# Further Results on Sum of Strong Efficient Domination Number and Chromatic Number

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**Abstract:** Let G = (V,E) be a graph. A subset S of V(G) is called a strong (weak) efficient dominating set of G if for every $v \in V(G)$, $|N_s[v] \cap S| = 1$ ($|N_w[v] \cap S| = 1$), where $N_s(v) = \{u \in V(G): uv \in E(G), \deg u \geq \deg v\}$ and $N_w(v) = \{u \in V(G) : uv \in E(G), \deg v \geq \deg u\}$, $N_s[v] = N_s(v) \cup \{v\}$, ($N_w[v] = N_w(v) \cup \{v\}$). The minimum cardinality of a strong (weak) efficient dominating set of G is called the strong (weak) efficient domination number of G and is denoted by $\gamma_{se}(G)$ ($\gamma_{we}(G)$). Let G be a graph on n vertices, containing a clique on $\chi(G)$ vertices and $\chi(G) = n - 3$. In this paper, the sum of strong efficient domination number and chromatic number of G equal to n+1 is studied

**Key words: Strong efficient dominating sets, Strong efficient domination number and chromatic number**

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## I Introduction:

Throughout this paper, only finite, undirected and simple graphs are considered. Let G = (V,E) be a graph. A subset S of V(G) of a graph G is called a dominating set of G if every vertex in V(G) \ S is adjacent to a vertex in S [9]. The domination number $\gamma$(G) is the minimum cardinality of a dominating set of G. Sampathkumar and Pushpalatha introduced the concepts of strong and weak domination in graphs [8]. A subset S of V(G) is called a strong dominating set of G if for every $v \in V - S$ there exists a $u \in S$ such that u and v are adjacent and deg u ≥ deg v. A subset S of V(G) is called an efficient dominating set of G if for every $v \in V(G)$, $|N[v] \cap S| = 1$[1,2]. The concept of strong (weak) efficient domination in graphs was introduced by Meena, Subramanian and Swaminathan [4]. A subset S of V(G) is called a strong (weak) efficient dominating set of G if for every $v \in V(G)$, $|N_s[v] \cap S| = 1$($|N_w[v] \cap S| = 1$). $N_s(v) = \{u \in V(G): uv \in E(G), \deg(u) \geq \deg(v)\}$.The minimum cardinality of a strong (weak) efficient dominating set is called strong (weak) efficient domination number and is denoted by $\gamma_{se}(G)$ ($\gamma_{we}(G)$ )A graph G is strong efficient if there exists a strong efficient dominating set of G. An *n* - colouring of a graph *G* uses n colours**.** The chromatic number $\chi(G)$ is defined as the minimum *n* for which *G* has an n - colouring. Several authors have studied the problem of obtaining an upper bound for the sum of a domination parameter and graph theoretic parameter and characterized the extremal graphs. In [6] Paulraj Joseph J and Arumugam S proved that $\gamma + \chi \leq n$. In [5], they proved that $\gamma + \chi \leq n+1$ and characterized the classes for which the upper bound is obtained. In [7], Paulraj. Joseph and Mahadevan G characterized the classes of graphs for which the sum of complementary connected domination number and chromatic number less than or equal to 2n – 6. Motivated by these results, an attempt has been made to find the sum of strong efficient domination number and chromatic number of a strong efficient graph. In this paper, let G be a graph on n vertices, containing a clique on $\chi(G)$ vertices and $\chi(G) = n - 3$. The sum of strong efficient domination number and chromatic number equal to n+1 is studied. For all graph theoretic terminologies and notations, Harary [3] is followed.

**Result 1.1** $\gamma_{se}(K_n) = 1$, for all $n \in N$.

**Theorem 1.2:** For any path $P_m$,

$$\gamma_{se}(P_m) = \begin{cases} n \text{ if } m = 3n, n \in N \\ n+1 \text{ if } m = 3n+1, n \in N \\ n+2 \text{ if } m = 3n+2, n \in N \end{cases}$$

**Theorem 1.3:** Let $G_1$and $G_2$ be strong efficient graphs with disjoint vertex sets. Then $\gamma_{se}(G_1 \cup G_2) = \gamma_{se}(G_1) + \gamma_{se}(G_2)$.

