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Free Vibration Analysis of Symmetric Angle-ply Composite Laminated Plate with Temperature Effect

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Abstract

The paper presents mainly on the natural frequency of symmetric angle ply composite laminated plates with temperature change. The design parameters of the laminates such as aspect ratio, thickness ratio, boundary conditions and lamination angle were investigated using classical laminated plate theory (CLPT) and Finite element coded by ANSYS. The experimental investigation is to fabricate the laminates and to find mechanical and thermal properties of glass-polyester such as longitudinal, transverse young modulus, thermal expansion and shear modulus. The main conclusion was the natural frequency could increase and decrease depending on the temperature change , boundary conditions, thickness ratio, lamination angle, and the aspect ratio of the plate.

Keywords: Composite laminated plate, Natural frequency, Classical laminated plate theory, ANSYS

1. Introduction

During the last years, needs for composite materials mixed of two or more types of materials together homogenously to produce best properties , the constituents are mixed at a macroscopic level, fiber is called the reinforcing phase and the embedded is the matrix. Ref.[Reddy J.N.]

The vibration analysis of the structural is necessary in order to calculate the natural frequencies of a structure. Ref. [Beards C.E]

Many researches had studied free vibration analysis, **Chorng-Fuh Liu and Chih-Hsing Huang,1996** performed a vibration analysis of laminated composite plates subjected to temperature change. The first order shear deformation theory of a plate is employed. The resulting finite element formulation leads to general nonlinear and coupled simulation equations and calculate the frequencies of vibration of a symmetric cross-ply plate. **Hui-Shen Shen, et.al**, **2003,** studied the dynamic response of laminated plates subjected to thermo-mechanical loading and resting on a two-parameter elastic foundation. The formulation is based on higher order shear deformable plate theory and includes the thermal effect. Effects of foundation stiffness, thickness ratio, and temperature change on the dynamic response are discussed. **Kullasup P. et al., 2010,** analysed free vibration of symmetrically laminated composite plates with various boundary conditions by Kantorovich method. The beam function is used as an initial trial function in the repeated calculation, which is employed to calculate the natural frequency. **Suresh K. J. et al., 2011,** developed an analytical procedure is to evaluate the free vibration characteristics of laminated composite plates based on higher order shear deformation with zig-zag function. Slope discontinuities improved by Zig-zag function at the interfaces of laminated composite plates. The solutions are obtained using Navier’s method. **Junaid Kameran Ahmed et al., 2013,** presented a static and dynamic analysis of Graphite /Epoxy composite plates. In this work the behavior of laminated composite plates under transverse loading using an eight-node diso-parametric quadratic element based on First Order Shear Deformation Theory was studied,. **Pushpendra k.**

kushwaha1 and jyoti vimal, 2014, the natural frequencies and mode shapes are compared for different boundary condition. Comparisons are made with the result for thin and thick composite laminated plate. Numerical results have been computed for the effect of temperature effect< number of layers, thickness ratio of plate, different boundary conditions, different aspect ratio, and different angle of fiber orientation of laminated composite plate.

The point of originality of the present work is how to derive the analytical solution of natural frequency for symmetric angle ply composite laminated plates by classical laminated plate theory with temperature effect. Thermal and mechanical properties for composite plate made from (glass-polyester) with fiber volume fraction (0.3) are determined experimentally. Also Finite element coded by ANSYS15.0 used to find natural frequency of composite laminate plate.

2. Analytical solution (classical laminate plate theory)

2.1 Displacement

Classical lamination theory (CLPT) based on the Kirchhoff hypothesis based on assuming the straight line perpendicular to the mid surface before deformation remains straight after deformation which means neglecting shear strains and transverse normal strain and stress in the analysis of laminated composite plates. Ref.[Reddy J.N.]

$$u(x,y,t)=u_o(x,y,t)-z\frac{\partial w_0}{\partial x} \qquad \textbf{(1. a)}$$

$$v(x,y,t)=v_o(x,y,t)-z\frac{\partial w_0}{\partial y} \qquad \textbf{(1. b)}$$

$$w(x,y,t)=w_o(x,y) \qquad \textbf{(1. c)}$$

Where $\frac{\partial w_0}{\partial x}$, $\frac{\partial w_0}{\partial y}$ denote the rotations about y and x axis respectively.

u_o , v_o and w_o denote the displacement components along (x, y, z) directions respectively of a point on the mid-plane (i.e.…z=0).

2.2. Stress and strain

The total strains can be written as follows

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_{xx}^{(0)}\\ \varepsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{Bmatrix}+z*$$

$$\begin{Bmatrix}\varepsilon_{xx}^{(1)}\\ \varepsilon_{yy}^{(1)}\\ \gamma_{xy}^{(1)}\end{Bmatrix}=\begin{Bmatrix}\frac{\partial u_0}{\partial x}-\alpha_{xx}\Delta T\\ \frac{\partial v_0}{\partial y}-\alpha_{yy}\Delta T\\ \frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}-\alpha_{xy}\Delta T\end{Bmatrix}+z\begin{Bmatrix}-\frac{\partial^2 w_0}{\partial x^2}\\ -\frac{\partial^2 w_0}{\partial y^2}\\ -2*\frac{\partial^2 w_0}{\partial x\,\partial y}\end{Bmatrix} \qquad \textbf{(2.a)}$$

