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Mean Time to Recruitment for a Two Graded Manpower System with Two Thresholds, Different Epochs for Exits and Correlated Inter-decisions

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Abstract

In this paper, the problem of time to recruitment is studied using a univariate policy of recruitment involving two thresholds for a two graded manpower system with attrition generated by its policy decisions. Assuming that the policy decisions and exits occur at different epochs, a stochastic model is constructed and the mean time to recruitment is obtained when inter-exit times form an ordinary renewal process and the inter-policy decision times are exchangeable and constantly correlated exponential random variables.

Keywords: *Two grade manpower system; decision and exit epochs; correlated inter-decision times; ordinary renewal process; univariate policy of recruitment with two thresholds, mean time to recruitment.*

1. Introduction

Attrition is common phenomenon in many organizations. This leads to the depletion of manpower. Recruitment on every occasion of depletion of manpower is not advisable since every recruitment involves cost. Hence the cumulative depletion of manpower is permitted till it reaches a level, called the threshold. If the total loss of manpower exceeds this threshold, the activities in the organization will be affected and hence recruitment becomes necessary. In [1, 2, 6] the authors have discussed the manpower planning models by Markovian and renewal theoretic approach. In [5] the author has initiated the study of the problem of time to recruitment for a single grade manpower system by incorporating alertness in the event of cumulative loss of manpower due to attrition crossing the threshold, by considering optional and mandatory threshold for the cumulative loss of manpower in this manpower system. In [11] the author has studied the problem of time to recruitment for a two graded manpower system, by considering optional and mandatory thresholds. In [7,8] the author has studied the problem of time to recruitment for a two graded manpower system, the loss of man hour are exchangeable and constantly correlated exponential random variables and inter-decision times follows an exponential distribution.

In all the above cited work, it is assumed that attrition takes place instantaneously at decision epochs. This assumption is not realistic as the actual attrition will take place only at exit points Which may or may not coincide with decision points. This aspect is taken into account for the first time in [3,4] and variance of time to recruitment is obtained when inter-decision times and exit times are independent and identically distributed exponential random variables using univariate policy for recruitment and Laplace transform in the analysis. Recently, in [9,10] the author has studied the work in [3,4] by considering optional and mandatory thresholds which considering non-instantaneous exits at decision epochs. In the present paper, for a two graded manpower system , a mathematical model is constructed in which attrition due to policy decision take place at exit points and there are optional and mandatory thresholds as control limits for the cumulative loss of manpower. A univariate policy of recruitment based on shock model approach is used to determine the expected time to recruitment when the system has different epochs for policy decisions and exits and the inter-policy decision times are exchangeable and constantly correlated exponential random variables.

2. Model Description

Consider an organization taking decisions at random epochs in $(0,\infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. It is assumed that the loss of manpower is linear and cumulative. Let X_i be the continuous random variable representing the amount of depletion of manpower (loss of man hours) caused at the i^{th} exit point . Let

S_k be the total loss of manpower up to the first k exit points. It is assumed that X_i 's are independent and identically distributed random variable with density function m(.), distribution function M(.) and Mean 1/α; α>0.Let U_k be the continuous random variable representing the time between the $(k-1)^{th}$ and k^{th} policy decisions. It is assumed that U_k's are exchangeable and constantly correlated exponential random variables with probability density function f(.) , distribution function F(.) and mean u.Let R be the correlation between U_i and U_j; i≠j and v=u(1-R). Let W_i be the continuous random variable representing the time between the $(i-1)^{th}$ and i^{th} exit times. It is assumed that W_i's are independent and identically distributed random variables with probability density function g(.), probability distribution function G(.). Let $N_e(t)$ be the number of exits points lying in (0,t] .Let Y_A , Y_B be the exponential random variable denoting the optional thresholds for grade A and B respectively. Let $H_A(.)(h_A(.))$, $H_B(.)(h_B(.))$ be the distribution (density) function of Y_A , Y_B with parameters λ_A , λ_B respectively. Let Z_A , Z_B be the exponential random variable denoting the mandatory thresholds for grade A and B respectively. Let $H_A(.)(h_A(.))$, $H_B(.)(h_B(.))$ be the distribution (density) function of Z_A , Z_B with parameters μ_A , μ_B respectively. Assume that $Y_A < Z_A$ & $Y_B < Z_B$. Let p be the probability that the organization is not going for recruitment when optional threshold is exceeded by the cumulative loss of manpower. Let q be the probability that every policy decision has exit of personnel. As q=0 corresponds to the case where exits are impossible, it is assumed that q≠0.Let T be the random variable denoting the time to recruitment with distribution function L(.), density function l(.) and mean E(T). Let $A^*(.)$, $\overline{a(.)}$ be the Laplace –Stieltjes and Laplace transform of A(.) and a(.) respectively.

