}} + \frac{1}{\gamma_{i-1}}, i = 1 \sim n-1, \; \gamma_n = \gamma_0 = \infty$$

In general the stability index is recommended as

$$\gamma_{n-1} \sim \gamma_2 = 2, \;\; \gamma_1 = 2.5 ----(6)$$

known as standard stability index.

Finally the characteristic polynomial known as the desired characteristic polynomial can be expressed as

$$P(s) = a_0\left[\left\{\sum_{i=2}^{n}\left(\prod_{j=1}^{i-1}\frac{1}{\gamma_{i-j}^{j}}\right)(\tau s)^i\right\} + \tau s + 1\right]$$
$$= a_n s^n + \cdots + a_1 s + a_0,$$

Where, $a_n, a_{n-1}, \ldots a_0$ are the coefficients of the desired characteristic polynomial.

B. *Pole coloring [4]:*

Consider the simple case of a third-order system where the nominal poles and perturbed poles for a fixed q (perturbations) are given in Fig1. Here, assume that big points represent perturbed poles and small points represent nominal poles corresponds to which of the perturbed poles is called '*pole coloring*'.

C. *Graphical approach for checking robustness [5]:*

Consider a real general polynomial *p(s)* of degree 'n' as given below:

$$p(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 ----(7)$$

The polynomial *p(s)* is said to be an interval polynomial if each coefficient is independent of the other and varies within an interval having a lower and upper bound [5]: i.e. $a_i = [a_i^-, a_i^+], i = 0,1,2,\ldots,n.$, such an uncertain polynomial is said to have an independent uncertainty structure.

Kharitonov Theorem: The interval polynomial *p(s)* is robustly stable if and only if the following four Kharitonov polynomials:

$$K_1(s) = a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + a_4^+ s^4 + a_5^+ s^5 + \cdots$$
$$K_2(s) = a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + a_4^- s^4 + a_5^- s^5 + \cdots$$
$$K_3(s) = a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + a_4^- s^4 + a_5^+ s^5 + \cdots$$
$$K_4(s) = a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + a_4^+ s^4 + a_5^- s^5 + \cdots$$
$$-----(8)$$

are stable [5].

Given the interval polynomial *p(s, a)* as defined in eq. (7) and a fixed frequency $\omega = \omega_0$, one can describe a set of possible values that $p(j\omega_0, a)$ can assume as *a* varies over the box Q which can be shown as:

$$p(j\omega_0, Q) = \{\, p(j\omega_0, a) : a \in Q\}$$

Then, $p\,(j\omega_0, Q)$ can be termed as the Kharitonov rectangle [5] at frequency $\omega = \omega_0$ with vertices which are obtained by evaluating the four Kharitonov polynomials, $K_i(s)$, i=1,2,3,4., as defined in eq. (8) , at $s = j\omega_0$. The rectangularity is proved in [5]. By varying the frequency from $\omega = 0$, and with ω increasing in discrete steps, results in the motion of the Kharitonov rectangle with the rectangle moving around the complex plane with vertices $K_i(j\omega)$. The dimensions (size) of this rectangle vary with the frequency *ω.*

Zero Exclusion Condition: Suppose that an interval polynomial family *p(s)* has invariant degree and at least one stable member, the *p(s)* is robustly stable if and only if *s=0* is excluded from the Kharitonov rectangle at all non-negative frequencies [5]; i.e. $0 \notin p\,(j\omega, Q)$

The zero exclusion condition suggests a simple graphical procedure for checking robust stability. By watching the motion of Kharitonov rectangle *p (jω, Q)* as ω varies from 0 to +∞, one can easily determine by inspection if the Zero

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Exclusion condition is satisfied. If it is satisfied, then one can say that the polynomial family *p(s)* is robustly stable.

III. SISO SYSTEM (PSS)

By varying the operating conditions over a range which includes almost all practical operating conditions for the generator and by varying lengths of transmission lines includes very weak to very strong transmission systems, so the operating conditions are chosen in the intervals P[0.4 1.0], Q[-0.2 0.5] and X[0.2,0.7].

By taking a step difference of 0.1 in the values of P, Q and X, totally 336 combinations are obtained which corresponds to 336 operating points. Using the Heffron-Philips linearized model [6] of a single machine connected to an infinite bus (SMIB) system, 336 linearized models of the plant is constructed. The problem considered in [2], [8] is to design a feedback controller that maintains a damping ratio of at least 0.1 and real parts of all closed-loop poles less than -0.5 simultaneously for all operating points.

A. Power system model:

The Heffron-Philips linearized model [6] of a single machine connected to an infinite bus (SMIB) system is considered in this work. Fig.1 shows the line diagram of a synchronous machine connected to an infinite bus through a transmission line having resistance R_e and reactance X

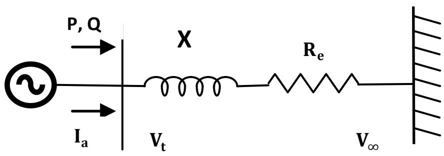

Fig.1. Line diagram of single machine connected to infinite bus.

The detailed derivation and assumptions are given in [7]. System data and state space representation are given in appendix.

