On The Comparison of Methods of Estimating Variance Components: A Case of Gudali Beef Cattle

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Abstract

A two-stage nested design with unequal replications was used in this study with an intention to capture the variability effects in the model. Moreover, five different methods of variance component estimation which were randomly chosen from the frequency approach were compared in this study. These methods which were Analysis of Variance method (ANOVA), Quasi-maximum-likelihood method (*QML*), Modified likelihood method (*ML*), Restricted maximum-likelihood method (*REML*), and Modified maximum likelihood method (*MML*), recommended "modified maximum likelihood method as the best since it has the smallest minimum variances".

Keywords: Two-way nested design, Analysis of variance method *(ANOVA*), Quasi-maximum-likelihood method (*QML*), Modified likelihood method (*ML*), Restricted maximum-likelihood method (*REML*), Modified maximum likelihood method (*MML*).

1.0 Introduction

Nested design, otherwise called hierarchical design, is a form of experimental design with a prominent characteristic such that each level of every factor occurs with all levels of the other factors, without interactions. For instance, in certain types of studies, the levels of one factor B, will not be identical across all levels of factor A, each level of factor A will contain different levels of factor B, therefore, levels of B are said to be nested within the levels of factor A.

However, the basic principle for estimating variance components has been, and to a large extent still is, that of equating quadratic functions of the observations to their expected values. Obvious candidates for such functions are those of the analysis of variance table. The first papers describing this procedure appear to be those of [7], whose interest was in weights of slubbings from the carding process in the woolen industry, and [28] who analyzed catches of different species in successive hauls of plankton nets.

Although [7] and [28] represent both sides of the Atlantic, it appears that major developments in variance components estimation subsequently took place mainly in the U. S. A. An exception to this was [11], dealing with nested classifications, and then came [6] concerned with randomized complete blocks, and [24] dealing with approximate sampling distributions of estimated variance components. This was followed by [9] who put a firm foundation to the distinction between the fixed effects model (Model I) and the random effects model (Model II), a distinction which [29] later took great exception to.

[2] is the first book that gives any treatment of variance components, its final four chapters being devoted to the topic entirely. This really set the subject on a firm footing and well and truly laid out the procedure of equating analysis of variance sums of squares to their expectations as a method of estimating variance components. The book deals very thoroughly with estimation from unbalanced data for both mixed and random models; it also deals with unbalanced data for nested classifications and, after considering incomplete blocks designs, it poses a number of pertinent research problems, many of which have still not been answered satisfactorily. In all, the book is a milestone in variance components estimation.

However, several researches have been carried out under the context of nested design. [27] described the use of nested designs in ruggedness testing in pharmaceutical work and proposed an alternative analysis method for use in ruggedness testing. See [17], [18], [3], [25], [19], [21], for more literatures on nested design.

Therefore the aim of this work is to compare five different methods of variance component estimation from the frequency approach, based on the properties of estimators adopted by [22]. The determination of the variability effects due to sires, dams within sire and estimation of variance components were the objectives.

1.1 Uses and Applications of Variance Components Estimation

Most statistical methods are developed in response to the demands of practical problems. Variance components estimation is no exception. The first papers, by [7] and [28], dealt with woolen industry and with plankton net data, respectively. [6] was interested in Drosophila egg production and he also refers to a variety of other applications of-variance components: enumeration sampling, cereal experiments, swine breeding (three papers), corn breeding and soil sampling. Papers by [13] on sheep breeding could be added to the list. Clearly, by the mid- 40’s, animal and plant breeders were making considerable use of variance components. The [2] book also contains references to numerous uses of variance components in subject-matter disciplines: industrial experimentation, corn trials, psychological testing, sample surveys (wheat fields, soybean trials and forest nurseries), the sampling of baled wool, studies of egg production and hog prices, and analyses of the efficacy of measuring instruments. Closely allied to the animal breeder's parameter of repeatability is the psychologist's and educationalist's measure of reliability of a test instrument, namely σ^2 respondent / (σ^2 respondent + σ^2residual), as, for example, in [1]. Geneticists and others who use the experimental design of the diallel cross (originating in genetics) also make great use of variance components – [20] provides a comprehensive review – and so do those designing sample surveys. Analyses of trajectory and orbital data in rocket flight testing have been based on the mixed model version of variance components models, [4] and so have analyses of data from clinical trials involving several clinics, [5]. Kalman filtering techniques of engineering, as described by [8], also utilize mixed model theory, as noted by [12]. And economists nowadays make very wide use of mixed models in combining cross-section with time series data [15], referring to their models as error components models. Variance components estimation continues, therefore, to be a technique that is quite widely used in data analysis, as well as receiving considerable attention on its theoretical side.