**II Main Result:**

**Theorem:** Let G = (V,E) be a simple graph on n vertices containing a clique on $\chi(G)$ vertices. If $\chi(G) = n - 3$, then $\gamma_{se}(G) + \chi(G) = n + 1$ if and only if $G \cong K_{n-3} \cup 3K_1$.

**Proof:** Suppose $G \cong K_{n-3} \cup 3K_1$ and $\chi(G) = n - 3$. Then $\gamma_{se}(G) = 4$. Therefore $\gamma_{se}(G) + \chi(G) = 4 + n - 3 = n + 1$.

Conversely, suppose $\gamma_{se}(G) + \chi(G) = n + 1$. By assumption, G contains a clique K on n – 3 vertices. Let $V(G) = V(K) \cup \{x, y, z\} = \{v_1, v_2, \ldots v_{n-3}, x, y, z\}$. Let p, q, r represent the number of vertices of K adjacent with x, y, and z respectively.

**Case (1):** Suppose the subgraph of G induced by $\{x, y, z\} = \overline{K_3}$ which is $3K_1$. Since $\chi(G) = n - 3$, $0 \le p, q, r \le n - 4$. If p = q= r = 0, then there is no edge between $K_{n-3}$ and $\langle\{x, y, z\}\rangle$. Therefore $G \cong K_{n-3} \cup 3K_1$. $\{v_i, x, y, z\}$, $1 \le i \le n - 3$, is a $\gamma_{se}$ - set of G. Then $\gamma_{se}(G) = 4$. Therefore $\gamma_{se}(G) + \chi(G) = 4 + n - 3 = n + 1$. Suppose $p \ne 0$, q = r = 0. Let $N(x) = \{u_1, u_2, \ldots u_p\}$. $\{u_i, y, z\}$ $1 \le i \le p$, is a $\gamma_{se}$ - set of G. Then $\gamma_{se}(G) = 3$. Therefore $\gamma_{se}(G) + \chi(G) = 3 + n - 3 = n < n + 1$ , a contradiction. Similar is the case with $q \ne 0$, p = r = 0 and $r \ne 0$, q = p = 0. Suppose p, $q \ne 0$, r = 0. Let $N(y) = \{v_1, v_2, \ldots v_q\}$. If $N(x) \cap N(y) = \phi$, then $\{u_i, y, z\}$ $1 \le i \le p$, and $\{v_j, x, z\}$ $1 \le j \le q$, are $\gamma_{se}$ - sets of G. Then $\gamma_{se}(G) = 3$. Therefore $\gamma_{se}(G) + \chi(G) = n < n + 1$, a contradiction. If $N(x) \cap N(y) \ne \phi$, then $u \in N(x) \cap N(y)$ for some $u \in V(K)$. $\{u, z\}$ is a $\gamma_{se}$ - set of G. Then $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = 2 + n - 3 = n - 1 < n + 1$, a contradiction. Similar is the case if p = 0, q, $r \ne 0$ and r = 0, q, $p \ne 0$. Suppose p, q, $r \ne 0$. Let $N(z) = \{w_1, w_2, \ldots w_r\}$. If $N(x) \cap N(y) \cap N(z) \ne \phi$, then $u \in N(x) \cap N(y) \cap N(z)$ for some $u \in V(K)$. Therefore u is the full degree vertex of G. Hence $\gamma_{se}(G) = 1$. Therefore $\gamma_{se}(G) + \chi(G) = 1 + n - 3 = n - 2 < n + 1$, a contradiction.

Suppose if $N(x) \cap N(y) \cap N(z) = \phi$.