Where ($\varepsilon_{xx}^{(0)}$, $\varepsilon_{yy}^{(0)}$, $\gamma_{xy}^{(0)}$) are the membrane strains and ($\varepsilon_{xx}^{(1)}$, $\varepsilon_{yy}^{(1)}$, $\gamma_{xy}^{(1)}$) are the flexural (bending) strains, known as the curvatures α_{xx} , α_{yy} and α_{xy} are thermal expansion coefficients defined

$$\alpha_{xx}=\alpha_{11}(\cos\theta)^2+\alpha_{22}(\sin\theta)^2 \qquad \textbf{(2.b)}$$

$$\alpha_{yy}=\alpha_{11}(\sin\theta)^2+\alpha_{22}(\cos\theta)^2 \qquad \textbf{(2.c)}$$

$$2\alpha_{xy}=2(\alpha_{11}-\alpha_{22})\sin\theta\cos\theta \qquad \textbf{(2.d)}$$

α_{11} and α_{22} Are longitudinal and transverse thermal expansions respectively. And θ is the lamination angle.

The change in temperature defined

ΔT= applied temperature– reference temperature **(2.e)**

Where reference temperature $T_{ref}=25C^{\circ}$ [Reddy J.N.]. The transformed stress-strain relations of an orthotropic lamina in a plane state of stress are; for $\bar{Q}_{ij}$ see [Reddy J.N.]

$$\begin{Bmatrix}\sigma_{xx}\\\sigma_{yy}\\\sigma_{xy}\end{Bmatrix}_k=\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\\bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\\bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}_k\begin{Bmatrix}\varepsilon_{xx}-\alpha_{xx}\Delta T\\\varepsilon_{yy}-\alpha_{yy}\Delta T\\\gamma_{xy}-2\alpha_{xy}\Delta T\end{Bmatrix} \quad \textbf{(3)}$$

The resultant of inplane force *Nxx* ,*Nyy* and *Nxy* and moments *Mxx*, *Myy* and *Mxy* acting on a laminate can be obtained from integration of the stress in each layer or lamina through the laminate thickness. Knowing the stressed in terms of the displacements, the inplane force resultants *Nxx*, *Nyy*, *Nxy*, *Mxx*, *Myy* and *Mxy* can be obtained.

The inplane force resultants are defined as

$$\begin{Bmatrix}N_{xx}\\N_{yy}\\N_{xy}\end{Bmatrix}=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\begin{Bmatrix}\sigma_{xx}\\\sigma_{yy}\\\sigma_{xy}\end{Bmatrix}_k dz \quad \textbf{(4.a)}$$

Where σ_x , σ_y and σ_{xy} are normal and shear stress.

$$\begin{Bmatrix}N_{xx}\\N_{yy}\\N_{xy}\end{Bmatrix}=\begin{bmatrix}A_{11} & A_{12} & A_{16}\\A_{12} & A_{22} & A_{26}\\A_{16} & A_{26} & A_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xy}^0\\\varepsilon_{xy}^0\\\gamma_{xy}^0\end{Bmatrix}+\begin{bmatrix}B_{11} & B_{12} & B_{16}\\B_{12} & B_{22} & B_{26}\\B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}^1\\\varepsilon_{yy}^1\\\gamma_{xy}^1\end{Bmatrix}-\begin{Bmatrix}N_{xx}^t\\N_{yy}^t\\N_{xy}^t\end{Bmatrix} \quad \textbf{(4.b)}$$

$$\begin{Bmatrix}M_{xx}\\M_{yy}\\M_{xy}\end{Bmatrix}=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\begin{Bmatrix}\sigma_{xx}\\\sigma_{yy}\\\sigma_{xy}\end{Bmatrix}_k zdz \quad \textbf{(5.a)}$$

$$\begin{Bmatrix}M_{xx}\\M_{yy}\\M_{xy}\end{Bmatrix}=\begin{bmatrix}B_{11} & B_{12} & B_{16}\\B_{12} & B_{22} & B_{26}\\B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xy}^0\\\varepsilon_{xy}^0\\\gamma_{xy}^0\end{Bmatrix}+\begin{bmatrix}D_{11} & D_{12} & D_{16}\\D_{12} & D_{22} & D_{26}\\D_{16} & D_{26} & D_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}^1\\\varepsilon_{yy}^1\\\gamma_{xy}^1\end{Bmatrix}-\begin{Bmatrix}M_{xx}^t\\M_{yy}^t\\M_{xy}^t\end{Bmatrix} \quad \textbf{(5.b)}$$

Here, A_{ij} are the extensional stiffness, B_{ij} the coupling stiffness, and D_{ij} the bending stiffness.

$$A_{ij}=\sum_{k=1}^{N}(\bar{Q}_{ij})_k(z_{k+1}-z_k) \quad \textbf{(6.a)}$$

$$B_{ij}=\frac{1}{2}\sum_{k=1}^{N}(\bar{Q}_{ij})_k({z^2}_{k+1}-{z^2}_k) \quad \textbf{(6.b)}$$

$$D_{ij}=\frac{1}{3}\sum_{k=1}^{N}(\bar{Q}_{ij})_k({z^3}_{k+1}-{z^3}_k) \quad \textbf{(6.c)}$$