B. PSS structure:

Case1: A simple PID-PSS is considered, when parameters are tuned using standard CDM only. The transfer function of PSS as:

$$K(s) = \frac{k_d s^2 + k_p s + k_i}{s} ----(1)$$

Case2: A PID amended with a pole-zero pair is considered when CDM & pole-coloring are used. The transfer function of PSS as:

$$K(s) = \left(\frac{s+a}{s+b}\right)\left(\frac{k_d s^2 + k_p s + k_i}{s}\right) ----(2)$$

The input to PSS is the machine shaft speed, $\Delta\omega$ and the output is ΔU, i.e.

$$\Delta U = K(s)\, \Delta\omega$$

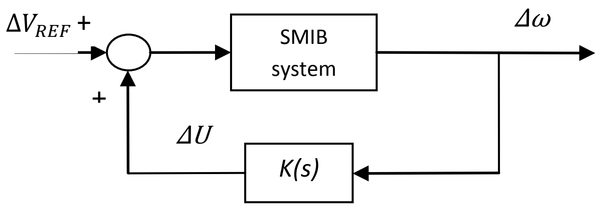

Fig. 2 Closed loop configuration of single-machine system.

The parameters of PSS viz. k_d, k_p, k_i, a and b are tuned through combination of CDM, GA and pole-coloring techniques to meet the desired objectives.

IV. CONTROLLER DESIGN FOR SISO SYSTEM

A family of 336 linearized models of the plants is constructed for grid of operating points as *P*,*Q* and X_e vary independently in steps of 0.1 over the interval [0.4, 1.0], [-0.2, 0.5] and [0.2, 0.7] respectively. The reference terminal voltage is kept as $\Delta V_{REF}=0.05$ and moment of inertia is calculated as *M=2H*. Open loop poles location: when P, Q, and X_e are varied independently in steps of 0.1 over the interval [0.4, 1.0], [-0.2, 0.5] and [0.2, 0.7] are shown in Fig.3.

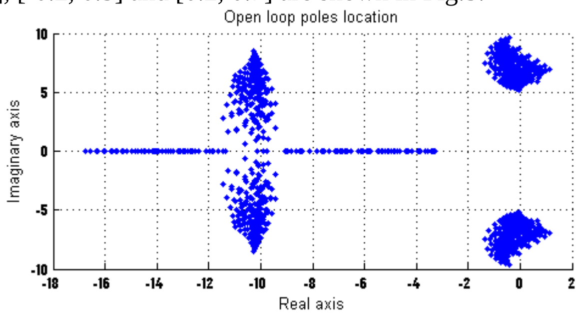

Fig. 3 Open loop poles locations for all chosen perturbations

A. Designing a robust PID-PSS:

Step1: Let the light loading condition P=0.4, Q=-0.2 and X=0.2 be the nominal operating point, then corresponding transfer function will be

$$G_{light}(s) = \frac{-44.3s}{s^4 + 21.1s^3 + 170.6s^2 + 1102.3s + 4371}$$

Step2: Choose controller to be designed as PID

Step3: Obtain the closed loop characteristic polynomial in terms of unknown controller parameters-

$$P(s) = s^4 + 21.1s^3 + (170.6s^2 + 44.3k_d)s^2 + (1102.3 + 44.3k_p)s + (4371 + 44.3k_i)$$

$$a_i = [a_5\ a_4\ a_3\ a_2\ a_1\ a_0] = [1\ \ 21.1\ \ (170.6s^2 + 44.3k_d)\ \ (1102.3 + 44.3k_p)\ \ (4371 + 44.3k_i)\ ] ----(9)$$

Step4: Choose the stability indices according to the CDM standard form Eq. (6)

As we have taken γ_i from standard form, stability conditions are satisfied, $\gamma_2 > \gamma_2^*$

Or from R-H criterion $a_2 > \left(\frac{a_1}{a_3}\right) a_4 + \left(\frac{a_3}{a_1}\right) a_0$ is also satisfied.

Step5: From equations (4), (6) & (9), k_d =1.1916, k_p = 1.7636, ki = -42.3410.

The interval characteristic polynomial of the closed loop system becomes:

$$G^{I}_{interval} = [1,1]s^4 + [20.66, 21.13]s^3 + [229.22, 233.84]s^2 + [708.82, 1974.81]s + [2726.75, 3686.83].$$

For this values of k_d, k_p, and ki closed loop poles location, closed-loop system response to 5% disturbance step, and Kharitonov rectangles are shown in figures 4a, 4b and 4c respectively.

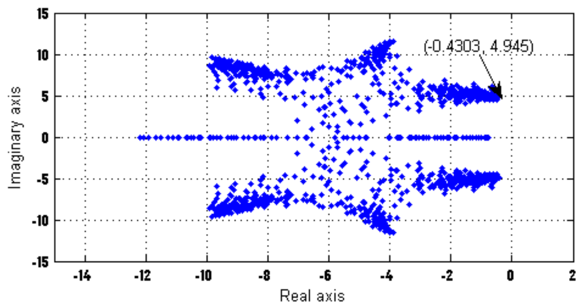

Fig.4a. Closed-loop poles location. Most dominant pole at -0.43, minimum damping ratio offered 5°.

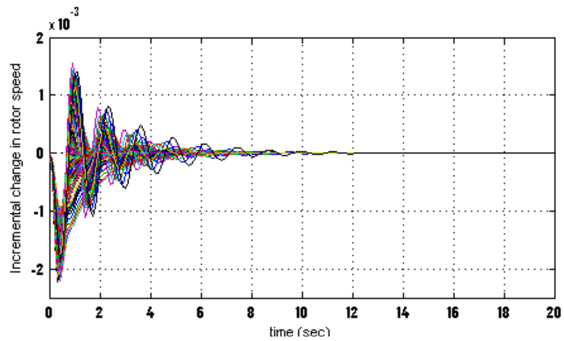

Fig.4b. Closed loop system response to a 5% disturbance step at all 336 operating points with k_d = 1.1916, k_p = 1.7636, k_i =-42.3410.