**Subcase (1a):** If N(x) , N(y) and N(z) are pairwise disjoint then $\{u_i, y, z\}$, $1 \le i \le p$, $\{v_i, x, z\}$, $1 \le i \le q$ and $\{w_i, x, y\}$, $1 \le i \le r$, are $\gamma_{se}$ –sets of G. Hence $\gamma_{se}(G) = 3$. Therefore $\gamma_{se}(G) + \chi(G) = 3 + n - 3 = n < n + 1$, a contradiction.

**Subcase (1b):** Suppose $N(x) \cap N(y) \ne \phi$. Let $u \in N(x) \cap N(y)$. $\{u, z\}$ is a $\gamma_{se}$ –set of G. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = 2 + n - 3 = n - 1 < n + 1$, a contradiction. Proof is similar if $N(y) \cap N(z) \ne \phi$ and $N(x) \cap N(z) \ne \phi$.

**Subcase (1c):** Suppose $N(x) \cap N(y) = \phi$ but $N(y) \cap N(z) \ne \phi$ and $N(x) \cap N(z) \ne \phi$. Let $u \in N(x) \cap N(z)$ and $y \in N(y) \cap N(z)$. {u, y} and {v, x} are $\gamma_{se}$ –sets of G. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = 2 + n - 3 = n - 1 < n + 1$, a contradiction. Proof is similar if $N(y) \cap N(z) = \phi$, $N(x) \cap N(y) \ne \phi$, $N(x) \cap N(z) \ne \phi$ and $N(x) \cap N(z) = \phi$, $N(x) \cap N(y) \ne \phi$, $N(y) \cap N(z) \ne \phi$.

**Case 2:** Suppose the subgraph of G induced by {x, y, z} is $K_2 \cup K_1$. Let x be adjacent with y.

**Subcase (2a):** Suppose p = q = r = 0. {u, x, z} and {u, y, z} for some $u \in V(K)$, is a $\gamma_{se}$ -set of G. Hence $\gamma_{se}(G) = 3$. Therefore $\gamma_{se}(G) + \chi(G) = 3 + n - 3 = n < n + 1$, a contradiction. Proof is similar if y is adjacent with z and x is adjacent with z.

**Subcase (2b):** Suppose $p \ne 0$, q = r = 0. $\{u_i, y, z\}$ for some $u_i \in N(x)$, $1 \le i \le p$, is a $\gamma_{se}$ -set of G. Hence $\gamma_{se}(G) = 3$. Therefore $\gamma_{se}(G) + \chi(G) = 3 + n - 3 = n < n + 1$, a contradiction. Proof is similar if $q \ne 0$, p = r = 0 and $r \ne 0$, q = p = 0.

**Subcase (2c):** Suppose p, $q \ne 0$ and r = 0. Suppose $N(x) \cap N(y) \ne \phi$. Let $u \in N(x) \cap N(y)$. {u, z} is a $\gamma_{se}$ -set of G. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = 2 + n - 3 = n - 1 < n + 1$, a contradiction. Suppose $N(x) \cap N(y) = \phi$. If p = q, then deg x = deg y. If $S_1 = \{u_i, y, z\}$, $1 \le i \le p$, is a strong efficient dominating set of G, then $|N_s[x] \cap S_1| = |\{u_i, y\}| = 2 > 1$, a contradiction. If $S_2 = \{v_j, x, z\}$, $1 \le j \le q$, is a strong efficient dominating set of G, then $|N_s[y] \cap S_2| = |\{v_j, x\}| = 2 > 1$, a contradiction. Hence if p = q, then strong efficient dominating set of G does not exist. ---- (1).

Assume that p < q. $N(x) = \{y, u_1, u_2, \ldots u_p\}$ and $N(y) = \{x, v_1, v_2, \ldots v_q\}$. Since p < q, deg x < deg y. $\{v_i, x, z\}$, $1 \le i \le q$, is a $\gamma_{se}$ -set of G. Hence $\gamma_{se}(G) = 3$. Therefore $\gamma_{se}(G) + \chi(G) = 3 + n - 3 = n < n + 1$, a contradiction. Similar is the case if p > q.