The univariate CUM policy of recruitment employed in this paper is stated as follows.

Recruitment is done whenever the cumulative loss of man power in the organization exceeds the mandatory threshold. The organization may or may not go for recruitment if the cumulative loss of man power exceeds the optional threshold.

3. Main Result:

P (T > t) = P{ Total loss of manpower at the exit points in (0,t] does not exceed Y or the total loss of manpower at the exit points in (0,t] exceeds Y but lies below Z and the organization is not making recruitment}

$$P(T>t)=P(S_{N_e(t)}\le Y)+P(Y<S_{N_e(t)}\le Z)p$$

$$P(T>t)=\sum_{k=0}^{\infty}P[N_e(t)=k]P(S_k\le Y)+p\sum_{k=0}^{\infty}P[N_e(t)=k]P(S_k>Y)P(S_k\le Z) \quad \text{-- (1)}$$

From Renewal theory, we have

$$P\{N_e(t)=k\}=G_k(t)-G_{k+1}(t) \text{ and } G_0(t)=1 \quad \text{---(2)}$$

From (1), we get

$$P(T>t)=\sum_{k=0}^{\infty}\{G_k(t)-G_{k+1}(t)\}P\{S_k\le Y\}$$
$$+p\sum_{k=0}^{\infty}\{G_k(t)-G_{k+1}(t)\}P\{S_k>Y\}P\{S_k\le Z\} \quad \text{-- (3)}$$

Note that

$$L(t)=1-P(T>t);E(T^r)=(-1)^r[\frac{d^r}{ds^r}\bar{l}(s)]_{s=0},r=1,2,... \quad \text{--- (4)}$$

3.1 Case-I: Y = max (Y_A , Y_B) & Z = max (Z_A , Z_B)

$$P\{S_k<Y\}=\int_0^{\infty}P\{Y>X\}g_k(x)dx$$

$$P(S_k\le Y)=a_1^k+a_2^k-a_3^k,$$

where $a_1=E[e^{-\lambda_A x}]$; $a_2=E[e^{-\lambda_B x}]$ & $a_3=E[e^{-(\lambda_A+\lambda_B)x}]$.

$$P(S_k\le Z)=b_1^k+b_2^k-b_3^k ,$$

where $b_1=E[e^{-\mu_A x}]$, $b_2=E[e^{-\mu_B x}]$ & $b_3=E[e^{-(\mu_A+\mu_B)x}]$ --- (5)

P(T>t)=

$$\sum_{k=0}^{\infty}\{G_k(t)-G_{k+1}(t)\}\{a_1^k+a_2^k-a_3^k\}$$
$$+p\sum_{k=0}^{\infty}\{G_k(t)-G_{k+1}(t)\}\{1-a_1^k-a_2^k+a_3^k\}\{b_1^k+b_2^k-b_3^k\} \quad \text{--- (6)}$$

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$$\bar{l}(s) = \overline{a_1}\sum_{k=1}^{\infty}[\overline{g(s)}]^k a_1^{k-1} + \overline{a_2}\sum_{k=1}^{\infty}[\overline{g(s)}]^k a_2^{k-1} - \overline{a_3}\sum_{k=1}^{\infty}[\overline{g(s)}]^k a_3^{k-1} + p\{\ \overline{b_1}\sum_{k=1}^{\infty}[\overline{g(s)}]^k b_1^{k-1} + \overline{b_2}\sum_{k=1}^{\infty}[\overline{g(s)}]^k b_2^{k-1} - \overline{b_3}\sum_{k=1}^{\infty}[\overline{g(s)}]^k b_3^{k-1}$$

$$-\overline{a_1b_2}\sum_{k=1}^{\infty}[\overline{g(s)}]^k (a_1b_2)^{k-1} + \overline{a_2b_3}\sum_{k=1}^{\infty}[\overline{g(s)}]^k (a_2b_3)^{k-1} - \overline{a_2b_1}\sum_{k=1}^{\infty}[\overline{g(s)}]^k (a_2b_1)^{k-1} - \overline{a_2b_2}\sum_{k=1}^{\infty}[\overline{g(s)}]^k (a_2b_2)^{k-1} + \overline{a_3b_1}\sum_{k=1}^{\infty}[\overline{g(s)}]^k (a_3b_1)^{k-1} + \overline{a_3b_2}\sum_{k=1}^{\infty}[\overline{g(s)}]^k (a_3b_2)^{k-1} - \overline{a_3b_3}\sum_{k=1}^{\infty}[\overline{g(s)}]^k (a_3b_3)^{k-1}\} \quad \text{--- (7)}$$