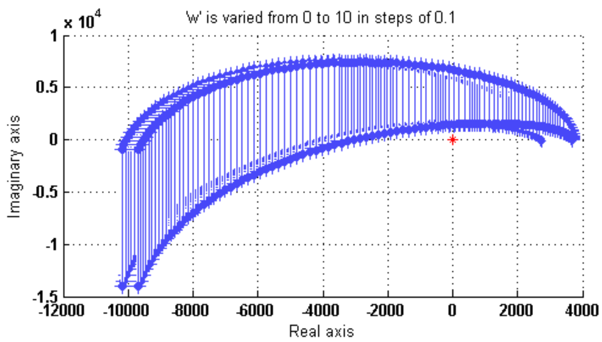

Fig.4c. The Kharitonov Rectangles satisfying zero exclusion condition

The heavy loading condition, P=1.0, Q=0.5, X=0.7 be the nominal operating point

$$G_{heavy}(s) = \frac{-37.23s}{s^4 + 20.66s^3 + 168.69s^2 + 569.62s + 5798.4}$$

Repeating steps 2 to 5, we finally obtain: k_d =1.2020, k_p=14.3134, k_i=-94.5537.

By observing the pole locations in Fig.5, we can say that the controller designed with nominal operating condition as heavy loading condition is not robust. It was already mentioned that CDM does not always guarantee robustness. To get the flexibility in choosing any operating point as our nominal operating condition, we include a pre-filter (Eq. 2) and search for the unknown parameters a & b by using GA such that the stability indices (standard CDM) are [2 2 … 2.5].

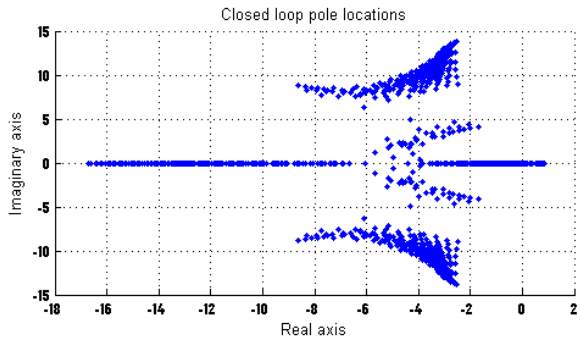

Fig.5 closed-loop poles location shows that the controller is not robustly stable.

This will give the robustness if the pole-zero pair is properly tuned. Out of 336 operating points we can choose anyone as the nominal operating point for designing the controller and we can attain robustness by proper tuning of pre-filter.

B. Fitness calculation for GA:

1. Using the nominal transfer function (with no perturbations), for a particular value of a and b (supplied by GA) find the values of k_d, k_p & k_i by using equations (4), (6) and characteristic polynomial. Find the roots of closed loop characteristic polynomial

2. By making perturbation in P, Q and X obtain the open loop transfer function from equations mentioned in appendix. Find the closed loop transfer function, with the same controller designed at nominal operating point, which gives perturbed pole locations.

3. Using the pole coloring technique, calculate the distance between the corresponding nominal poles and the perturbed poles let sum of the distances be d_{ij} (distance corresponding to i^{th} pertubation and j^{th} iteration)

4. Repeat the steps 2 & 3 for 336 times, add all d'_{ij}s, $D_j = \sum_{i=1}^{336} d_{ij}$, ($D_j$ *is to be minimised*).

C. Robust PSS (PID and a pre-filter):

Step1: Let the heavy loading condition P=1.0, Q=0.5, X=0.7, be the nominal operating point

$$G_{heavy}(s) = \frac{-37.23s}{s^4 + 20.66s^3 + 168.69s^2 + 569.62s + 5798.4}$$

Step2: Choose controller to be designed as in Eq. (2)

Step3: Obtain closed-loop Characteristic polynomial coefficients:

$$a_5 = 1;$$
$$a_4 = 20.66 + b;$$
$$a_3 = 20.66b + 168.69 + 37.23k_d$$
$$a_2 = 168.69b + 569.62 + 37.23ak_d + 37.23k_p$$
$$a_1 = 569.62b + 5798.4 + 37.23ak_p + 37.23k_i$$
$$a_0 = 5798.4b + 37.23ak_i$$

Step4: Choose the stability indices according to the standard CDM as in Eq. (6)

Step5: From equations (4), (6) and step3, we can get the values of k_d, k_p & k_i, for every given values of a and b. Using GA tune a and b by minimizing D_j.

k_d =2.93, k_p = 15.54, k_i = -36.23, a = 9.51, b = 11.36.

The interval characteristic polynomial of the closed loop system becomes:

$G^{I}_{interval} = [1,1]s^5 + [32.02,\ 32.49]s^4 + [502.97,\ 605.57]s^3 + [5123.73,\ 5645.31]s^2 + [19024.95,\ 31328.33]s + [12350.93, 74198.11]$.

