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The Study of the Dynamics of Electoral Sub-Networks in Albania Using Q-Functions.

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Abstract

Here we consider some practical mathematical tools and algorithms to analyse the state and the dynamics for a specific opinion system consisting in the differences of votes between to main subjects. Analysing q-distributions fitted to the empiric distributions we acknowledge the level of the stability, the rate of dynamics and relaxation according to Tsallis statistics. We reveal the importance of the use of mathematical tools of physical statistics in the study of some non physical systems. Another aspect consists in the efficiency of combined algorithms in assisting statistical analysis.

Key words: *Socio-dynamics, networks, complexity, algorithm, Tsallis triplet.*

1. Introduction

The models of opinion formation have been firstly developed in the framework of mathematical structures that could offer some quantitative approaches in social system analysis. Usually they are based in networks that consist in graphs of nodes characterized by a dynamic value, which can be updated by a given rule. The mathematical voter's model introduced by Ligett is considered as a pioneering work in such aspects, followed by intensive work last decades usually in the framework of statistical mechanics and detailed explanation could be found given in reference [1] etc. Opinion system models have been generally considered in the framework of stochastic systems or mechanical physics works as described in [2]. Problems socio-dynamics as cultural and linguistic dynamics, voting processes, financial index behavior, networking-based systems etc, have ben routinely addressed in this context. Such structures are known as "complex systems" and exhibit particular properties as critical dynamics, self-organization behavior as detailed in [3], and therefore their treatments and analysis have been developed in an interdisciplinary perspective based in specific mathematical tools and numerical calculation. Sometimes, the condensed maters physics methods are used successfully but statistical mechanics and methods and tools have been largely accepted as very useful techniques. In this case the distribution of an measurable observable and the physical state of the system are considered in some sense identical, and hence, the improvement of techniques in the analysis of distributions will help in the study of the dynamical behavior itself, and we've used it in similar studies of our systems as in [6] [7] or in the more complicated mathematical modeling as in [5]. It happens that in such systems and especially in opinion dynamics the distribution of the observable as number of links per n odes for example could be not stationary, hence and the system is assumed to be in an out of equilibrium state [10] and therefore the initiating point should be the non-equilibrium mechanical statistics. This last have been considered thoroughly by C Tsallis and his followers, which introduced q-functions to describe the distributions as detailed in [4]. In this case, q-Gaussians of the form $p(x) = C_q\left[1-(1-q)\beta x^2\right]^{\frac{1}{1-q}}$ and q-Gibbs forms $e_q(x) = (1-(1-q)\beta x)^{\frac{1}{1-q}}$ have been obtained as result of optimization of the q-entropy defined $S_q = \left(1 - \int_{\sup ort} p^q(x)dx\right)/(q-1)$

In a more theoretical consideration, by a natural extension of the q-sum and q-product respectively

$$x \oplus_q y = x + y + (1-q)xy;\ x \otimes b = [x^{1-q} + y^{1-q} - 1]_+^{\frac{1}{1-q}}$$

making use of two basic q-functions $\ln_q x = \frac{x^{1-q}-1}{1-q}$, and $e_q^x = (1+(1-q)x)^{\frac{1}{1-q}}$, a q-Fourier transform have been introduced in the reference [8], and therefore an alternative Central Limit Theorem is deduced. Thereof, q-Gaussian has been acknowledged as attractor of the distributions for the sum of q-independent identically distributed random variables (q-IID variables). According to those arguments, the mathematical basis of q-distribution is solidified in the case of correlated random variables. Next, the dynamics of such system will be analyzed in the same framework. The parameter that estimates the rate of dynamics is introduced in [4] by the formula $q_{sens} \equiv 1 - \frac{\alpha_{\max} - \alpha_{\min}}{\alpha_{\max} * \alpha_{\min}}$ where $\alpha_{\min}, \alpha_{\max}$ are the zeros of the spectrum power. The self-similarity will be checked by the analysis of the multi-fractal power spectrum and Hurst exponents. Finally to count for competition, in our systems we refer the mathematical analysis provided in [5] which proposed that again distribution of an physical quantity could be a q-function, composed by q-exponential $p(x) \sim [(3-2q)e_q(-b(x-\mu)) - 2(1-q)e_q(-2b(x-\mu))]h(x)$ where b is a positive constant and h is Heaviside function [5]. To adopt such a model which call a scalar (y) to be characteristic, we make use of an empiric energy form $k(j) = \frac{NumberVotes(j)}{TotalVotes}$ as discussed recently in [6].