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2.0 Material and Methods

A two-stage nested design with unequal replication model (unbalanced data), proposed by [22] was applied in a population of Gudali beef cattle, to estimate the significant effects of variability in sires (male cattle) and dams (female cattle) within sire and also compute its variance components. In a population of 110 sires, a random sample of 20 sires was drawn and a further sample of 3 dams out of 3565 dams, were randomly selected to be nested within the sires. Then a total of 60 dams were drawn to be nested within the 20 selected sires, and the weaning weights of their progeny (offspring) measured. The selection was facilitated by the tags on their body and randomness was a case of lottery method of randomization. However, according to [22] the model equation is given as

$$y_{ijk} = \mu + A_i + B_{j(i)} + \varepsilon_{ijk}; \quad i = 1, 2, \ldots, p; \quad j = 1, 2, \ldots, q_i; \quad k = 1,2, \ldots, n_{ij}$$

$$A_i \sim N(0, \sigma_A^2), \quad B_{j(i)} \sim N\left(0, \sigma_{B(A)}^2\right), \quad \varepsilon_{ijk} \sim N(0, \sigma_\varepsilon^2)$$

where y_{ijk} is the k^{th} observed response in ij^{th} cell
μ is the universal constant, which is also called the overall mean
A_i is the effect of the i^{th} level of sire
$B_{j(i)}$ is the effect of the j^{th} level of dam nested within the i^{th} level of sire
ε_{ijk} is the random observational error associated with y_{ijk}; n_{ij} is the number of replication
p is the i^{th} level of sire; q_i is the j^{th} level of dam. However, it is noted that the total number of observations is denoted by $N = \sum_i N_i = \sum_i n_{ij}$ for an unbalanced case and $N = an$ for a balanced case by [22].

2.1 Layout of the design

The layout of the design is given by

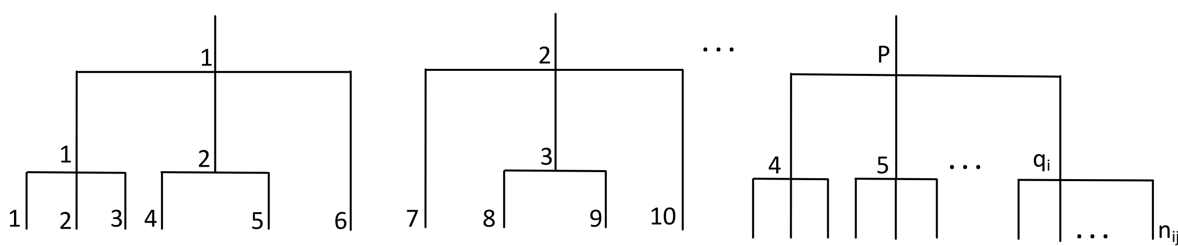

Figure 1

Figure 1 above shows the layout of the nested design with unbalanced arrangement. Each of the parameters has their explained meaning. The sire was represented by p. q_{ij} represented the dams within the sire and the weights of the progeny were represented by n_{ij}.