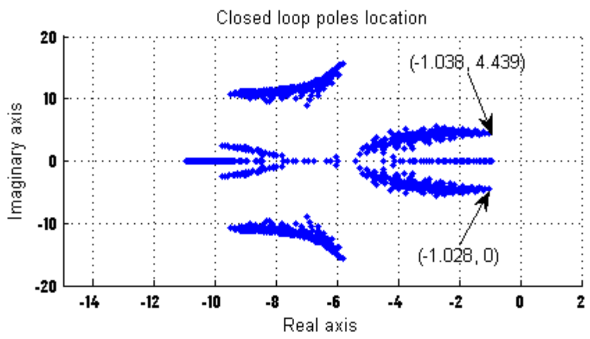

Fig.6a. Closed-loop poles location. Most dominant pole is at -1.028, minimum damping ratio offered is 13.16°.

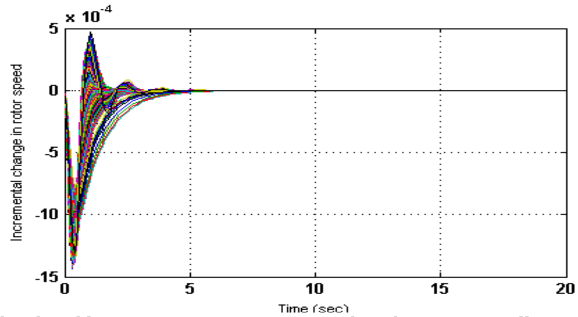

Fig.6b. Closed loop system response to a 5% disturbance step at all 336 operating points with k_d = 2.93, k_p = 15.54, k_i = -36.23, a = 9.51, b= 11.36.

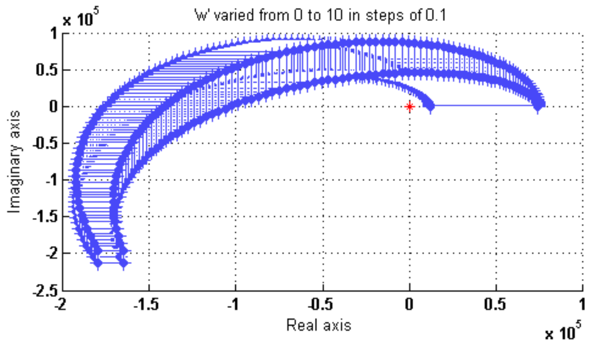

Fig.6c The Kharitonov Rectangles satisfying zero exclusion condition

In table-1 we can find robust PSS designed at different operating points (light, average, heavy and worst loading conditions) as our nominal operating condition, which are satisfying all the closed loop requirements as mentioned in the problem.

TABLE I
ROBUST PSS DESIGNED BY CHOOSING DIFFERENT OPERATING POINTS AS NOMINAL OPERATING CONDITIONS

Loading Condition: [P,Q,X]	PSS $[k_d, k_p, k_i, a, b]$	Most Dominant Pole	Min. Damping Ratio (factor)
Light: [0.4, -0.2,0.2]	[3.58,20.75,-42.75,7.41,14.54]	-0.77	9.67° (0.17)
Average: [0.7,0.15,0.45]	[3.08,15.60,-61.17,7.66,14.66]	-0.67	8.57° (0.15)

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Heavy: [1.0,0.5,0.7]	[2.93,15.54,-36.23,9.51,11.36]	-1.03	13.16° (0.23)
Worst: [1.0,0.0,0.7]	[3.18,6.33,-45.38,10.81,12.15]	-0.71	9.61° (0.17)

In all the cases most dominant pole is always less than -0.5, and the minimum damping factor offered is always greater than 0.1. Thus the performance requirements are achieved.

V. CONTROLLER DESIGN FOR MIMO SYSTEMS

A. Stable systems:

Unity negative feedback configuration using diagonal controller and ZFD:

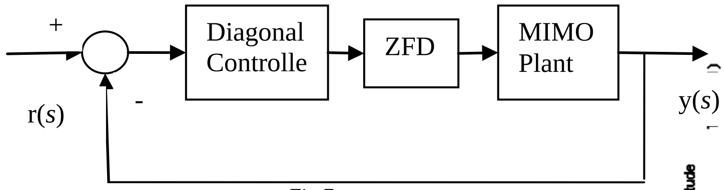

Fig.7

ZFD is the inverse of the zero frequency (steady-state) gain matrix of the plant P(s), i.e. $(P(0))^{-1}$.

Example1: [Ref.11] The open loop transfer function of a pressurized flow-box is considered in the example: Plant TFM:

$$P(s) = \begin{bmatrix} \frac{0.0336}{s+0.395} & \frac{1.03s}{s^2+0.395s+1.26e-4} \\ \frac{9.66e-4s+0.117e-4}{s^2+0.395s+1.26e-4} & \frac{-0.0114}{s^2+0.395s+1.26e-4} \end{bmatrix}$$

$$ZFD = \begin{bmatrix} 11.7560 & 0 \\ 0.012065 & -0.011053 \end{bmatrix}$$

Choosing controller as diagonal PI controller:

$$C_d(s) = \begin{bmatrix} \frac{as+b}{s} & 0 \\ 0 & \frac{cs+d}{s} \end{bmatrix}$$