. 2. Material and method.

We considered the results of parliamentary election in Albania at 2013 and election results for majors voting in administrative election at 2015. Results for polling station provided from the Central Election Commission, were normalized to avoid the sizes effect of the elements of the statistical ensemble. The first and second subjects as appear ranked in final result are assumed to compete to each one, over limited votes available and hence we considered such systems as competitive in the sense of the reference [6]. By those results, we produced the series of differences between the winner and follower for every polling station and we keep the same arguments given in reference [10] for physical aspects of this system. Analyzing the stability and the dynamics of such system, using tolls described above we acknowledges the relationship between the winner and its competitor. The key preliminary step is the optimization of the bins-size beginning from Scot or Freedman-Diaconic rules and trying the fit to a q-Gaussian as in the algorithm we've proposed in [11]. If the q-Gaussian found represent an non-stable distribution that is q>5/3, we changes slightly the bin size and try a couple of common distributions to fit the empiric data, and after that we selected few ones that were fitted better. The goodness of fit is evaluated by standard statistical tools and error analysis. We stop varying the bins size when the candidate's distribution shows its best fit and fix the bins width. If the best-fitted curves are a q-Gaussian, we evaluated the distance from the stability by the acknowledgment of the q-parameter. If not, the stability issue is only approached by the best q-Gaussian fitted. Next we estimate the relaxation rate and fit it to the q-exponential, so we have the q-relaxation parameter as prescribed in [4],[8]. Performing a multifractal power spectrum measure we identified the q-sensitive parameter as described by Tsallis triplet analysis [4]. Finally the fit is tried to the q-competition function as introduced in [5] and they result are analyzed. According to the properties of q-functions, in limit q→1 we will obtain the classic forms, so by identification of q in the last function we estimate the rate of competition too. For a distribution form $p(k) \sim a[1 - b(1-q)(k-\mu)^n]^{\frac{1}{1-q}}$ we argue that if n > 2, the system is considered a meta-stable state according to the arguments in [4]. If exponent is 2 and the goodness of fit is admissible, we continue with triplet Tsallis evaluation to see what prognosis we can do about the dynamics of the system. Our fitting algorithms are realized generally using *Non Linear Least Squares*, and sometimes we used an *ad hoc Genetic Algorithm* to better specify the boundaries in the first algorithm. Multifractal spectrum analysis is realized by the prescription of Multifractal Detrended Fluctuation Analysis algorithm. The trend of the process is analyzed using an Empirical Mode Decomposition algortihm as described in [9].

3. Data administration and results

The election results consist on the largest real opinion system possible with real and full measurement of the variables. Therefore, nor problem about sampling or whatever statically tools affect the analysis and we deal with the system as a whole. In administrative elections, one vote for the majors and for councils at the time and electoral coalitions manage themselves the inner distributions of the seats, therefore many hidden factors contribute in the net result. The system seems to be really complex. According to the results from polling stations for the elections of 2013 we tested firstly the competitive function of the form $p(k) \sim \left[(3-2q)e_q\left(-b(k-k_0)\right)-2(1-q)e_q\left(-2b(k-k_0)\right)\right]$ and a q-Gaussian. . We see that a q-Gaussian is better fitted to the empiric data (fig 1). It is logical that a q-Gaussian is always a candidate, but in our case it can provide the information of competition in stabilities issues.

