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Application of Fuzzy Sets Theory in Students' Performance Appraisal

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Abstract

This paper presents a method for evaluating students' performance based on fuzzy sets theory. This evaluation approach takes into account two major factors: Continuous Assessment Tests (CATs) and Final Exam. The results achieved by this method are compared to the results of the existing arithmetical method of evaluating students' performance. This paper also describes the fuzzy logic basic concepts applied in evaluating students' performance.

Keywords: *Fuzzy Logic, Fuzzy sets theory, Fuzzy Inference System, Membership Functions, Students' Performance.*

1. Introduction

Promotion and certification of students through different education levels is entirely bounded to students' performance evaluation. Different arithmetical and statistical methods of evaluating students' performance have been in use for a long time. However, new approaches are emerging such as Fuzzy sets theory based approach of student performance evaluation to enhance existing evaluation methods.

The classical way of evaluating student's performance is to calculate the aggregate mark of continuous and final assessment results. However, the performance evaluated in such classical way omits some important details to achieve an accurate evaluation of performance.

In their study, Servaas & Debra (2010) distinguish two types of assessment inaccuracies, in terms of their statistical qualities, with different signaling dimensions:

- **Assessment leniency** (where Continuous Assessments marks are much higher than exam marks): An inflated CA mark, where it is much higher than examination marks, can give students a false sense of security about how well they are prepared for the exams in that subject.
- **Low Assessment reliability** (where performance measured by CATs and examination marks is only weakly correlated): A poor correlation between CATs and examination marks indicates that the former is also an unreliable indicator of the individual's relative ability compared to classmates in a particular subject.

e.g. a student who passed with a mark 50% (42/60 in CATs and 8/40 in final exam) is evaluated as having a better performance regardless his lower performance in the final exam.

It is in that perspective that this paper describes a fuzzy logic based approach adopted to enhance the measurement of students' performance.

2. Fuzzy Logic

The Fuzzy Logic (FL) was initated in 1965, by Lofti A.Zadeh, professor for computer science at the University of California in Berkeley. Basically, Fuzzy Logic (FL) is a multivalued logic that allows intermediate values to be defined between conventional evaluations like true/false, yes/no, high/low, etc. (Zadeh.,L.A., 1984). Fuzzy system is an alternative to traditional notions of set membership and logic that has its origins in the ancient Greek philosophy, and applications at the leading edge of Artificial Intelligence. Yet, despite its long-standing origins, it is a relatively new field, and as such leaves much room for development (Ramot.,D., et al, 2003).

Brief information about fuzzy logic systems is given below based on the studies of Mendel. (Mendel. J., et al., 1997).

3. Fuzzy set theory

3.1. Crisp sets

Fuzzy logic is a concept that is founded on classical or crisp set theory. Crisp set theory defines a universe, say Z, in which a collection of objects, also known as elements, exist within this universe. Often these elements share a similarity that allows them to be grouped together for simplicity or convenience. For example, in a universe whose characteristic elements are whole numbers ranging from one to ten inclusive; there exists a set titled ‘prime numbers’. Therefore the elements that are unique to this set are two, three, five and seven. Every other number would fall outside the boundary of this set. This is classified as a classical or crisp set, as known ‘without-a-doubt’ that these four numbers belong to this set (Marcus, 2011).

Classical set theory studies the properties of sets. Its methods span over vast fields in mathematics and are applied to a variety of applications (e.g. fuzzy logic). Set theory uses a language based on a single fundamental relation called membership, denoted “∈”. If an element “x” is a member of set “A” then the relationship can be expressed as follows:

$$x \in A \quad (3)$$

In classical set theory the membership of elements in a set is assessed in binary terms (1 or 0) according to a bivalent condition (Marcus, 2011).

3.2. Fuzzy Sets

Fuzzy sets can be considered as an extension of crisp sets. For a fuzzy set A, the function μA represents the membership function for which μA (x) measures the degree to which an absolute value x, of the universal set Z, belongs to set A . For a classical set, the membership function follows conventional Boolean logic in that an element either does or does not belong to a set. Therefore, its membership value will be either 1 (true) or 0 (false) (Marcus, 2011).