By finding the closed loop transfer function of the system Fig7: we get the closed loop characteristic polynomial coefficients as,

a0= 2733*b*d;
a1= 2733*a*d+2733*b+0.8814e7*b*d+2508*d+2733*b*c;
a2=-0.2240e9*b+2508*c+0.8108e7*d+2733*a*c+0.8814e7*b*c+0.4287e8*b*d+2733*a+0.8814e7*a*d;
a3=0 .4287e8*a*d+0.5205e8*b*d+0.8108e7*c+0.4287e8*b*c+0.8814e7*a*c+0.7893e9*d-0.2240e9*a-0.7295e12*b;

a4=0.7893e9*c+0.3842e10*d+0.5205e8*b*c-0.3691e13*b+0.5205e8*a*d-0.7295e12*a+0.4287e8*a*c;

a5= -0.4672e13*b+0.3842e10*c+0.5205e8*a*c-0.3691e13*a+0.4800e10*d;

a6= 0.4800e10*c-0.4672e13*a;

By using equations (4) & (6)

[a,b,c,d] = 1.0e+005* [-2.420, -0.0567, -4.3036, -0.01161];

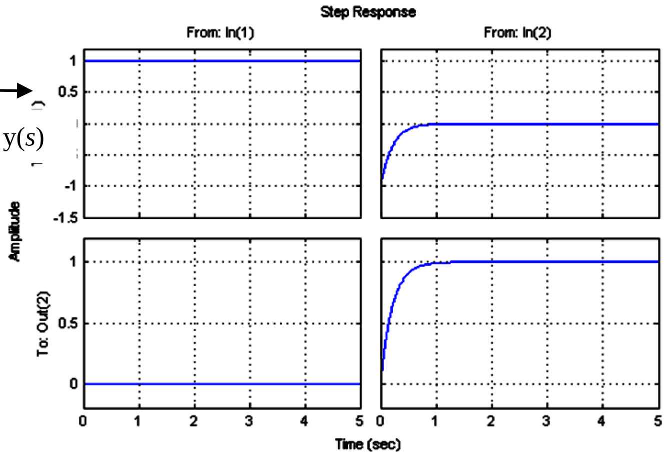

Fig.8

Example 2: [Ref. 12]

In this example, we consider the four-input four-output gas fired furnace problem. The furnace has the following transfer function matrix.

$$P(s) = \begin{bmatrix} \frac{1}{4s+1} & \frac{0.7}{5s+1} & \frac{0.3}{5s+1} & \frac{0.2}{5s+1} \\ \frac{0.6}{5s+1} & \frac{1}{4s+1} & \frac{0.4}{5s+1} & \frac{0.35}{5s+1} \\ \frac{0.35}{5s+1} & \frac{0.4}{5s+1} & \frac{1}{4s+1} & \frac{0.6}{5s+1} \\ \frac{0.2}{5s+1} & \frac{0.3}{5s+1} & \frac{0.7}{5s+1} & \frac{1}{4s+1} \end{bmatrix}$$

$$ZFD = \begin{bmatrix} 1.7484 & -1.2112 & -0.1586 & 0.1694 \\ -0.9796 & 1.8745 & -0.2307 & -0.3217 \\ -0.3217 & -0.2307 & 1.8745 & -0.9796 \\ 0.1694 & -0.1586 & -1.2112 & 1.7484 \end{bmatrix}$$

Choosing controller as diagonal PI controller:

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$$C_d(s) = \begin{bmatrix} \frac{kp1*s+ki1}{s} & 0 & 0 & 0 \\ 0 & \frac{kp2*s+ki2}{s} & 0 & 0 \\ 0 & 0 & \frac{kp3*s+ki3}{s} & 0 \\ 0 & 0 & 0 & \frac{kp4*s+ki}{s} \end{bmatrix}$$

The closed loop characteristic polynomial coefficients:

a5=.2500e5*kp4*kp2*kp3+.2500e5*kp1*kp4*kp2+.2500e5*kp1*kp2*kp3+.2500e5*kp1*kp4*kp3.

a4=4999.*kp1*kp4*kp2+.2500e5*kp1*ki4*kp3+5001.*kp1*kp2*kp3+.2500e5*kp1*kp4*ki2+.2500e5*kp4*ki2*kp3+.2500e5*kp4*ki1*kp3+.2500e5*kp1*kp2*ki3+.2500e5*ki4*kp2*kp3+4999.*kp1*kp4*kp3+.2500e5*kp1*ki2*kp3+.2500e5*kp4*kp2*ki3+.2500e5*ki1*kp2*kp3+.2500e5*kp4*kp2*ki1+.2500e5*kp1*kp4*ki3+5002.*kp1*kp4*kp2*kp3+5001.*kp4*kp2*kp3+.2500e5*kp1*ki4*kp2.

a3=.2500e5*ki4*kp2*ki3+.2500e5*ki4*ki2*kp3+5001.*ki4*kp2*kp3+5001.*kp4*ki2*kp3+5001.*kp4*kp2*ki3+.2500e5*kp4*ki2*ki1+4999.*kp1*ki4*kp3+.2500e5*kp1*ki4*ki2+4999.*kp1*ki4*kp2+.2500e5*ki4*ki1*kp3+.2500e5*ki1*kp2*ki3+.2500e5*ki1*ki2*kp3+.2500e5*kp4*ki1*ki3+4999.*kp4*ki1*kp3+5001.*ki1*kp2*kp3+4999.*kp1*kp4*ki2+.2500e5*ki4*kp2*ki1+5001.*kp1*kp2*ki3+.2500e5*kp1*ki4*ki3+5001.*kp1*ki2*kp3+.2500e5*kp1*ki2*ki3+4999.*kp1*kp4*ki3+.2500e5*kp4*ki2*ki3+5002.*kp1*kp4*kp2*ki3+5002.*kp1*kp4*ki2*kp3+5002.*kp1*ki4*kp2*kp3+5002.*kp4*kp2*ki1*kp3+4999.*kp4*kp2*ki1.