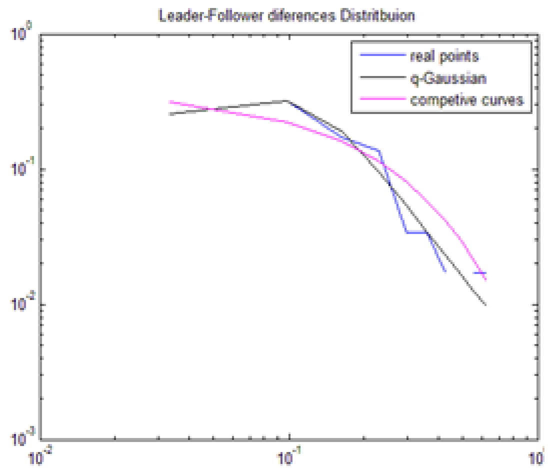

Fig.1 a) results by election zone

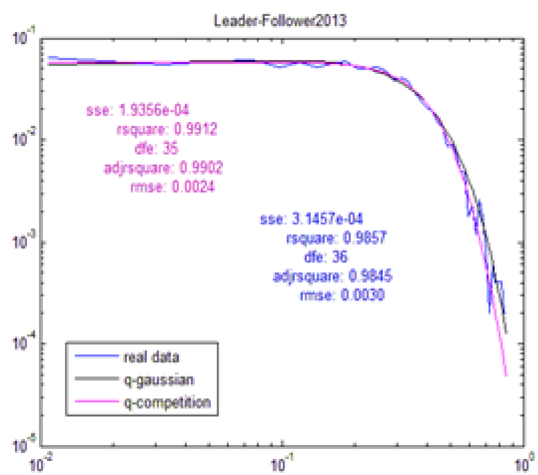

Fig.1 b) results by polling stations results.
Fig.1. Distribution of leader - follower differences

In a recent consideration [5] is reported that the best fitted distribution in the case of the unstable states is found q-exponential with argument ~k^3, where k is the fraction of votes , therefore we propose to change the scalar that reports the competition parameter in the model [5], and try a double q-exponential of the general form

$$p(k) \sim \left[(3-2q)e_q\left(-b(k-k_0)^3\right)-2(1-q)e_q\left(-2b(k-k_0)^3\right)\right].$$

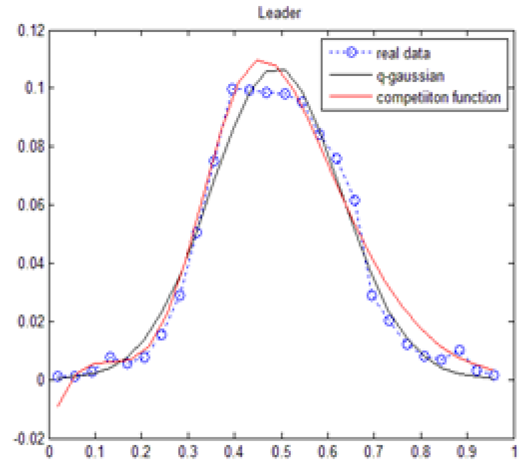

Fig. 2.b). The election of 2015, voting for majors.

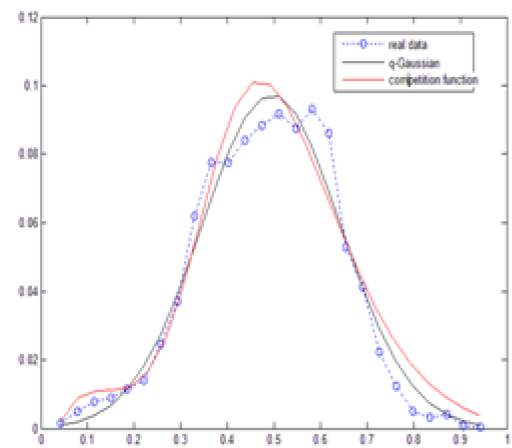

Fig. 2.a). Distribution of the leader-followers.