In their paper Ramjeet & Vijendra (2011) highlighted how Fuzzy sets are defined:

A fuzzy set A in a universe of discourse X is defined as the following set pairs

$$A = \{(\mu_A(x) : x \in X\} \qquad (1)$$

Where, $\mu A: X \to [0,1]$ is a mapping called the membership function of fuzzy set A and $\mu_A(x)$ is called the degree of belongingness or membership value or degree of membership of $x \in X$ in the fuzzy set A. It can be written in the following form:

$$A = \{(\mu_A(x)/x : x \in X\} \quad (2)$$

4. Description of fuzzy logic system

A Fuzzy logic system (FLS) consists of four basic elements (Fig. 1): the fuzzifier, the fuzzy rulebase, the inference engine, and the defuzzifier. The fuzzy rulebase is a collection of rules of the form of R^l, which are combined in the inference engine, to produce a fuzzy output (in essence, the inference engine produces mappings from fuzzy sets to fuzzy sets). The fuzzifier maps the crisp inputs into fuzzy sets, which are subsequently used as inputs to the inference engine, whereas the defuzzifier maps the fuzzy sets produced by the inference engine into crisp numbers. Fuzzy sets can be interpreted as membership functions μX that associate with each element x of the universe of discourse, U, a number μX *(x)* in the interval [0.1].

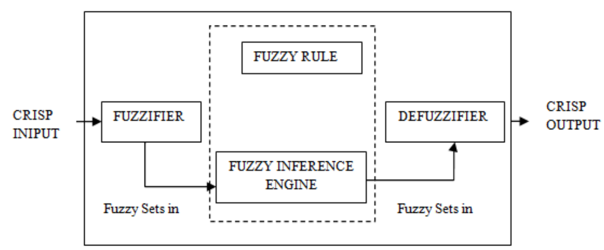

Fig.1. Structure of fuzzy logic system (Mendel & Mouzouris, 1997).

There are two basic types of fuzzy inference systems: Mamdani-Assilian (or Mamdani), 1975 by Ebrahim Mamdani (Mamdani & Allian, 1975) and Takagi-Sugeno-Kang (or Sugeno), introduced in 1985 (Takagi & Sugeno, 1985). These two types of inference systems vary somewhat in the way outputs are determined. In Mamdani systems, which is the most common methodology (Fuzzy logic toolbox, fuzzy inference systems), both the input and output are represented with linguistic terms (such as “good”, “poor”, “excellent”). The antecedent and consequent of an if-then rule are typically Bollean expression of simple clauses. A simple form of the Mamdani system is of the form:

If x is A, then y is B.

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In which A is a linguistic term defined by a fuzzy sets on the ranges (universe of discourse) *X* and *Y* respectively.

In Sugeno systems, the antecedent is a Bollean expression of simple clauses, but the consequent is a function of the input (usually a polynomial). This can be represented in the form:

If *x* is *A*, then y is *f(x).*

In which *A* is a linguistic term defined by a fuzzy set on the universe of discourse *X* and *f(x)* is a function of the input *x*.

The Sugeno fuzzy inference systems are faster and work better with linear techniques. The Mamdani systems however, are intuitive and suitable for human inputs. Therefore, we employed the Mamdani FISs to improve the students' performance evaluation.

We will have a brief overview of the process of fuzzy inference, and explain the design process of Mamdani-type FISs in the following sections.

5. Evaluating students' performance using Fuzzy Logic System

The Fuzzy System provided in this paper was built using a FIS Editor tool of MATLAB Software. Mamdani's fuzzy inference method as mentioned earlier.

Three steps were involved during the application of fuzzy model. The steps included:

1. Fuzzification of input variables (CATs and EXAM) and output variable (Performance value);
2. Definition of Fuzzy rules for the fuzzy logic system and inference method;
3. Defuzzificatin of performance value.