a2=5002.*kp1*kp4*ki2*ki3+5002.*kp4*kp2*ki1*ki3+.2500e5*ki4*ki2*ki3+5002.*ki4*kp2*ki1*kp3+5001.*ki1*ki2*kp3+5001.*ki1*kp2*ki3+5002.*kp1*ki4*ki2*kp3+5002.*kp1*ki4*kp2*ki3+5001.*ki4*ki2*kp3+4999.*kp1*ki4*ki2+.2500e5*ki2*ki1*ki4+5001.*kp1*ki2*ki3+4999.*kp1*ki4*ki3+4999.*kp4*ki2*ki1+4999.*ki4*kp2*ki1+5001.*kp4*ki2*ki3+4999.*kp4*ki1*ki3+.2500e5*ki4*ki1*ki3+5001.*ki4*kp2*ki3+4999.*ki4*ki1*kp3+.2500e5*ki2*ki1*ki3+5002.*kp4*ki2*ki1*kp3.

a1=5002.*ki1*ki4*ki2*kp3+4999.*ki4*ki1*ki3+5002.*kp4*ki2*ki1*ki3+4999.*ki2*ki1*ki4+5001.*ki4*ki2*ki3+5002.*kp1*ki4*ki2*ki3+5002.*ki4*kp2*ki1*ki3+5001.*ki2*ki1*ki3.

a0=5002.*ki4*ki2*ki1*ki3

By using equations (2) & (4)

$$C_d(s) = \begin{bmatrix} \frac{22.1s+4.68}{s} & 0 & 0 & 0 \\ 0 & \frac{1.741s+1.501}{s} & 0 & 0 \\ 0 & 0 & \frac{17.37s+3.675}{s} & 0 \\ 0 & 0 & 0 & \frac{22.1s+4.68}{s} \end{bmatrix}$$

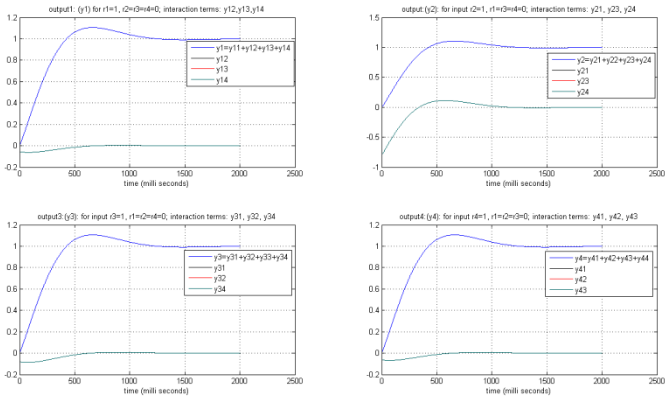

Fig.9

Example 3: [Ref. 13]

The plant considered is Mueller's two-shaft aircraft gas turbine.

Plant TFM:

$$P(s) = \frac{1}{\Delta(s)} \begin{bmatrix} 14.96s^2 + 1521.432s + 2543.2 & 95150s^2 + 1132094.7s + 1 \\ 85.2s^2 + 8642.688s + 12268.8 & 124000s^2 + 1492588s + 2 \end{bmatrix}$$

$$\Delta(s) = s^4 + 113.225s^3 + 1357.275s^2 + 3502.75s + 2525$$

$$ZFD = \begin{bmatrix} -0.4054 & 0.2898 \\ 0.0020 & -0.0004 \end{bmatrix}$$

$$C_d(s) = \begin{bmatrix} \frac{as+b}{s} & 0 \\ 0 & \frac{cs+d}{s} \end{bmatrix}$$

The closed loop characteristic polynomial coefficients:

a8=.3322e5*a+1709.*c;
a7=1709.*d+.4112e7*a+.3322e5*b+.2137e6*c;
a6=.4643e7*c+.3166e5*c*a+.2137e6*d+.4112e7*b+.8491e8*a;
a5=.8491e8*b+.3585e7*c*a+.4643e7*d+.3166e5*c*b+.3166e5*d*a+.3704e8*c+.5972e9*a;
a4=.4295e8*c*a+.3585e7*d*a+.3166e5*d*b+.3585e7*c*b+.5972e9*b+.3704e8*d+.1360e10*a+.1181e9*c;
a3=.1617e9*c+.4295e8*c*b+.1107e9*c*a+.1004e10*a+.1360e10*b+.3585e7*d*b+.4295e8*d*a+.1181e9*d;
a2=.8340e8*a+.1107e9*d*a+.1004e10*b+.1107e9*c*b+.7969e8*c*a+.1617e9*d+.7969e8*c+.4295e8*d*b;
a1=.7969e8*d+.7969e8*c*b+.1107e9*d*b+.8340e8*b+.7969e8*d*a;

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a0=.7969e8*d*b;

By using equations (2) & (4) [a, b, c, d] = [0.8547, 1.4018, -0.004, 330.8]