And Where$\{N^t\}$ and $\{M^t\}$ are thermal stress and bending results, respectively

$$\begin{Bmatrix} N_{xx}^t, M_{xx}^t \\ N_{yy}^t, M_{yy}^t \\ N_{xy}^t, M_{xy}^t \end{Bmatrix} = \sum_{k=1}^{N} \int_{-h/2}^{h/2} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \alpha_{xx} \\ \alpha_{yy} \\ 2\alpha_{xy} \end{Bmatrix} (1\,, z)\, \Delta T dz \quad \textbf{(6.d)}$$

2.3. Equation of motion

The equations of motion are obtained by setting the coefficient of δu_0 , δv_0 , δw_0 to zero separately

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \frac{\partial^2 u}{\partial t^2} - I_1 \frac{\partial^3 w}{\partial x \partial t^2} \quad \textbf{(7.a)}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = I_0 \frac{\partial^2 v}{\partial t^2} - I_1 \frac{\partial^3 w}{\partial y \partial t^2} \quad \textbf{(7.b)}$$

$$\frac{\partial^2 M_{xx}}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} + \widehat{N}_{xx}\frac{\partial^2 w}{\partial x^2} + \widehat{N}_{yy}\frac{\partial^2 w}{\partial y^2} + \widehat{N}_{xy}\frac{\partial^2 w}{\partial x \partial y} = I_0\frac{\partial^2 w}{\partial t^2} + I_1\left(\frac{\partial^3 u}{\partial x \partial t^2} + \frac{\partial^3 v}{\partial y \partial t^2}\right) - I_2\left(\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{\partial^4 w}{\partial y^2 \partial t^2}\right) - q(x,y,t) \quad \textbf{(7.c)}$$

Where

$$(I_0, I_1 . I_2) = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \rho^{(k)} (1, z, z^2) dz \quad \textbf{(8)}$$

$\rho^{(k)}$ being the material density of k^{th} layer and q(x,y,t) is a dynamic force subjected on a system (here equal to zero because its natural frequency). $\widehat{N}_{xx}, \widehat{N}_{yy}\ and \widehat{N}_{xy}$ equal to zero because there were no buckling.

These equations of motion (7 a-c) can be expressed in terms of displacements **(**δu_0 , δv_0 , δw_0) by substituting the forces results from Eqs. (4 ,5,8) into Eq. (**7.a**) to**(7.c)** and get partial differential equations,

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{12} & c_{22} & c_{23} \\ c_{13} & c_{23} & c_{33} \end{bmatrix} \begin{Bmatrix} u_0 \\ v_0 \\ w_0 \end{Bmatrix} + \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix} \begin{Bmatrix} \ddot{u}_0 \\ \ddot{v}_0 \\ \ddot{w}_0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \textbf{(9.a)}$$

$$c_{11} = A_{11} d_x^2 + 2A_{16} d_x d_y + A_{66} d_y^2 \quad \textbf{(9.b)}$$

$$c_{12} = A_{16} d_x^2 + (A_{12} + A_{66}) d_x d_y + A_{26} d_y^2 \quad \textbf{(9.c)}$$

$$c_{13} = -\left[B_{11} d_x^3 + 3B_{16} d_x^2 d_y + (B_{12} + 2B_{66}) d_x d_y^2 + B_{26} d_y^3\right] \quad \textbf{(9.d)}$$

$$c_{22} = A_{66} d_x^2 + 2A_{26} d_x d_y + A_{22} d_y^2 \quad \textbf{(9.e)}$$

$$c_{23} = -\left[B_{16}d_x^3 + (B_{12} + 2B_{66})d_x^2 d_y + 3B_{26}d_x d_y^2 + B_{22}d_y^3\right] \quad \textbf{(9.f)}$$

$$c_{33} = -D_{11}d_x^4 - 4D_{16}d_x^3 d_y - 2(D_{12} + 2D_{66})d_x^2 d_y^2 - 4D_{26}d_x - D_{22}d_y^4 - (A_{11}\alpha_{xx} + A_{12}\alpha_{yy} + 2A_{16}\alpha_{xy})\Delta T d_x^2 - (A_{16}\alpha_{xx} + A_{26}\alpha_{yy} + 4A_{66}\alpha_{xy})\Delta T d_x d_y - (A_{12}\alpha_{xx} + A_{22}\alpha_{yy} + 2A_{26}\alpha_{xy})\Delta T d_y^2 \quad \textbf{(9.g)}$$

And the coefficients m_{ij} is defined by

$$m_{11} = -I_0 d_t^2 \;, m_{13} = I_1 d_x d_t^2, m_{22} = -I_0 d_t^2;\; m_{23} = I_1 d_y d_t^2 \;, m_{33} = I_0 d_t^2 - I_2 d_t^2(d_x^2 + d_y^2) \quad \textbf{(9.k)}$$

To solve equation (9-a) used Levy method with state space approach.