It can be shown that distribution function G(.) of the inter exit times satisfy the relation

$$G^*(s) = \sum_{n=1}^{\infty}(1-q_A)^{n-1}q_A F_n(x) + \sum_{n=1}^{\infty}(1-q_B)^{n-1}q_B F_n(x) \quad \text{--- (8)}$$

$$\bar{g}(s) = q_A\sum_{n=1}^{\infty}(1-q_A)^{n-1}F_n^*(s) + q_B\sum_{n=1}^{\infty}(1-q_B)^{n-1}F_n^*(s),$$

where $F_n^*(s) = \dfrac{(1-R)(1+vs)^{1-n}}{(1-R)(1+vs)+nRvs}$ --- (9)

$\bar{g}(0) = 2$ and $\bar{g}'(0) = \dfrac{-u}{q_A q_B}$ --- (10)

$$E(T)= \left(\frac{v}{1-R}\right)\frac{1}{q_Aq_B}\ \{\ \frac{\overline{a_1}}{(1-2a_1)^2} + \frac{\overline{a_2}}{(1-2a_2)^2} - \frac{\overline{a_3}}{(1-2a_3)^2} + p\ \{\ \frac{\overline{b_1}}{(1-2b_1)^2} + \frac{\overline{b_2}}{(1-2b_2)^2} - \frac{\overline{b_3}}{(1-2b_3)^2} - \frac{\overline{a_1b_1}}{(1-2a_1b_1)^2} - \frac{\overline{a_1b_2}}{(1-2a_1b_2)^2} + \frac{\overline{a_1b_3}}{(1-2a_1b_3)^2} - \frac{\overline{a_2b_1}}{(1-2a_2b_1)^2} - \frac{\overline{a_2b_2}}{(1-2a_2b_2)^2} + \frac{\overline{a_2b_3}}{(1-2a_2b_3)^2} + \frac{\overline{a_3b_1}}{(1-2a_3b_1)^2} + \frac{\overline{a_3b_2}}{(1-2a_3b_2)^2} - \frac{\overline{a_3b_3}}{(1-2a_3b_3)^2}\}\} \quad \text{--- (11)}$$

This Equation (11) gives the mean time to recruitment for case - I

3.2Case-II: Y= min(Y_A, Y_B) & Z= min(Z_A, Z_B)

$$P\{S_k < Y\} = \int_0^{\infty} P\{Y > X\} g_k(x)dx$$

$P(S_k \le Y) = a_3^k$ where $a_3 = E[e^{-(\lambda_A+\lambda_B)x}]$.

$P(S_k \le Z) = b_3^k$ where $b_3 = E[e^{-(\mu_A+\mu_B)x}]$

$$P(T > t) = 1 + \overline{a_3}\sum_{k=1}^{\infty}G_k(t)a_3^{k-1} + p\{\ \overline{a_3b_3}\sum_{k=1}^{\infty}G_k(t)(a_3b_3)^{k-1} - \overline{b_3}\sum_{k=1}^{\infty}G_k(t)b_3^{k-1}\}$$

$$\bar{l}(s) = \overline{a_3}\sum_{k=1}^{\infty}[\overline{g(s)}]^k a_3^{k-1} + p\{\ \overline{b_3}\sum_{k=1}^{\infty}[\overline{g(s)}]^k b_3^{k-1} - \overline{a_3b_3}\sum_{k=1}^{\infty}[\overline{g(s)}]^k (a_3b_3)^{k-1}\} \quad \text{--- (12)}$$

$$E(T)= \left(\frac{v}{1-R}\right)\frac{1}{q_Aq_B}\ \{\ \frac{\overline{a_3}}{(1-2a_3)^2} + p\{\ \frac{\overline{b_3}}{(1-2b_3)^2} - \frac{\overline{a_3b_3}}{(1-2a_3b_3)^2}\}\} \quad \text{--- (13)}$$

Equation (13) gives the mean time to recruitment for case - II

4 Remark:

Computation of E(T) for extended exponential and SCBZ property possessing thresholds is similar for both the cases as their distribution will have just additional terms.

4.1Note:

When c=1, $p \neq 0$, $q \neq 1$ our results agree with the results for the manpower system having the inter-decision times as an ordinary renewal process.

5 Findings:

From the above results, the observation are presented which agree with reality.

.As λ increases, on the average , the inter-decision time decreases and consequently the mean of time to recruitment decreases when the other parameters are fixed.

6 Conclusion:

The models discussed in this paper are found to be more realistic and new in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs and (ii) associating a probability for any decision to have exit points (iii)provision of optional and mandatory thresholds. From the organization's point of view, our models are more suitable than the corresponding models with instantaneous attrition at decision epochs, as the provision of exit points at which attrition actually takes place, postpone the time to recruitment.

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