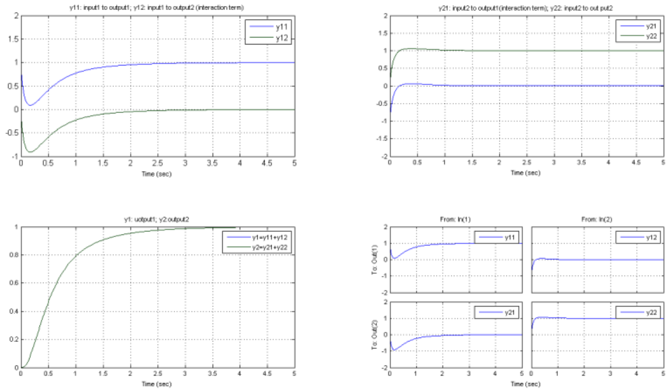
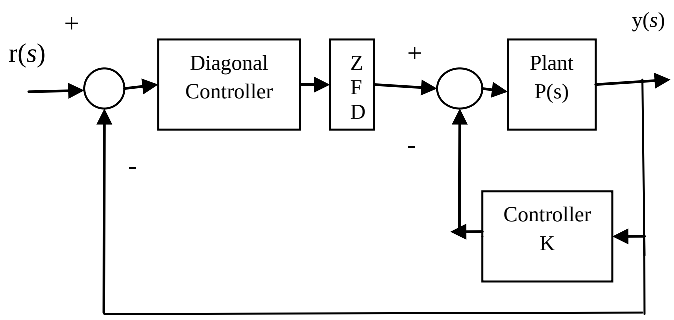


Fig.10

B.Unstable systems:

Two-Loop Unity Negative Feedback Configuration: Fig.11



Example 4: [Ref. 14]

The state space matrices of an unstable system are given by

$$A=\begin{bmatrix}2.375 & 0.857 & 1.000\\ -17.719 & -5.50 & -5.250\\ -14.766 & -6.75 & -7.375\end{bmatrix};\ B=\begin{bmatrix}0 & 0\\ 1 & 0\\ -1 & 1\end{bmatrix};$$

$$C=\begin{bmatrix}0 & 0.3 & 1.8\\ 0 & 0 & -4\end{bmatrix};\ D=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix};$$

The transfer function matrix is

$$\frac{1}{s^3+10.5s^2+4.5s-5}\begin{bmatrix}-1.5s^2-14.7s+3 & 1.8s^2+4.05s+2.25\\ 4s^2+39.5s-5 & -4s^2-12.5s-8.5\end{bmatrix}$$

The poles are at: -1,-10 and +0.5.

Eigen value assignment by the method given in [224].

The desired characteristic polynomial is

(s+1)(s+10)(s+2)

The gain matrix for stabilization is obtained as

$$K=\begin{bmatrix}h_1\\ h_2\end{bmatrix}\begin{bmatrix}f_1 & f_2\end{bmatrix}=\begin{bmatrix}24.975 & 10.1\\ 0 & 0\end{bmatrix}.$$

The closed-loop TFM, Z(s) of the stabilized system is found to be

$$Z(s)=\frac{1}{d(s)}\begin{bmatrix}-1.5s^2-14.7s+3.0076 & 1.8s^2+16.17s-26.539\\ 4s^2+39.5s-5.0164 & -4s^2-42.47s+62.688\end{bmatrix}$$

$$d(s)=s^3+13.438s^2+36.316s+19.441$$

This stabilized closed-loop TFM (minor loop) will serve as the plant TFM in the next step, i.e. design of outer loop for performance.

$$ZFD=\begin{bmatrix}21.9945 & 9.3114\\ 1.76 & 1.0552\end{bmatrix}$$

$$C_d(s)=\begin{bmatrix}\frac{k1(s+a1)}{s+b1} & 0\\ 0 & \frac{k2(s+a2)}{s+b2}\end{bmatrix}$$

The closed loop characteristic polynomial coefficients:

a7=-2401.*k1+3290.*k2;
a6=3290.*k2*a2+3290.*k2*b1+.8510e6*k1-2401.*k1*b2-.2762e5*k2-2401.*k1*a1+.1022e5*k1*k2;
a5=3290.*k2*a2*b1-.2762e5*k2*a2+.8510e6*k1*a1+.1198e7*k1*k2-.8700e6*k2-.2762e5*k2*b1+.1022e5*k1*k2*a2+.8510e6*k1*b2-2401.*k1*a1*b2+.1022e5*k1*a1*k2+.1176e8*k1;
a4=-.8700e6*k2*a2+.1176e8*k1*a1+.8510e6*k1*a1*b2-.2872e7*k2+.1022e5*k1*a1*k2*a2-.8700e6*k2*b1+.1176e8*k1*b2+.1198e7*k1*k2*a2+.3170e8*k1-.2762e5*k2*a2*b1+.1068e8*k1*k2+.1198e7*k1*a1*k2;
a3=.3170e8*k1*a1+.3170e8*k1*b2+.1198e7*k1*a1*k2*a2+.1176e8*k1*a1*b2-.2872e7*k2*a2+.1068e8*k1*k2*a2-.2278e7*k2-.8700e6*k2*a2*b1-.2858e7*k1*k2+.1068e8*k1*a1*k2-.2872e7*k2*b1+.1629e8*k1;
a2=.1629e8*k1*a1-.2858e7*k1*a1*k2-.4720e6*k2-.2278e7*k2*b1-.2858e7*k1*k2*a2+.1629e8*k1*b2-.4722e6*k1*k2-.4702e6*k1-.2278e7*k2*a2-.2872e7*k2*a2*b1+.1068e8*k1*a1*k2*a2+.3170e8*k1*a1*b2;
a1=-.4720e6*k2*a2-.4720e6*k2*b1+.1629e8*k1*a1*b2-.4702e6*k1*b2-.4702e6*k1*a1-.2858e7*k1*a1*k2*a2-.2278e7*k2*a2*b1-.4722e6*k1*a1*k2-.4722e6*k1*k2*a2;