The next figure illustrate the Fuzzy model structure adopted:

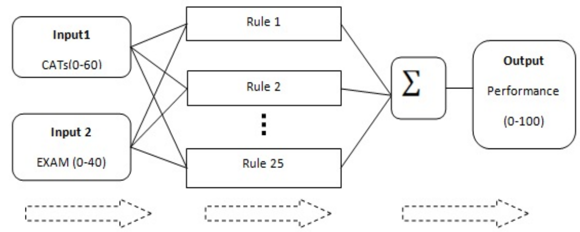

Fig.2. Fuzzy model structure of students' performance evaluation

In the above figure, information flows from left to right, from two inputs to a single output. According to MATLAB user guide (2013), the parallel nature of the rules is one of the most important aspects of fuzzy logic systems. Instead of sharp switching between modes based on breakpoints, logic flows smoothly from regions where the system's behavior is dominated by either one role or another.

Step1: Fuzzification of input variables and output variable

The following linguistic variables were used: "Very Low", "Low", "Average", "High", and "Very High". In this paper the simplest membership functions were formed using triangular membership function. This function is nothing more than a collection of three points forming a triangle. The inputs and outputs were fuzzified according to each of the linguistic sets mentioned above as shown by the next figure 3.

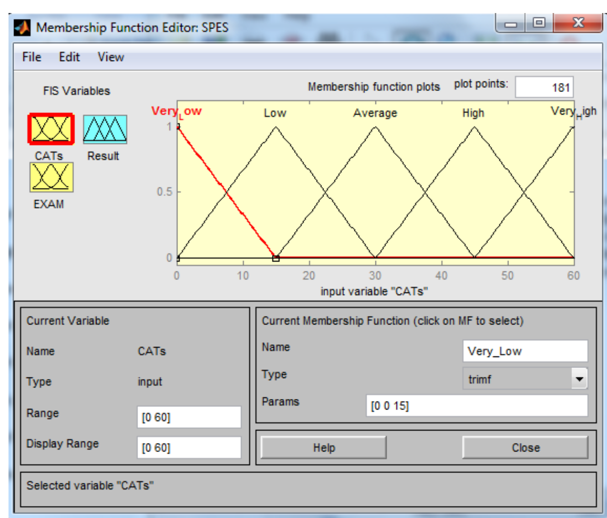

Fig.3. Membership functions for input variable "CATs"

Step 2: Definition of Fuzzy rules for the fuzzy logic system and inference method

Fuzzy sets and fuzzy operators are subjects and verves of fuzzy logic. These if-then rule statements were used to formulate the conditional statements that comprise fuzzy logic. CATs and EXAM were taken as parts of antecedent. All parts of antecedent were calculated simultaneously and resolved to a single number using the logical operator AND. The performance of the student was considered as the consequent affected by the antecedent. Next is sample of 10 rules taken from 25 if-then rules generated for the system :

1. If (CATs is Very_Low) and (EXAM is Very_Low) then (Performance is Very_Low) (1)
2. If (CATs is Very_Low) and (EXAM is Low) then (Performance is Very_Low) (1)
3. If (CATs is Very_Low) and (EXAM is Average) then (Performance is Low) (1)
4. If (CATs is Very_Low) and (EXAM is High) then (Performance is Low) (1)
5. If (CATs is Very_Low) and (EXAM is Very_High) then (Performance is Average) (1)
6. If (CATs is Low) and (EXAM is Very_Low) then (Performance is Very_Low) (1)
7. If (CATs is Low) and (EXAM is Low) then (Performance is Low) (1)
8. If (CATs is Low) and (EXAM is Average) then (Performance is Low) (1)
9. If (CATs is Low) and (EXAM is High) then (Performance is Average) (1)
10. If (CATs is Low) and (EXAM is Very_High) then (Performance is Average) (1)

Step 3: Defuzzification of performance value

The most popular defuzzification method was adopted which is centroid calculation. According to Ross (1995), centroid calculation is computationally faster and easier and gives fairly accurate result. The defuzzification helps to resolve a single output value from the set given that the input for the deffuzification process is a fuzzy set (the aggregate output fuzzy set).