The fit is very good as seen in (Fig1, b). Physically it supports the idea that the best scalar or the energy-like function to description such system could be in the cubic form against the grade of nodes (links per nodes). For the election of the 2015 we found another picture. First, we considered the q-distribution on the case of absolute differences between leaders and followers for 61 election unities to perform a qualitative view because the number of points is small for quantitative description. Here the q-parameter read form the q-Gaussian fitted is obtained at the value 1.9935, so higher than the limit of the stability 5/3. The distribution of the differences using q-competitive function of first power shows no good fit. Considering the q-exponentials of the third power we obtain acceptable fit and the parameter q is near to the boundary value (found 1.0023) that is to say that function itself looks mostly with an exponential forms. Q-Gaussian form results a high unstable distribution, practically near to the indefinite q-variance limit (q~2). In this case we expect intensive activity in the network of leader–followers. When considering the differences of votes of corresponding parties, the q-Gaussian is even more unstable, with a parameter value at q~2.6. As for comparison, in the results of the elections in 2013, we have found that quite differently the q-Gaussian fitted to the absolute differences is almost the classic Gaussian as q=1.026. Q-Gaussian of real differences has the parameter q~1.2 whereas the best fitted q-distribution that corresponds to the assumed energy or strength of the nodes to be of the form $y \sim k^3$ has the parameter q=1.314. It provided that the competition network is found in a state far from "equilibrium", hence the strength of it will change drastically from zone to zone. Moreover we see that in this case the double q-exponential gave the best fit. If mathematical approaches herein has a physical significance, again the q-parameter obtained therein reports a high unstable state because q~2. The property considered is more heterogeneous and the overall state is less stable in the sense of the arguments in references [10]. We underline that a more meaningful comparison with 2013 should be only in the elections of the 2017, if the rules should remain unchanged. Therefore the analysis is limited in the actual system itself. We've checked the same idea on the case of particular results for each major parties, assuming an upper limit to be the goal, and compute the difference which is used as variable of the interests. We see that the competition function and even q-Gaussian does not fit very well with the distribution of the differences of the result for each subject to a constant level assumed (0.5 is considered the limit)