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a0=-.4722e6*k1*a1*k2*a2-.4720e6*k2*a2*b1-.4702e6*k1*a1*b2; By using equations (2) & (4)

$$C_d(s) = \begin{bmatrix} \frac{0.1379s + 0.394}{s + 3.93} & 0 \\ 0 & \frac{1.367s - 1.889}{s + 0.4375} \end{bmatrix}$$

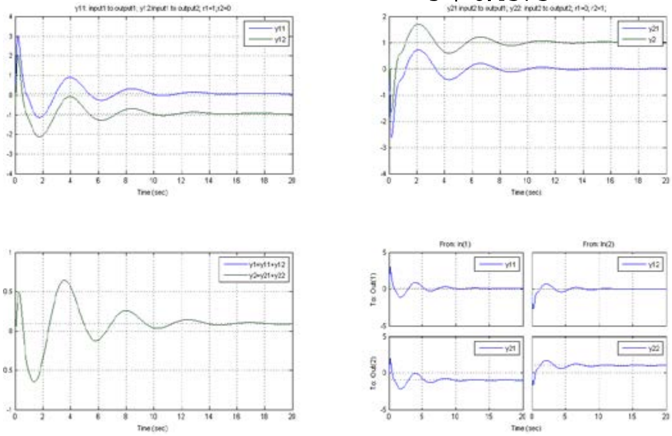

Fig.10

VI. CONCLUSIONS

The resultant controller is: Implementable, Low Order, All Stabilizing and Robust. Only output feedback is used and all given closed loop specifications are satisfied.

Using CDM and GA robust PSS can be designed by choosing any operating point (in the specified range) as our nominal operating condition. The method is flexible to choose nominal operating point, where the plant is running most of the time, at which we desire better performance.

APPENDIX

A. *Open loop state space representation:*

The state equation of a single machine connected with infinite bus (SMIB) system may be derived from the linearized transfer function model (Fig.2) as:

$$\dot{x} = Ax + bu$$
$$\Delta\omega = c_1 x$$
$$\Delta V_t = c_2 x$$

Where,

$$A = \begin{bmatrix} 0 & \omega_r & 0 & 0 \\ -\frac{k_1}{2H} & -\frac{D}{2H} & -\frac{k_2}{2H} & 0 \\ -\frac{k_4}{\tau'_{d0}} & 0 & -\frac{1}{k_3\tau'_{d0}} & \frac{1}{\tau'_{d0}} \\ -\frac{K_a k_5}{T_a} & 0 & -\frac{K_a k_6}{T_a} & -\frac{1}{T_a} \end{bmatrix}$$

$$b = \begin{bmatrix} 0 & 0 & 0 & \frac{K_a}{T_a} \end{bmatrix}^t$$

$$c_1 = [0 \quad 1 \quad 0 \quad 0]$$
$$c_2 = [k_5 \quad 0 \quad k_6 \quad 0]$$

State vector x is defined as, $x = [\Delta\delta \quad \Delta\omega \quad \Delta E'_q \quad \Delta E_{fd}]^t$

Where $\Delta\delta$, $\Delta\omega$, $\Delta E'_q$ and ΔE_{fd} are the incremental changes in rotor speed, rotor angle, voltage proportional to field flux linkage and field voltage respectively. k_i, i=1, 2… 6 are the k-parameters whose value depends on the operating conditions.

B. *Equations for k-parameters [7]*

$$k_1 = K_t V_\infty [E_{qa}\{R_e \sin(\delta-\alpha) + (X_e + x'_d)\cos(\delta-\alpha)\} + I_q(x_q - x'_d)\{(X_e + x_q)\sin(\delta-\alpha) - R_e\cos(\delta-\alpha)\}]$$

$$k_2 = K_t\left\{R_e E_{qa} + I_q\left(R_e^2 + (X_e + x_q)^2\right)\right\}$$

$$k_3 = \left(1 + \left(K_t(x_q - x'_d)(X_e + x_q)\right)\right)^{-1}$$

$$k_4 = K_t V_\infty (x_d - x'_d)\{(X_e + x_q)\sin(\delta-\alpha) - R_e\cos(\delta-\alpha)\}$$

$$k_5 = \frac{K_t V_\infty}{V_t}[V_q x'_d\{R_e\cos(\delta-\alpha) - (X_e + x_q)\sin(\delta-\alpha)\} - V_d x_q\{R_e\sin(\delta-\alpha) + (X_e + x'_d)\cos(\delta-\alpha)\}]$$

$$k_6 = \frac{V_q}{V_t}(1 - K_t(X_e + x_q)x'_d) - \frac{K_t V_d}{V_t} R_e x_q$$

Where

$$K_t = \frac{1}{R_e^2 + (X_e + x'_d)(X_e + x_q)}$$

C. *System parameter values [2]*

TABLE II
SYSTEM PARAMETERS

H	ω_r	K_a	T_a	R_e	r	ΔV_t	D
3.25	314.15	50	0.05	0	0	1	0
x_d	x_q	x'_d	τ'_{d0}				
2.0	1.91	0.224	4.18				

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