For angle-ply rectangular laminates with edges x=0 and x=a simply supported and the other two edges y=b/2, having arbitrary boundary conditions The state space approach for angle ply lamination shown below

$$\left\{\frac{\partial Z}{\partial y}\right\} = [T]\{Z\} \quad \textbf{(10)}$$

$$\{Z\} = \begin{Bmatrix} U \\ U' \\ V \\ V' \\ W \\ W' \\ W'' \\ W''' \end{Bmatrix} \;;\; [T] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_1 & 0 & 0 & C_2 & 0 & C_3 & 0 & C_4 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & C_5 & C_6 & 0 & C_7 & 0 & C_8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & C_9 & C_{10} & 0 & C_{11} & 0 & C_{12} & 0 \end{bmatrix}$$

$C_1 = (A_{11}\alpha^2 + I_0 w_m^2)/A_{66}$; $C_2 = \alpha(1 + A_{12}/A_{66})$;
$C_3 = -3\alpha^2 B_{16}/A_{66}$; $C_4 = B_{26}/A_{66}$; $C_5 = -\alpha(A_{66} + A_{12})/A_{22}$;
$C_6 = (A_{66}\alpha^2 + I_0 w_m^2)/A_{22}$; $C_7 = -B_{16}\alpha^3/A_{22}$; $C_8 = 3B_{26}\alpha/A_{22}$
$C_0 = 1/(D_{22} - C_4 B_{26})$; $C_9 = \left(-3B_{16}\alpha^2 + B_{26}(C_1 + C_2C_5 - C_5 - 2\alpha C_5)\right)C_0$; $C_{10} = \left(\alpha^3 B_{16} + B_{26}C_6(C_2 - 1 - 2\alpha)\right)C_0$;

$$C_{11} = \left(-D_{11}\alpha^4 + A_{11}\alpha_{xx}\Delta T\alpha^2 + A_{12}\alpha_{yy}\Delta T\alpha^2 + I_0 w_m^2 + I_2 w_m^2\alpha^2 + B_{26}C_7(C_2 - 1 - 2\alpha)\right)C_0 \;;$$

$$C_{12} = \left(D_{12}\alpha^2 + 4D_{66}\alpha^2 + D_{16}\alpha^2 - A_{12}\alpha_{xx}\Delta T - A_{22}\alpha_{yy}\Delta T - I_2 w_m^2 + B_{26}(C_2C_8 + C_3 - C_8 - 2\alpha C_8)\right)C_0$$

3. Numerical analysis

3.1 Element selection and modeling

An element called shell281 as shown in **Fig.1** is selected which is suitable for analyzing thin to moderately thick shell structures. The element has eight nodes with six degrees of freedom at each node: translations in the x, y, and z axes, and rotations about the x, y, and z axes. It may be used for layered applications for modeling composite shells. It is include the effects of transverse shear deformation. The accuracy in modeling composite shells is governed by the first order shear deformation theory. The shell section allows for layered shell definition, options are available for specifying the thickness, material, orientation through the thickness of the layers. But to insert the temperature effect in calculations must be to adding degree of freedom (T). Then, the degrees of freedom change from (6 to7) in each node.

3.2 Verification Case Study

In the present study, Series of preselected cases are modeled to verify the accuracy of the method of analysis. The results are compared to analytical solution and numerical solution (Finite element method). To validate the present methods on the natural frequency of laminated plates, the natural frequency are calculated and compared in Table.1.

4. EXPERIMENTAL WORK

In the present work, three- purposes were investigated. First, to outline the general steps to design and fabricate the rectangular test models from fiber (E-glass) and polyester resin to form laminate composite materials. Second, the manufactured models are then used to evaluate the mechanical properties (E_1, E_2, G_{12}) with temperature change of unidirectional composite material. Third, evaluate coefficient of thermal expansion (CTE) of the composite plate.

4.1. Materials and specimen preparation

In the present work, the laminates which are handled lay-up include E-glass fiber/polyester laminate with fiber volume fraction approximately (0.34). Flat panels were fabricated from this material using (320 mm * 320 mm) wood open mold with two X-ray photo sheets covered with wax matrix to avoid abrasive and insure flattening.

The X-ray photo sheet were placed on the surface of wood mold and mixed polyester (which has 0.85 g/cm^3 as density) and hardener together (assume each 300g polyester = 1 g hardener), and for ensuring air removal and wet out, it should cover the base surface completely especially at the end edges as shown in Fig.(2.a.b).

The catalyzed resin was applied to the fiber layer by using brushes and rollers the fiber layer would saturate by resin. Small blade was used to take the bubble out of the fibers. The X-ray photo sheet will cover the composite and with rolling over layer to insure complete air removal. The pressure applied through the formation of the specimen is (three ceramic block) to get rid of the excess resin and remove air bubble. The panels were cured in temperature 37° C and for (24 hr) period of time.

4.2. Thermo-Mechanical Analyzer

Thermo-mechanical Analysis (TMA) determines dimensional changes of materials as a function of temperature under a defined mechanical force. Irrespective of the selected type of deformation (expansion, compression, penetration, tension or bending), every change of length in the sample is communicated to a highly sensitive inductive displacement transducer (LVDT) via a push rod and transformed into a digital signal. The push rod and corresponding sample holders of fused silica or aluminum oxide can be quickly and easily interchanged to optimize the system to the respective application. **Figs.3 and 4.**

The dimension of sample is (5*20*4) mm as shown in **Fig. 5.** The thermal properties which obtain from this test shown in **Table 2.**

5. Results and discussion

The present study focused mainly on the natural frequency of composite laminated plates subjected to temperature change (three cases of temperature (without temperature effect, T=50°C and T=100°C)). The natural frequency of composite plate discussed for different parameters such as temperature change, Boundary condition, lamination angle for symmetric angle ply composite laminated plate analytically by CLPT with Levy method and numerically by ANSYS.