The next figure illustrates the defuzzification in rules viewer of FIS Editor:

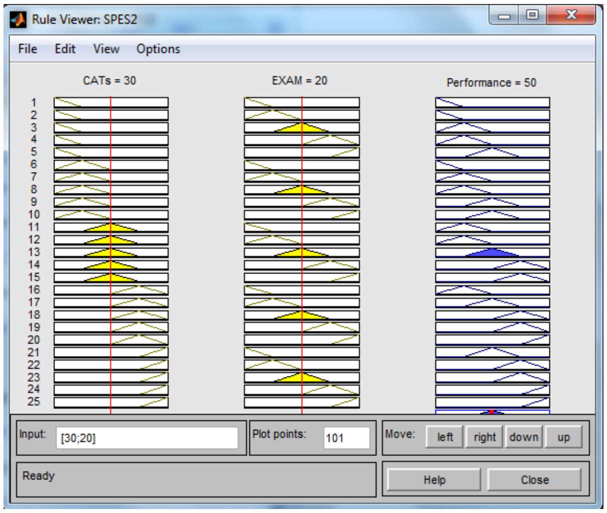

Fig.4. Performance analysis with fuzzy logic rule based system

6. Results and Discussion

The research used only secondary data collected from the Independent Institute of Lay Adventists of Kigali. A copy of marks record of General Mathematics course was used. Marks obtained in CATs and in Final Exam were used as inputs of the system to produce a single mark output which is the final performance of the student.

This FISs method of evaluating students' performance was tested on a representative sample of 77 grades (performance record) of students obtained in the course of General Mathematics. Only 20 students record are presented in this paper for discussion.

The next table shows the overall performance value calculated by classic method and FIS method. The observed difference between the two methods in terms of marks is shown as well.

Table 1: Students' performance calculated using Classic method and Fuzzy Inference System method

S/N	Reg. No	CATs	EX	Classic Total	FIS Total	Dif.
1	**03	28.5	12.5	**44**	**38.5**	5.5
2	**81	31.5	19.5	**50.5**	**50.5**	0
3	**86	34	16	**50**	**47.4**	2.6
4	**60	27.5	10	**37.5**	**25**	12.5
5	**62	35	24	**59**	**60.5**	-1.5
6	**65	45.5	35	**80.5**	**78.5**	2
7	**71	35.5	27.5	**63**	**67.5**	-4.5
8	**73	31	25	**56**	**62.5**	-6.5
9	**74	35.5	24.5	**60**	**61.5**	-1.5
10	**77	35	26.5	**61.5**	**65.6**	-4.1
11	**76	29	24	**53**	**58.1**	-5.1
12	**77	37.5	20.5	**58**	**62.5**	-4.5
13	**95	33.5	13.5	**47**	**42.5**	4.5
14	**69	52	38.5	**90.5**	**83.5**	7
15	**79	42.5	8	**50.5**	**44**	6.5
16	**33	38	12.5	**50.5**	**45.9**	4.6
17	**56	53	30	**83**	**78.7**	4.3
18	**21	42.5	9.5	**52**	**44.8**	7.2
19	**32	34.5	8	**42.5**	**33.4**	9.1
20	**86	36.5	22	**58.5**	**61.4**	-2.9

By looking closely to the data displayed in the above table, it is observed that:

-When classical method was used to evaluate students' performance, students with very poor performance in one component, especially in exam, since CATs have more weight (60%) than exam (40%), can pass the module as long as their overall performance exceeds 50%. E.g. a student who obtained 42.5 over 60 in CATs with 9.5 over 40 in exam passed the module with 52 over 100. In addition, students who had different gaps of performance in their respective components were assessed to have the same overall performance since their grade sum was the same. E.g. a student who got 31.5 over 60 in CATs and 19.5 over 40 in exam was evaluated to have the same overall performance with a student who got 42.5 over 60 in CATs and 8 over 40 in exam since overall performance summed up to 50.5 over 100.