2.2 An Analysis of Variance table with all the necessary description

Source of Variation	Degree of Freedom	Sum of Squares	Mean Squares	Approximate F-test	E (MS)
Sire	*p - 1*	SS_A	MS_A	$F = \frac{MS_A}{MS_\theta} \sim F_{(p-1),\ F_\theta}$	$\sigma_{res}^2 + K_{B(A)}\,\sigma_{B(A)}^2 + K_A\ \sigma_A^2$

Dam(sire)	$F_B = \sum_i q_i - p$	$SS_{B(A)}$	$MS_{B(A)}$	$F = \frac{MS_{B(A)}}{MS_{res}} \sim F_{F_B,\, F_\theta}$	$\sigma^2_{res} + K_1\sigma^2_{B(A)}$
Residual	$F_Z = N - \sum_i q_i$	SS_{res}	MS_{res}		σ^2_{res}
Total	$N-1$	SS_T			

Table1: ANOVA table for two –stage nested design random effect model

From the table 1, $K_{A,}$ $K_{B(A),}$ K_1 , MS_θ , $F_{\theta,}$ was derived with the denotations below and approximate F test was derived too since there is no immediate F ratios for the main effects in a case of unbalanced nested design.

$$K_A = N - N^{-1}\sum_i N_i^2 \quad ; \qquad K_{B(A)} = \frac{\sum_i N_i^2 \sum_i n_{ij}^2 - N^{-1}\sum_{ij} n_{ij}^2}{\sum_i q_i - p} \quad ; \qquad K_1 = \frac{N - \sum_i N_i^{-1}\sum_{ij} n_{ij}^2}{\sum_i q_i - p} \ ;$$

$$F_\theta = (MS_\theta)^2 \left[\theta^2 \frac{[MS_{B(A)}]^2}{F_B} + \frac{(1-\theta)^2[MS_{res}]}{F_Z}\right]^{-1} \ ; \quad where\ \theta = \frac{K_{B(A)}}{K_1} \quad , \alpha = 0.05$$

$$also;\ MS_\theta = \theta MS_{B(A)} + (1-\theta)MS_{res}\ ;$$

$$we\ also\ define\ \ F_Z = N - \sum_i q_i\ ; \qquad F_B = \sum_i q_i - p$$

SS_A is sum of square of sire; $SS_{B(A)}$ is the sum of square of dam nested with sire. SS_{res} is the residual sum of square; SS_T is the sum of square total. MS_A is the mean square of sire; $MS_{B(A)}$ is the mean square of dam nested within sire. MS_{res} is the mean square of the residual; σ^2_{res} is the variance of the residual. σ^2_A is the variance of sire; $\sigma^2_{B(A)}$ is the variance of dam nested within sire.

3.0 Methods of Variance Component Estimation

Estimation of variance components is a method often used in population genetics and applied in animal breeding. Even experienced population geneticists nowadays feel lost if confronted with the huge set of different methods of variance component estimation. Especially because there exists no uniformly best method, a decision which method should be used is often difficult to take, [22]. This work considered only the frequency approach. This approach assumes that the parameters of the distribution and by this especially the overall mean and the variance components are fixed but unknown real values or vectors. Quasi Maximum Likelihood Method (QML), Analysis of Variance Method (ANOVA), Restricted Maximum Likelihood Method (REML), and Maximum Likelihood Method (ML), Modified Maximum-Likelihood Method (MML) were those five methods of variance components estimation from the frequency approach, considered in this work. These methods were chosen on no criteria.

3.1 Analysis of Variance Method (ANOVA-method)

According to [10], analysis of variance method is the one of the oldest and commonest methods of estimating variance components. In this method we replace in the column *E* (*MS*) of the ANOVA table the variance

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components of each σ (population standard deviation) by its estimate S (sample standard deviation) and we put the resulting expressions equal to the observed Mean Square component and finally we solve the resulting equations by substitution. In this case where we considered unbalanced two- stage classification this leads to:

$$S^2 = MS_{res} \quad (1)$$

$$S^2 + K_{B(A)}S_{B(A)}^2 + K_A S_A^2 = MS_A \quad (2)$$

$$S^2 + K_1 S_{B(A)}^2 = MS_{B(A)} \quad (3)$$

On solving equation (1) – (3) above by substitution from the ANOVA table gives

$$S^2 = \widehat{\sigma}_{ANOVA} = MS_{res} \quad (4)$$

$$S_A^2 = \widehat{\sigma}_{A.\ ANOVA} = \frac{1}{K_A}\left[MS_A - MS_{res} - \frac{K_{B(A)}}{K_1}\left(MS_{B(A)} - MS_{res}\right)\right] \quad (5)$$

$$S_{B(A)}^2 = \widehat{\sigma}_{B(A).ANOVA} = \frac{1}{K_1}\left[MS_{B(A)} - MS_{res}\right] \quad (6)$$