**Subcase (2d):** Suppose p, q, $r \ne 0$. If $N(x) \cap N(y) \cap N(z) \ne \phi$, then $u \in N(x) \cap N(y) \cap N(z)$. Therefore u is a full degree vertex of G. Hence $\gamma_{se}(G) = 1$. Therefore $\gamma_{se}(G) + \chi(G) = n - 2 < n + 1$, a contradiction. Suppose $N(x) \cap N(y) \cap N(z) = \phi$. If N(x), N(y) and N(z) are pairwise disjoint and if p = q, then deg x = deg y. As discussed earlier (1), strong efficient dominating set of G does not exist. Assume that p < q. $\{v_i, x, z\}$, $1 \le i \le q$, is a $\gamma_{se}$ -set of G. Hence $\gamma_{se}(G) = 3$. Therefore $\gamma_{se}(G) + \chi(G) = 3 + n - 3 = n < n + 1$, a contradiction. If $N(x) \cap N(y) \neq \phi$, N(z) has no element with N(x), N(y) or $N(z) \cap N(y) \neq \phi$, $N(z) \cap N(x) = \phi$, then {u, z} for some $u \in N(x) \cap N(y)$ is a $\gamma_{se}$ -set of G. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = 2 + n - 3 = n - 1 < n + 1$, a contradiction. Suppose $N(x) \cap N(z) \neq \phi$ and N(y) is disjoint with N(x) and N(z). Let $u \in N(x) \cap N(z)$. Therefore deg u = n – 2. Hence u is the maximum degree vertex. If S = {u, y} is a strong efficient dominating set of G. Hence $|N_s[x] \cap S| = |\{u, y\}| = 2 > 1$, a contradiction. Hence strong efficient dominating set of G does not exist. Similar is the case with $N(y) \cap N(z) \neq \phi$ and N(x) is disjoint with N(y) and N(z).

**Case 3:** Suppose the subgraph of G induced by {x, y, z} is $P_3$. Let without loss of generality assume that x and z are pendent vertices.

**Subcase (3a):** Suppose p = q = r = 0. Then $G = K_{n-3} \cup P_3$. $\gamma_{se}(G) = \gamma_{se}(K_{n-3}) + \gamma_{se}(P_3)$, by theorem (1.3). Since $\gamma_{se}(K_{n-3}) = 1$ and $\gamma_{se}(P_3) = 1$ by theorem (1.2), $\gamma_{se}(G) + \chi(G) = 2 + n - 3 = n - 1 < n + 1$, a contradiction.

**Subcase (3b):** Suppose p ≠ 0, q = r = 0. If p = 1, then $N(x) = \{u_i, y\}$ for some i, $1 \le i \le p$ . Since deg $u_i$ = n – 3, if $S = \{u_i, y\}$is a strong efficient dominating set of G, then $|N_s[x] \cap S| = |\{u_i, y\}| = 2 > 1$, a contradiction. Hence if p = 1, strong efficient dominating set of G does not exist. Suppose p ≥ 2. Then deg x > deg y. Therefore $\{u_i, y\}$, $1 \le i \le p$, is a $\gamma_{se}$ -set of G. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = 2 + n - 3 = n - 1 < n + 1$, a contradiction. Similar is the case with r ≠ 0, p = q = 0.

**Subcase (3c):** Suppose q ≠ 0, p = r = 0. deg x = deg z = 1 < deg y. $\{v_i, x, z\}$, $1 \le i \le q$, is a $\gamma_{se}$ -set of G. Hence $\gamma_{se}(G) = 3$. Therefore $\gamma_{se}(G) + \chi(G) = 3 + n - 3 = n < n + 1$, a contradiction.