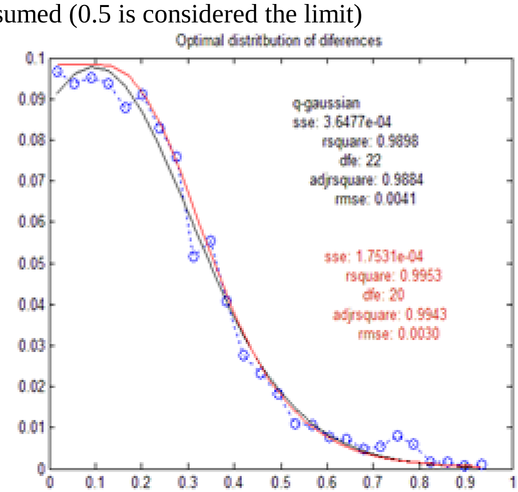

Fig.3.a). Distribution of absolute differences

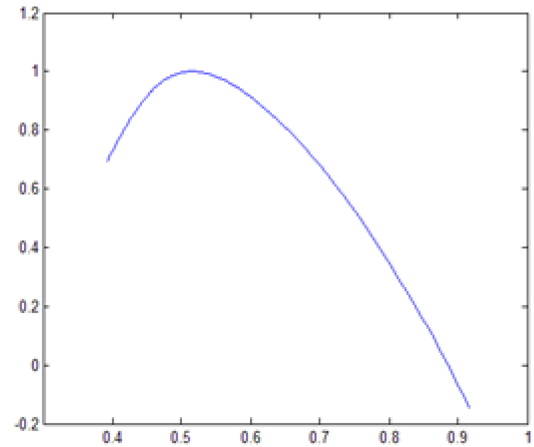

Fig. 3.b). Multi-fractal power spectrum

Another argument about the stability/instability issues could be based on multifractal analysis or EMD decomposition. To make use of time coordinate, one idea comes from the network aging. Here we assume that if the network is young (polling station not maturated), the fraction of the links for two first competitors should be low, because few voters were approached so far hence, the time is considered insufficient for this process to happen. As the inner time goes on, more nodes will attach the sub-networks of the two first candidates. Hence, we rank the value of differences according to the sum of first two subjects. The new series now is time series and we can speak for the evolution in such time reference. This idea has ben developed physically in [10] and we apply herein another techniques to make use of it. Performing the multi-fractal analysis we can estimate the sensitivity parameter that helps us to understand the rate of changes on this system. As seen on (Fig.3,b), the multi fractal power spectrum is a smooth function. So we can say that the system is organised with self-similar structure. The $q_{sens} \sim 0.734 << 1$, therefore, the system will be exposed to low rate dynamics, including aging in terms above. Analyzing the multifractal curve itself we see that the self-similarities are characteristic in our ensemble, and therefore, many results related to this properties will be concluded. Finally by applying the EMD as in many similar systems [9] we considered the trend behavior of our system, matched to the inner time as stated in this paragraph. It correspond to the last intrinsic mode function (IMF), which is presumed to maintain the overall trend of the system. As seen in (Fig.4), according to the aging process that consisted in the absolute dominance of the two major parties, the trend of the system is approached by a decreasing function. This is clear in the last frame that represent the residuals after intricacy mode separation.

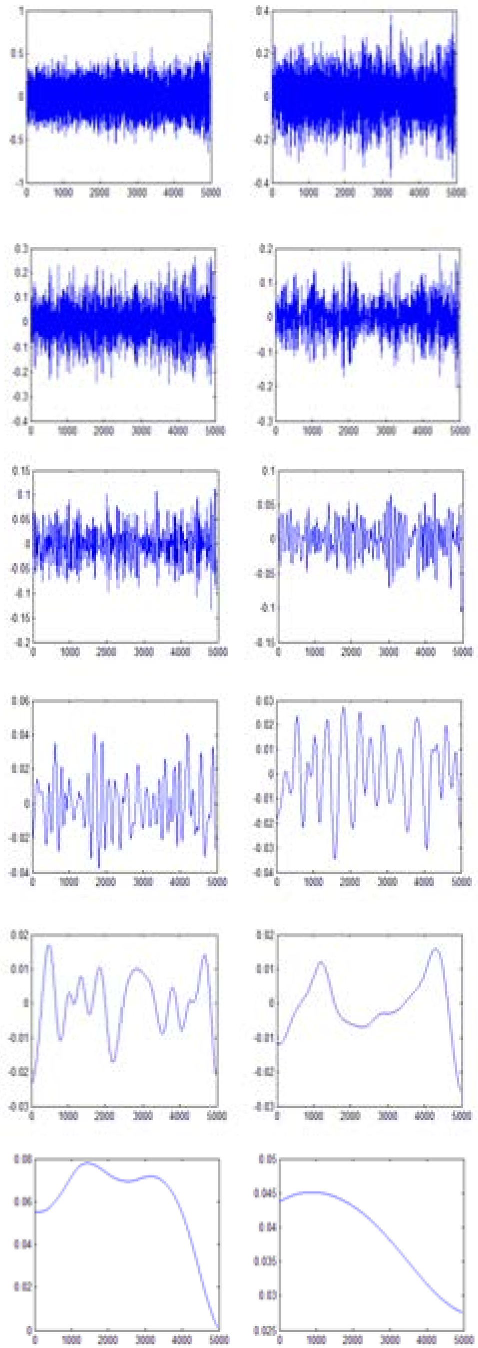

Fig.4. IMF of the direct differences leader-follower.

The detailed analysis according to the EMD techniques is not covered fully here, and we hope to reconsider it soon. Basically it needs more physical arguments, whereas this paper is focused on the practical proves for alternated mathematical and algorithmic tools in the study of complex systems following our recent view in [11] .

4. Conclusion.

The distribution of the differences between Leader and Followers for the election of the 2013 and 2015 has been approached under competition analysis as presented in considered literature. We observe that under this framework, the scalar related to the competitive game could be assumed to proportional to the third power of fraction. The dynamics of such network is found moderate, so, little changes occur by the time. According to the q-statistical analysis, the system of absolute differences is somewhat more stable whereas the system of real differences is highly unstable. It seems that the position of leaders and follower may switch more easily, but their differences will remain nearly robust. This work support the idea that a carefully use of q-functions in specific circumstances will provide to the researchers more flexible analysis, and the implantation of simple algorithms is possible and fruitful for such studies.

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