Therefore, from the computation carried out;

$$S^2 = 464.2744, \quad S_A^2 = 178.0210\,, \quad and \quad S_{B(A)}^2 = 32.4560$$

3.2 Quasi- Maximum-Likelihood Method (QML)

A quasi-maximum likelihood estimate is obtained by minimizing the likelihood function of the sample under the restriction that the solution lies in the parameter space. Without this restriction some variance components may be negative. For more details, see [23]. We estimate the variances using these equations:

$$S^2 = \hat{\sigma}_{QML}^2 = MS_{res} \quad (7)$$

$$S_A^2 = \hat{\sigma}_{A.\ QML}^2 = \frac{1}{K_A}\left[\frac{(p-1)}{p}MS_A - MS_{res}\right] \quad (8)$$

$$S_{B(A)}^2 = \hat{\sigma}_{B(A).\ QML}^2 = \frac{1}{K_{B(A)}}\left[\frac{y^*}{x^*}MS_{B(A)} - MS_{res}\right] \quad (9)$$

$$S^2 = 464.2743\,, \quad S_A^2 = 170.6439\,, \quad and \quad S_{B(A)}^2 = 32.1370, \quad .$$

$$x^* = N - \sum_i q_i\ ; \quad y^* = \left(N - \sum_i q_i\right) - 1\ ; \quad N = 90; \quad \sum_i q_i = 60\ ;$$

$$p = 20; \quad x^* = 31; \quad y^* = 30$$

3.3 Maximum Likelihood Method (ML)

According to [14] a real maximum likelihood estimator was derived. The restriction being imposed on it is to enable the solution to lie within the parameter space. The estimators here are biased. Variances can be computed like:

$$S^2 = \hat{\sigma}^2_{ML} = Min\left\{MS_{res}, \frac{SS_{res} + SS_A}{x^*}\right\} \quad (10),$$

$where\ x^*\ is\ known$

$$S_A^2 = \hat{\sigma}^2_{A.\ ML} = \frac{1}{K_A} Max\left\{\left(\frac{(p-1)}{p} MS_A - MS_{res}\right); 0\right\} \quad (11)$$

$$S^2_{B(A)} = \hat{\sigma}^2_{B(A).\ ML} = \frac{1}{K_{B(A)}} Max\left\{\left(\frac{y^*}{x^*} MS_{B(A)} - MS_{res}\right); 0\right\} \quad (12)$$

$S^2 = 464.2742,\ S_A^2 = 170.6439,\ and\ S^2_{B(A)} = 32.1370,$

3.4 Restricted Maximum-Likelihood Method (REML)

According to [2], restricted maximum-likelihood uses a translation invariant restricted likelihood function depending on the variance components to be estimated only and not on the fixed effects like μ . This restricted likelihood function as a function of the sufficient statistics for the variance components. The latter is then derived with respect to the variance components under the restriction that the solutions are non- negative. The solutions are:

$$S^2 = \hat{\sigma}^2_{REML} = Min\left\{MS_{res}, \frac{SS_{res} + SS_A}{y^*}\right\} \quad (13)$$

$where\ y^*\ is\ known$

$$S_A^2 = \hat{\sigma}^2_{A.\ REML} = \frac{1}{K_A} Max\{(MS_A - MS_{res}); 0\} \quad (14)$$

$$S^2_{B(A)} = \sigma^2_{B(A).\ REML} = \frac{1}{K_1} Max\{(MS_{B(A)} - MS_{res}); 0\} \quad (15)$$