5.1. Effect of boundary condition

From the results listed in tables **3**, it can be observed that the boundary conditions always effect on the fundamental natural frequency. It's worth mention that the natural frequencies in CSCS and CSSS for angle ply are higher than other cases because of B.C′S. effect. That effect is increasing the stiffness matrix thus increased natural frequency. When the temperature increases caused reduction in the natural frequency, for angle ply the percentage reach to (35.2%) for CSSS and temperature 50°C and the percentage reach to (91.7%) for CSFS and temperature 100°C. The dimensions used are: a=b=200mm, h=4mm the four layers have the same thickness.

5.2. Effect of aspect ratio under different temperature condition

Fig. 6 for SSSS angle ply shows that the natural frequency decreases when a/b increase with high percentage reaches to 55% when there isn't applied temperature for symmetric angle ply. On the other hand for symmetric angle ply when T=100 natural frequency decreases with high percentage reaches to 80% when a/b varies from 0.5 to 1.5, Then, it′s decreases with small percentage 28% when a/b varies from 1.5 to 2. The maximum natural frequency in case of symmetric angle ply without any temperature changes is at a/b=0.5. While the minimum is at a/b=2.5 also for symmetric angle ply when T=100. The reason behind the decreasing is the increase of temperature causes the high drop of mechanical properties and also ΔT insert in the equation of motion in stiffness matrix and has the negative sign (the equations are shown above). Thus when its value became larger the decrease became larger too. Changing a/b equal to 1.5 to 2.5 the reduction in natural frequency is smaller than the reduction in natural frequency when a/b changing from 0.5 to 1.5 for all cases.

5.3. Effect of lamination angle

Figs (7), (8) shows the effect of angle (θ) for SSSS and SSCC symmetric angle ply for different applied temperature that the natural frequency decreases and increase with different percentage when θ varies from 10 to 80. In case of SSSS the natural frequency increases when θ varies from 10 to 40, then it decreases at θ varies from 50 to 80. The natural frequency at θequal to 40 is same as θequal to 50 because the effect of boundary conditions. The maximum natural frequency is when $\theta = 45$ and there were no applied temperature because the B.C's are the same for all sides thus the angle between two bordering layers is 90 that causes the orthognal mechanical properties lead to maximum stiffeness which proportional with square root of natural frequency. And the minimum frequency for this cases is when lamination angle 10 & 80 and temperature is 100°C. The maximum percentage of decreasing natural frequency when the temperature became 50°C when lamination angle is (23.83%) when lamination angle is 10 and for temperature equal to 100°C is (84.5%) for the same angle.

In the case SSCC the natural frequency increase when θ varies from 10 to 80, the maximum natural frequency is when$\theta = 80$ and there where no applied temperature. And the minimum frequency for this cases occurs at lamination angle 10 and temperature is 100°C. The maximum percentage of decreasing natural frequency when the temperature became 50°C when lamination angle is (28.8%) when lamination angle is 10 and for temperature equal to 100°C is (82.1%) for the same angle.

5.4. Effect of thickness ratio under different temperature condition

It is shown from fig.9 that the natural frequency decreases when thickness ratio increases. It can be observed that the natural frequency is decrease with high percentage when b/h varies from 10 to 20 reach to (70.15%). Then, this percentage gets smaller when b/h varies from 20 to 50 equal to (53.96%). the maximum natural frequency accrues when no applied temperature on symmetric angle ply and a/h=10.

The maximum percentage of reduction in the natural frequency when temperature is 50°C reaches to (27%) and for temperature is 100°C reach to (76.7%).

6. CONCLUSION

This study considers the free vibration analysis of symmetric angle-ply composite laminate plate. From the present study, the following conclusions can be made:

1-The Young and shear modulus decrease when temperature increases with high percentages reach to96.3% when temperature changes from (20 °C to

100°C) for longitudinal young modulus, for transverse young modulus is 96.53% and for shear modulus is 91.1%. The longitudinal and transverse coefficient of thermal expansion also decrease when temperature increase with percentage 80% and73.7% respectively for the same temperature.

2- The temperature increases causes decreasing the natural frequency at the temperature became 50°C the maximum percentage of decrease natural frequency reaches to(35.2%) for CSSS and the percentage reaches to(91.7%) for CSFS and temperature $100C^0$. the maximum frequency occurs at clamped boundary condition of plate. while the B.C's.

3- The fundamental natural frequency of composite laminated plate is decreasing when a/b increases with high percentage reaches to 55% at there isn't applied temperature for symmetric angle ply. On the other hand for symmetric cross ply when T=100 natural frequency decrease with high percentage reaches to 80% when a/b varies from 0.5 to 1.5

4- In the case SSSS the natural frequency increase when θ varies from 10 to 40 with percentage equal to10.9%, then it decreases when θ varies from 50 to 80. The natural frequency when θequal to 40 is the same as θequal to 50 because the effect of boundary conditions. And In the case SSCC the natural frequency increase while θ varing from 10 to 80 with percentage equal to32.8%, the maximum natural frequency is at $\theta = 80$ and there were no applied temperature.