**Subcase (3d):** Suppose p, q ≠ 0 and r = 0. If $N(x) \cap N(y) = \emptyset$ and p ≤ q, then $\{v_i, x, z\}$ $1 \le i \le q$, is a $\gamma_{se}$ -set of G. Hence $\gamma_{se}(G) = 3$. Therefore $\gamma_{se}(G) + \chi(G) = 3 + n - 3 = n < n + 1$, a contradiction. If q = p - 1, then $\{u_i, y\}$ is a $\gamma_{se}$ -set of G. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = 2 + n - 3 = n - 1 < n + 1$, a contradiction. If $N(x) \cap N(y) \neq \emptyset$, then $u \in N(x) \cap N(y)$. Therefore {u, z} is a $\gamma_{se}$ -set of G. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = 2 + n - 3 = n - 1 < n + 1$, a contradiction. Similar is the case with q, r ≠ 0, p = 0.

**Subcase (3e):** Suppose p, r ≠ 0 and q = 0. Suppose $N(x) \cap N(z) = \emptyset$. If p = r, $\{u_i, z\}$, $1 \le i \le p$, $\{w_j, x\}$, $1 \le j \le r$, are $\gamma_{se}$ -sets of G. If p < r, then $\{w_j, x\}$, $1 \le j \le r$, is the $\gamma_{se}$ -set of G. Similar is the case if p > r. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = n - 1 < n + 1$, a contradiction. Suppose $N(x) \cap N(z) \neq \emptyset$. Let $u \in N(x) \cap N(z)$ . If p = r = 1, then u is the only maximum degree vertex of G. If S = {u, y} is a strong efficient dominating set of G, then $|N_s[x] \cap S| = |N_s[z] \cap S| = |\{u, y\}| = 2 > 1$, a contradiction. Therefore if p = r =1, then strong efficient dominating set of G does not exist. Let p, r ≥ 2. Hence deg y < deg x, deg z. Also {u, y} is a $\gamma_{se}$ -set of G. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = n - 1 < n + 1$, a contradiction.

**Subcase (3f):** Suppose p, q, r ≠ 0. If $N(x) \cap N(y) \cap N(z) \neq \emptyset$. Then $u \in N(x) \cap N(y) \cap N(z)$. Therefore u is a full degree vertex of G. Hence $\gamma_{se}(G) = 1$. Therefore $\gamma_{se}(G) + \chi(G) = n - 2 < n + 1$, a contradiction. Suppose $N(x) \cap N(y) \cap N(z) = \emptyset$. If N(x), N(y) and N(z) are pairwise disjoint, then $N(x) = \{y, u_1, u_2, . . . u_p\}$, $N(y) = \{x, z, v_1, v_2, . . . v_q\}$ and $N(z) = \{y, w_1, w_2, . . . w_r\}$. Suppose p = q + 1 and r < p, q. Then deg x = deg y and deg z < deg x, deg y. $S = \{u_i, y\}$, $1 \le i \le p$, is a strong efficient dominating set of G. $|N_s[x] \cap S| = |\{u_i, y\}| = 2 > 1$, a contradiction. Hence if p = q + 1 and r < p, q, then strong efficient dominating set of G does not exist. Similar is the case with r = q + 1, and p < q, r. If q < p, r then $\{u_i, z\}$, $1 \le i \le p$ and $\{w_j, z\}$, $1 \le j \le r$ are the $\gamma_{se}$ -sets of G. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = n - 1 < n + 1$, a contradiction. If p = q = r, then $\{v_i, x, z\}$, $1 \le i \le q$ is the $\gamma_{se}$ -set of G. Hence $\gamma_{se}(G) = 3$. Therefore $\gamma_{se}(G) + \chi(G) = n < n + 1$, a contradiction. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = n - 1 < n + 1$, a contradiction.

Suppose $N(x) \cap N(y) \neq \emptyset$. Let $u \in N(x) \cap N(y)$. If $N(y) \cap N(z) = \emptyset$ or not, {u, z} is a $\gamma_{se}$ -set of G. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = n - 1 < n + 1$, a contradiction.