$S^2 = 464.2741,\ S_A^2 = 184.5122, and\ S^2_{B(A)} = 32.4560.$

3.5 Modified Maximum-Likelihood Method (MML)

Details on the estimation processes can be seen in [26] and [16]. Hence the solutions are given as:

$$S^2 = \hat{\sigma}^2_{MML} = Min\left\{MS_{res}, \frac{SS_{res}+SS_A}{x^*+1}, \frac{SS_{res}+SS_A+x^*\bar{y}^2}{y^*+2}, \dots\right\} \quad (16)$$

$$S_A^2 = \hat{\sigma}^2_{A.\ MML} = \frac{1}{K_A} Min \left\langle \left[Max\left(\left\{\frac{(P-1)}{P+1} MS_{res}\right\}; 0\right)\right]; Max\left[\left(\frac{SS_A+x^*\bar{y}^2\dots}{x^*+2} - MS_{res}\right); 0\right]\right\rangle \quad (17)$$

$$S^2_{B(A)} = \hat{\sigma}^2_{B(A).\ MML} = \frac{1}{K_{B(A)}} Min \left\langle Max\left\{\left(\frac{y^*}{x^*+1} MS_{B(A)} - MS_{res}\right); 0\right\}; Max\left\{\left(\frac{SS_{B(A)}+ x^*\bar{y}^2 \dots - MS_{res}}{x^*+2}\right); 0\right\}\right\rangle \quad (18)$$

$S^2 = 464.2740$, $S_A^2 = 95.4440$, $and\ S_{B(A)}^2 = 16.6741.$

4.0 Evaluation of the Analysis of Variance Table

The calculated values of the parameters in the ANOVA Table were given below. See appendix for details.

$SS_A = SS_{sire} = 26.349.857$; $SS_{B(A)} = SS_{dam(sire)} = 20,516.324$; $SS_{Total} = 2,170,418$

$SS_{res} =$ 14,392.50; $MS_{sire} = 1386.835$; $MS_{dam(sire)} = 512.958$; $MS_{res} = 464.274$ $K_A = 5.0$;

$K_{B(A)} =$ 1.0; $K_1 = 1.5$; $F_\theta =$ 66.00; $F_Z = 31$; $F_B =$ 40

Evaluated Analysis of Variance Table for Two-Stage Nested Design; Unbalanced Case

Source of Variation	Degree of Freedom	Sum of Squares	Mean Square	Approximate. - Test	Sig-value
Sire	20	26349.857	1386.836	1.660	0.020
Dam(sire)	40	20518.324	512.958	1.690	0.060
Res	31	14392.500	464.274		
Total	90	2170418			

Table 2: Computed ANOVA table for two stage nested design random effect model.

4.1 Table of Results

The results of comparative study of different methods of variance components estimation is presented below:

Method	Estimate of $\hat{\sigma}_{sire}^2$	Estimate of $\hat{\sigma}_{dam(sire)}^2$	Estimate of $\hat{\sigma}_{res}^2$	Total Variance
MML	95.444	16.674	464.274	576.392
ML	170.644	32.137	464.274	667.055
QML	170.644	32.137	464.274	667.055
ANOVA	178.021	32.456	464.274	674.751
REML	184.512	32.456	464.274	681.242

Table 3: Comparative table of variance components

Based on the result above, the five different methods of variance components estimation considered from the frequency approach differ only slightly from one another. But it might be quite different for higher classification or hierarchy (more than two stage classification) and covariates in the models. Such conclusions can be drawn only if we apply the different methods to a data set for which the parameter are known like in our collected data. Otherwise we can only see differences between the methods but we do not know which of them is good. But based on the results from the numerical example in this study, *MML* gave the smallest minimum variance among others, and hence was recommended as the best, among the considered methods from the frequency approach. Moreover, the effect due to sires is significant whereas that of dams within sire is not.

5.0 Conclusion

In this work where application of two-stage nested design unbalanced case was applied on a population of Gudali beef Cattle, observing weight of the progeny under dams nested with the sires with a view to observe the significant effects of variability and the variance components, conclusion was that the variability effects of sires was significant and that of dams within sire was not. Moreover, modified maximum likelihood method of variance component estimation was recommended as the one with the smallest minimum variance. This smallest minimum variance, which Modified maximum likelihood method has placed it first before the other considered methods, in line with the properties of estimators [22].