5- The natural frequency decreases when thickness ratio increases. It can be observed that the natural frequency is decreasing with high percentage when b/h varies from 10 to 20 reaches to (70.15%). Then, this percentage gets smaller when b/h varies from 20 to 50 the maximum percentage reach to (53.96%).

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- "*Theory, Analysis, and Element Manuals*" ANSYS 15 Program.

Nomenclature

Symbol	Description	Unit
a, b	Dimension of plate in x and y coordinate	m
A_{ij} , B_{ij} ,D_{ij}	Extensional stiffness, the coupling stiffness, and the bending stiffness	-
E_1, E_2 , E_3	Elastic modulus of composite material	GPa
G_{12}, G_{23} , G_{13}	Shear modulus of composite material	GPa
h	Thickness	m
I_0, I_1, I_2	Mass moment of inertia	kg.m^2
[MA]	Mass matrix	kg
M_{xx} , M_{yy} ,M_{xy}	Moment resultant per unit length	N.m/m
N	Total number of plate layers	-
Nxx, Nyy , Nxy	The resultant of in-plane force per unit length	N/m
$N_{xx}^t, N_{yy}^t, N_{xy}^t$	The resultant of in-plane force per unit length with thermal effect	N/m
$\hat{N}xx, \hat{N}yy , \hat{N}xy$	Applied edge force	N/m
q(x, y, t)	Dynamic force subjected on a system	N/m^2
$\bar{Q}_{ij}^{(k)}$	Transformed lamina stiffness	N/m
R	Vector of externally applied loads	N
t	Time	min or s
Δt	Time Interval	min or s
t_1	The end time of load	sec
T	Temperature	C^0

ΔT	Temperature increment	C^0
T_{ref}	Reference temperature	C^0
$U, \dot{U}, \ddot{U}$	Displacement , velocity and acceleration vectors	m, m/s, m/ s^2
u_o , v_o , w_o	Displacement components along (x,y,z) directions respectively	m
U_{mn} , V_{mn} , W_{mn}	Amplitudes of (u_o , v_o , w_o) respectively	-
x , y , z	Cartesian coordinate system	m
z	Distance from neutral axis	m
θ	Fiber orientation angle	Degree
α_1, α_2	Coefficient of thermal expansion of composite material	C^{-1} or K^{-1}
ρ	Density	(kg/m3)
ϵ_{xx} , ϵ_{yy} , ϵ_{xy}	Strain components	m/m
σ_{xx} , σ_{yy} , σ_{xy}	Stress components	GPa

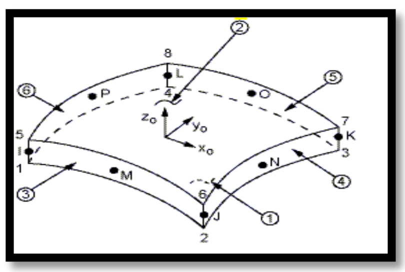

Figure 1. Shell281 Geometry [ANSYS 15 Program]

Table 1. Dimensionless natural frequency $\overline{w} = \frac{wb^2}{h}\sqrt{\frac{\rho}{E_2}}$ of simply support symmetric cross ply laminates Ref. (Ebtihal Abbas Sadiq, 2009)

ΔT	HOST	Present CLPT(levy method) (Error %)	ANSYS15.0 (Error %)	*E1/E2=40 E2=1MPa* *G12=G13=0.6E2* *G23=0.5 E2*
-50	14.8991411	13.99 (6.10%)	15.1733 (1.84%)	ν*12=0.25* *a =b=10h* *1kg*m^{-3}=ρ
0	14.7620091	13.7777 (6.67%)	15.07957 (2.15%)	*h=0.1m* $\alpha_{11} = 1.14 * 10^{-6} C^{-1}$

50	14.4679306	13.653 (5.63%)	14.977857 (3.52%)	$\alpha_{22} = 11.4 * 10^{-6} C^{-1}$ Orientation 0/90/90/0

Figure. (3. a) **Figure. (2. b)**

Figure. (2. a, b): specimen fabricate

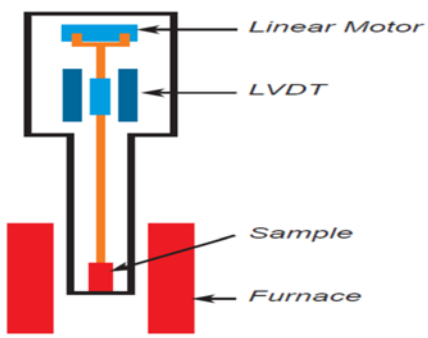

Figure 3. Operating principle of TMA.

Figure 4. TMA PT1000 device**.**

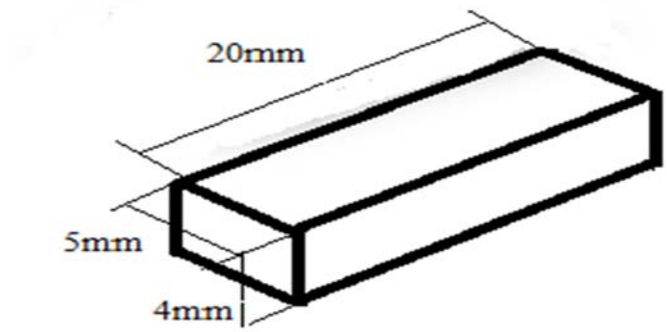

Figure.5. Dimension of sample used in TMA

Table2. Experimental value of mechanical and thermal properties of fiber –polyester composite plate for fiber volume fraction= 0.3 changed with temperature.