If $N(x) \cap N(z) \neq \emptyset$, $N(y) \cap N(z) = \emptyset$ and if r = q + 1, then deg y = deg z. Since u is the maximum degree vertex, if S = {u, z} is the strong efficient dominating set of G, then $|N_s[y] \cap S| = |\{u, y\}| = 2 > 1$, a contradiction. Hence no strong efficient dominating set of G exist. Similarly if p = q and r = q + 1, then no strong efficient dominating set of G exist, since deg y = deg z and deg x < deg y, deg z. If p = q and r > q + 1, then $\{w_i, z\}$, $1 \leq i \leq r$ is a $\gamma_{se}$ -set of G. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = n - 1 < n + 1$, a contradiction. If p = q and r < q, then $\{u_i, z\}$, $1 \leq i \leq q$ are the $\gamma_{se}$ -set of G. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = n - 1 < n + 1$, a contradiction. Similar is the case if $N(y) \cap N(z) \neq \emptyset$ and N(x) is disjoint from N(y) and N(z).

Suppose $N(x) \cap N(z) \neq \emptyset$ and N(y) is disjoint from N(x) and N(z). If $p \leq q + 1$ or $r \leq q + 1$, then deg x = deg y or deg y = deg z. Let $u \in N(x) \cap N(z)$. If S = {u, y} is the strong efficient dominating set of G, then $|N_s[x] \cap S| = |\{u, y\}| = 2 > 1$, a contradiction. Similarly $|N_s[z] \cap S| = |\{u, y\}| = 2 > 1$, a contradiction. Therefore no strong efficient dominating set of G exist. If p > q + 1 and r > q + 1, then {u, y} is a $\gamma_{se}$ -set of G, since deg y < deg x, deg z. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = n - 1 < n + 1$, a contradiction.

**Case 4:** Suppose the subgraph of G induced by {x, y, z} is $K_3$.

**Subcase 4a:** Suppose p = q = r = 0. Then $G = K_{n-3} \cup K_3$. Therefore $\gamma_{se}(G) = \gamma_{se}(K_{n-3}) + \gamma_{se}(K_3)$. $\gamma_{se}(G) = 2$. Hence $\gamma_{se}(G) + \chi(G) = n - 1 < n + 1$, a contradiction.

Suppose $p \neq 0$, q = r = 0. If p = n – 4, then deg x = n – 2, since x is not adjacent with some $u_i \in V(K)$, say $u_m$. Hence x is the maximum degree vertex. If $\{x, u_m\}$is a strong efficient dominating set of G, then all the vertices of $K_{n-3}$ other than $u_m$ are strongly dominated by two vertices x and $u_m$. Therefore no strong efficient dominating set of G exist. If $1 \leq p \leq n - 5$, then $N(x) = \{ y, z, u_1, u_2, . . . u_p\}$. For $1 \leq i \leq p$, $\{u_i, y\}$ or $\{u_i, z\}$ is a $\gamma_{se}$ -set of G. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = n - 1 < n + 1$, a contradiction. Similar is the case with $q \neq 0$, p = r = 0 and $r \neq 0$, p = q = 0.

**Subcase 4b:** Suppose p, $q \neq 0$, r = 0. If $N(x) \cap N(y) \neq \emptyset$, then let $u \in N(x) \cap N(y)$. {u, z} is a $\gamma_{se}$ -set of G. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = n - 1 < n + 1$, a contradiction.

Suppose $N(x) \cap N(y) = \emptyset$. $N(x) = \{ y, z, u_1, u_2, . . . u_p\}$ and $N(y) = \{ x, z, v_1, v_2, . . . v_q\}$. If p = n – 4 or q = n – 4, then as discussed earlier, no strong efficient dominating set of G does not exist. If p < q, then $\{u_i, x\}$ is a $\gamma_{se}$ –set of G. If p > q, then $\{u_i, y\}$ is a $\gamma_{se}$ –set of G. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = n - 1 < n + 1$, a contradiction. Similar is the case with q, $r \neq 0$, p = 0 and p, $r \neq 0$, q = 0.