Appendixes

App.1

		Weaning Weight		
Sire	**Dam**	**1**	**2**	**3**
1.	1	189	180	
	2	194	168	
	3	155	203	145
2.	4	150	163	
	5	160	123	
	6	170	131	
3.	7	112		
	8	107	182	
	9	146	133	
4.	10	163	130	
	11	140	149	
	12	156	179	
5.	13	149	109	
	14	139		
	15	113		
6.	16	166	166	140
	17	169	165	
	18	150	150	
7.	19	170	149	
	20	139		
	21	152		
8.	22	202	164	
	23	205		
	24	149	170	
9.	25	146	129	
	26	160	167	
	27	200	160	
10.	28	171		
	29	170		
	30	145		
11.	31	89	100	
	32	128	144	
	33	130	72	
12.	34	128		
	35	224		
	36	160		
13.	37	169		
	38	146		
	39	144		
14.	40	113		
	41	169		
	42	187		
15.	43	149	168	
	44	150	139	
	45	198		
16.	46	140		
	47	163		
	48	180		
17.	49	120		
	50	145		
	51	113		
18.	52	139		
	53	143		
	54	157		
19.	55	166		
	56	149	142	
	57	126		
20.	58	163	150	
	59	159		
	60	163		

App. Table 1: table showing the weaning weight of progeny produced by dams nested within sire.

App.2

	Dam			
Sire	**1**	**2**	**3**	n_{ij}^2
1.	4	4	9	17
2.	4	4	4	12
3.	1	4	4	9
4.	4	4	4	12
5.	4	1	1	6
6.	9	4	4	17
7.	4	1	1	6
8.	4	1	4	9
9.	4	4	4	12
10.	9	1	4	14
11.	1	4	4	9
12.	1	1	1	3
13.	1	1	4	6
14.	4	1	1	6
15.	1	4	1	6
16.	1	1	1	3
17.	1	1	1	3
18.	1	1	1	3
19.	1	4	1	6
20.	4	1	1	6

Table 2: n_{ij}^2 table.

Calculation of sum of squares

$SS_\mu = \frac{Y_{...}^2}{N} = 2109157.319$

$$SS_A = \sum_{ij}^{n_{ij}} Y_{ij.}^2 - \frac{Y_{...}^2}{N} = 26349.857$$

$$SS_{B(A)} = \sum_{ij}^{n_{ij}} Y_{ij.}^2 - \sum_{i}^{n_{ij}} \frac{Y_{i..}^2}{N_i} = 20516.324$$

$$SS_T = \sum_{ij}^{n_{ij}} Y_{ijk}^2 = 2170418$$

$$SS_\varepsilon = SS_T - SS_\mu - SS_A - SS_{B(A)} = 14392.50$$

Estimation of K_A, $K_{B(A)}, K_1$,

$$K_A = N - N^{-1} \sum_{i}^{n_{ij}} N_i^2 = 4.525 \approx 5.0$$

$$K_{B(A)} = \frac{\sum_i^{n_{ij}} N^{-1} \sum_{ij}^{n_{ij}} n_{ij}^2 - N^{-1} \sum_{ij}^{n} n_{ij}^2}{\sum_i^{n_{ij}} q_i - p} = 0.74846 \approx 1.0$$

$$K_1 = \frac{N^{-1} \sum_i^{n_{ij}} N_i^{-1} \sum_{ij}^{n_{ij}} n_{ij}^2}{\sum_i^{n_{ij}} q_i - p} = 1.48286 \approx 1.5$$

Estimation of F_θ

$$F_\theta = (MS_\theta)^2 \left[\theta^2 \frac{\{MS_{B(A)}\}^2}{F_B} + \frac{(1-\theta)^2 [MS_\varepsilon]}{F_Z} \right]^2 = 66.1365 \approx 66.0$$

$$MS_\theta = \theta MS_{B(A)} + (1 - \theta) MS_\varepsilon = 496.892$$

Where $\theta = \frac{K_{B(A)}}{K_1} = 0.6$

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