- While a FIS method was used to evaluate students' performance, students who had a wide gap of performance in evaluation components failed to achieve the pass mark. E.g. In the previous example the student with 42.5 over 60 in CATs and 8 over 40 in exam was evaluated to have an overall performance of 44 out of 100. Furthermore, students who performed differently in CATs and Exam but with the same total were evaluated differently by the FIS method. The case of a student who got 31 out of 60 in CATs and 19.5 out of 40 in exam was evaluated to have an overall performance of 50.5 out of 100 while his classmate who got 42.5 out of 60 in CATs and 8 out of 40 in exam was evaluated to have an overall performance of 44 out of 100.

Contrarily to classic method, FIS method shown its ability to make a distinction of the performance level by taking into account the deepness of the gap observed between the components of evaluation. We can deduct that narrow is the gap between performance level in CATs and EXAM better is the overall performance level. Wide gap resulted into poor overall performance.

A paired t-test was used to compare the significance of the difference observed between the two methods of measurement. SPSS was used to compute the t and p value. Input variables consisted of two sets of 77 students' final performance, one containing grades obtained by classic method and the second containing grades obtained by FIS method. The next table highlights the paired samples statistics.

Table 2: Paired Samples Statistics

	Variables	Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Classical Method	57.2727	77	17.64946	2.01134
	FIS Method	55.7156	77	18.89060	2.15278

The above table shows that the FIS method mean is approximately 2 marks less than classic method mean. The next table shows the paired samples test results.

Table 3: Paired Samples Test results

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	Classical_Method - FIS_Method	1.55714	4.94428	.56345	.43493	2.67936	2.764	76	**.007**

From the table above, the most interesting value is the final column “sig. (2-tailed). It contains a value named **p** value of **.007**. From the last row of the table, it is observed that t statistic, t=2.764 and p=0.007; that is to say a very small probability of this result occurring by chance, since $p<0.05$ (in fact p=0.007). It implies that there is strong evidence (t=2.764, p=0.007) that the difference of two methods of evaluating students’ performance is statistically significant.

In these date sets, the FIS method reduced total marks at an average of 2 marks approximately compared to classic method of evaluating students’ performance. Though the difference was statistically significant, it was actually relatively small. Thus, it was necessary to consider if the noticed difference in marks was practically important.

According to NCHE (2013), the assessment of each module shall generate a single mark between 0 and 100% expressing the extent to which the learning outcomes have been achieved. The pass mark for all modules of each level shall be 50% in undergraduate programmes. Based on the above statement, the next table shows the observation made based on the final results of 77 students evaluated using both methods.

Table 4: Practical implications of evaluation methods

Method	**No of passes ≥50%**	**No of fails <50%**	**Tot of students**	**Pass %**	**Fail %**
Classic Method	54	23	77	70.1%	29.9%
FIS Method	46	31	77	59.7%	40.3%
Difference				**-10%**	

The findings from table 4, reveals that the use of classic method allowed only 70% of students to pass the module while the use of FIS method allowed approximately 60% of students. A practical difference of 10% was due to the efficiency of FIS method in tracking unbalanced performance as far as evaluation components were concerned.

Definitely, the significant statistical difference observed between Classic method and FIS method of evaluating students’ performance corresponded to a practical implication given that FIS method could filter more unsuccessful performances than classic method at 10%.

7. Conclusion

Based on the results, evaluating students’ performance using Fuzzy Inference System showed to be more effective than classic method. Fuzzy inference System can be used to balance the performance of students in both Continuous assessments tests and exam, by therefore avoiding overlooking poor performance in one component given better performance in another. Finally, at the University level, the new approach has an emphasis of encouraging students to become active in all aspects of assessment components (CATs and Exam) to obtain better grades.

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