**Subcase 4c:** Suppose p, q, $r \neq 0$. If $N(x) \cap N(y) \cap N(z) \neq \phi$, then $u \in N(x) \cap N(y) \cap N(z)$, for some $u \in V(K)$. u becomes a full degree vertex of G. Hence $\gamma_{se}(G) = 1$. Therefore $\gamma_{se}(G) + \chi(G) = n - 2 < n + 1$, a contradiction.

Suppose $N(x) \cap N(y) \cap N(z) = \phi$ and N(x), N(y) and N(z) are pairwise disjoint. If p = q= r, then deg x = deg y = deg z. If $S = \{u_i, y\}$, $1 \leq i \leq p$ is a strong efficient dominating set of G, then $|N_s[x] \cap S| = |\{ u_i, y \}| = |N_s[z] \cap S| = 2 > 1$, a contradiction. Therefore no strong efficient dominating set of G exist. Suppose $p \neq q \neq r$. If p = n – 4 or q = n – 4 or r = n – 4 then as discussed earlier, no strong efficient dominating set of G exist. Let $1 \leq p, q, r \leq n - 5$. Suppose p > q, r. If q = r, then $\{ u_i, y \}$, $\{ u_i, z\}$, $1 \leq i \leq p$ are $\gamma_{se}$ –sets of G. If q > r, then $\{ u_i, y \}$, $1 \leq i \leq p$ is a $\gamma_{se}$ –set of G. If q < r, then$\{ u_i, z\}$, $1 \leq i \leq p$ is a $\gamma_{se}$ –set of G. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = n - 1 < n + 1$, a contradiction. Similar is the case with q > p, r and r > p, q. Suppose $N(x) \cap N(y) \neq \phi$ and N(z) is disjoint from N(x) and N(y). Let $u \in N(x) \cap N(y)$. Suppose p = q = r. Then S = {u, z} is a $\gamma_{se}$ –set of G. Therefore $|N_s[x] \cap S| = |N_s[y] \cap S| = |\{u, z\}| = 2 > 1$, a contradiction. Hence no strong efficient dominating set of G exist. If p > q, r or q > p, r then {u, z} is a $\gamma_{se}$ –set of G. Hence $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = n - 1 < n + 1$, a contradiction. If r > p, q then deg z > deg x, deg y. Then S = {u, z} is a $\gamma_{se}$ –set of G. Therefore $|N_s[x] \cap S| = |N_s[y] \cap S| = |\{u, z\}| = 2 > 1$, a contradiction. Hence no strong efficient dominating set of G exist. Similar is the case with $N(y) \cap N(z) \neq \phi$ and N(x) is disjoint from N(y) and N(z) and $N(x) \cap N(z) \neq \phi$ and N(y) is disjoint from N(x) and N(z). Suppose $N(x) \cap N(y) \neq \phi$, $N(y) \cap N(z) \neq \phi$ but $N(x) \cap N(z) = \phi$. Let $u \in N(x) \cap N(y)$ and $v \in N(y) \cap N(z)$ where u, $v \in V(K)$. If p = q = r, then as discussed earlier no strong efficient dominating set of G exist. If r < p, q then {u, z} is a $\gamma_{se}$ –set of G. If p < q, r then {v, x} is a $\gamma_{se}$ –set of G. Therefore

$\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = n - 1 < n + 1$, a contradiction. If $p > q$, $r$ and $q = r$, then deg $x >$ deg $y$, deg $z$. If $\{u, y\}$ or $\{u, z\}$ is a $\gamma_{se}$ –set of G, then $\gamma_{se}(G) = 2$. Therefore $\gamma_{se}(G) + \chi(G) = n - 1 < n + 1$, a contradiction. Similar is the case with $N(x) \cap N(y) \neq \phi$, $N(x) \cap N(z) \neq \phi$ but $N(y) \cap N(z) = \phi$ and also with $N(y) \cap N(z) \neq \phi$ $N(x) \cap N(z) \neq \phi$ but $N(x) \cap N(y) = \phi$. Therefore If $\chi(G) = n - 3$ and $\gamma_{se}(G) + \chi(G) = n + 1$, then $G \cong K_{n-3} \cup 3K_1$.

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