T C°	E_1 **Mpa**	E_2 **Mpa**	G_{12}=G_{13}= G_{23} **Mpa**	α_1 **E-6/K**	α_2 **E-6/K**
20	24627.0	5588.04	1551.77	14.57	47.81
30	23343.30	4123.11	1618.65	9.03	31.7
40	21775.40	1550.22	2505.75	7.20	29.36
50	15219.80	1515.37	623.423	4.79	25.79
60	6475.41	566.8	114.2336	3.20	21.38
70	2990.82	458.59	113.535	3.18	15.60
80	2555.71	289.27	130.48	3.22	15.59
90	1471.90	210.49	158.83	3.08	15.19
100	903.90	193.84	138.48	2.91	12.58
110	741.31	191.75	131.74	2.75	11.57
120	674.40	187.53	125.23	2.57	10.47
130	644.70	186.51	122.56	2.48	9.34
140	629.01	185.19	117.59	2.45	7.67
150	612.02	182.66	107.81	2.44	5.28
160	597.61	181.88	100.28	2.44	4.19
170	592.41	173.50	95.414	2.44	3.90
180	592.22	164.55	95.04	2.45	3.88
190	591.02	163.91	85.71	2.46	3.66
200	590.57	153.3	83.64	2.47	3

Table 3. Effect of boundary condition on natural frequency in (HZ) (45/-45/-45/45) with or without temperature changes

Boundary condition	Without temperature change		With applied T=50°C		With applied T=100°C	
	Analytical Levy	F.E.M ANSYS (Error%)	Analytical Levy	F.E.M ANSYS (Error%)	Analytical Levy	F.E.M ANSYS (Error%)
F-S-F-S	105.7	96.513 (8.7%)	78.1	70.72 (9.4%)	17.55	18.863 (7.5%)
C-S-F-S	150.999	143.96 (5%)	110.68	107.26 (3%)	12.586	12.3 (2.3%)
C-S-C-S	380.067	345.19 (9.2%)	264.24	262.57 (0.6%)	62.9	64.19 (2%)
S-S-S-S	259.18	249.24 (4%)	176.083	190.7 (8%)	35.1997	37.56 (6.7%)
C-S-S-S	322.2	290.48 (9.8%)	208.72	221.6 (6.2%)	47.75	49.1 (3%)
F-S-S-S	145.94	132.7 (9.1%)	108.2	99.03 (8.5%)	8.15	7.43 (8.8%)

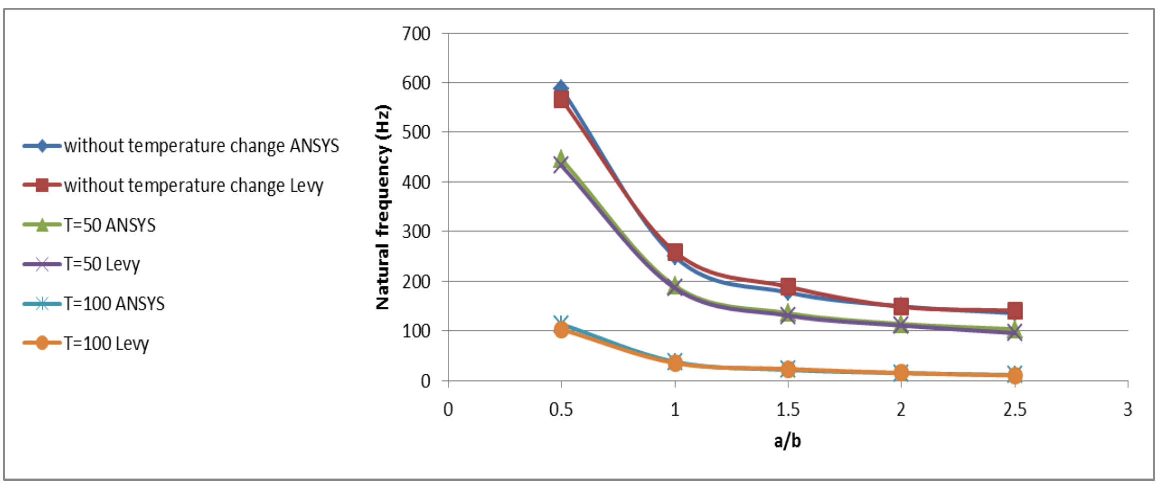

Figure 6. Effect of aspect ratio on Natural frequency for simply supported (45/-45/-45/45) for different applied temperature.

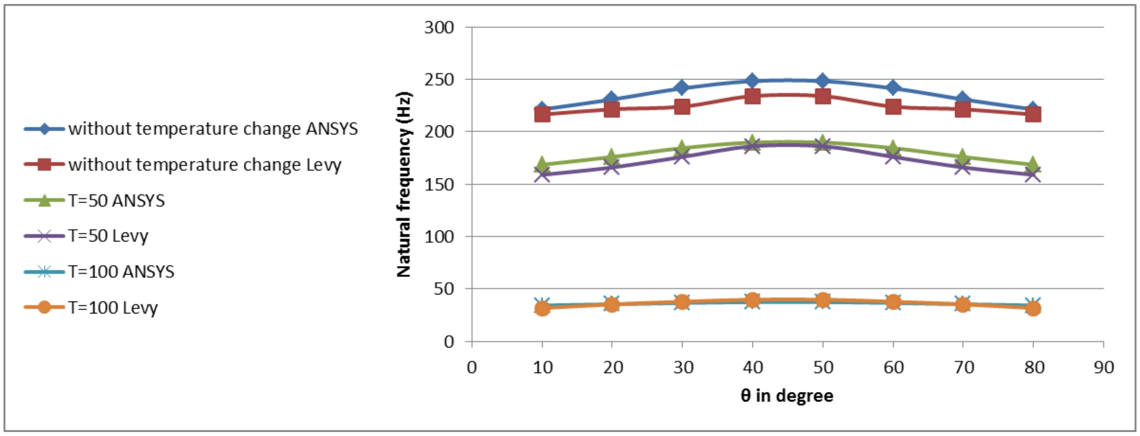

Figure 7. Effect of lamination angle on Natural frequency for simply supported (*θ*/-*θ*/-*θ*/*θ*) for different applied temperature.

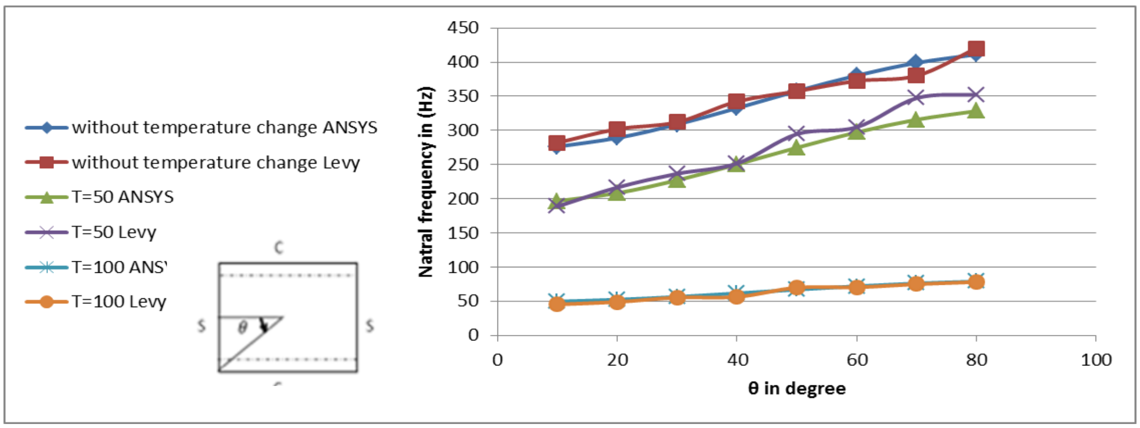

Figure 8. Effect of lamination angle on Natural frequency for simply-clamped supported (*θ*/-*θ*/-*θ*/*θ*) for different applied temperature.

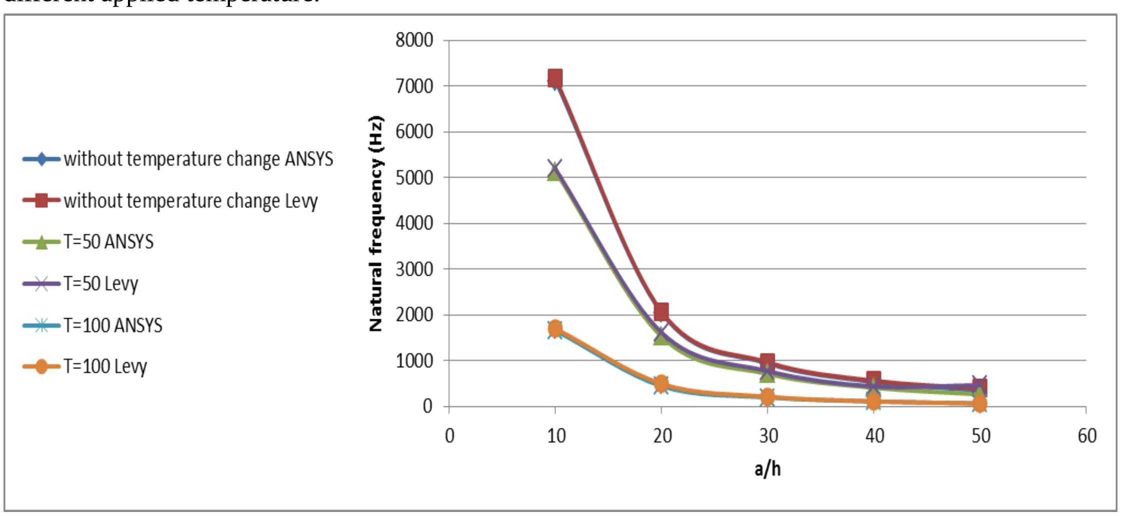

Figure 9. Effect of thickness ratio on Natural frequency for simply-clamped supported (45/-45/-45/45) for different applied temperature.