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Effect of Viscosity on Energy of Electron in Hydrogen Atom

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Abstract: Particles that are moving within a bulk matter are affected by the surrounding atoms. This effect can be recognized by treating matter as a viscous medium. The expression of energy loss by particle in viscous medium was derived by relating it to the orbital angular momentum. This energy expression is used to find a new quantum law that accounts for the effect of viscosity. It was found that this equation is reduced to ordinary Schrodinger equation in the absence of friction. The solution of this equation shows that both energy and viscosity coefficients are quantized and are related to the orbital quantum number. The total energy reduces to that of ordinary one in the absence of viscosity. The Reynolds number agrees with the known one.

Key wards: Law of Stock Force, Laminar and Non Laminar, Flow, Hydrogen Atom.

I. Introduction

Quantum mechanics describe the microscope particle like electrons, protons, atom and molecules, and subatomic particles [1].The most important early experiments involved light nature. The black body radiation spectra indicate that the light cannot behave as a wave. This for Max Planck to propose that; light waves are quantized into particles called photons as suggested by Einstein [2].The dual nature of light encourages De Broglie to propose that matter particles can also behaves as waves. This dual nature of matter was confirmed experimentally by Davisson and Germer. Historically, the laws of quantum mechanics have been having based on the dual nature of atomic world building blocks. This dual nature beside the energy relations represents the corner stone's of quantum laws [3]. There are many fields which are developed by quantum mechanics like electronic, spectroscopy, solid state physics, biology as well as medicine, chemistry and laser. Recently quantum mechanics is used to develop quantum computers and it also include quantum tunneling which is uses in operation of many devices like flash memory chips, but these researches need multi researches in quantum mechanics [4]. This needs to modify quantum laws to account for some medium properties like viscosity. The viscosity of a fluid is a measure of its resistance to gradual deformation by shear stress or tensile stress, it is transforms kinetic energy of a particle into heat energy. The flow of particle in viscous media is either laminar or non laminar flow according to satisfy critical value called Reynolds number [5].The aim of present study is modify energy of electron in

Hydrogen atom using Stock's force to drive an expression for energy lost by the particles due to viscosity. This expression of energy is used to derive quantum equation that accounts for the effect of viscosity. The discussion and conclusions are respectively in section (3) and (4).

II. Electron Energy Loss Due to Viscosity of Fluid Around Nuclear of Hydrogen Atom

To get electron energy loss due to viscosity of fluid around nuclear of hydrogen atom E_{vis}, one must obtain the Stock force that act on electron in its orbit for laminar and non laminar flow. The Stock force act on electrons revolving in its orbit, for laminar flow is given by

$$F_{Stock} = 6\pi\gamma\eta\upsilon = \beta_0\upsilon \quad (1)$$

Where $\beta_0 = 6\pi\gamma\eta$ (2)

Where η is viscosity coefficient, γ is radius of the electron orbit and υ is velocity of electron. For non laminar flow the Stock force become

$$F_{Stock} = \frac{1}{4}\pi\eta\, N_g\gamma C_D\upsilon = \beta_1\upsilon \quad (3)$$

The termC_D is the coefficient resistance of air stands and $N_g$represents Reynolds number

$$\beta_1 = \frac{1}{4}\pi\eta\, N_g\gamma C_D \quad (4)$$

Then let the Stock force in general has the form

$$F_{Stock} = \beta\upsilon \quad (5)$$

Thus the electron energy loss due to viscosity of fluid around nuclear of hydrogen atom is given by

$$E_{vis} = \int \beta\upsilon \,.\, ds \quad (6)$$

But the equation of velocity in $r, \theta, \phi -$ direction takes the form

$$\upsilon = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + rsin\theta\;\dot{\phi}\hat{\phi} \quad (7)$$

And its equation of the path in $r, \theta, \phi -$ direction

$$ds \;=\; dr\hat{r} \;+ rd\theta\;\hat{\theta} + rsin\theta\; d\phi\hat{\phi} \quad (8)$$

Sub Eqn. (7) and Eqn. (8) in Eqn. (6) to obtain electron energy by a viscose media as

$$E_{vis} = \int \beta\left(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + rsin\theta\;\dot{\phi}\hat{\phi}\right).\left(dr\hat{r} \;+ rd\theta\;\hat{\theta} + rsin\theta\; d\phi\hat{\phi}\right) \quad (9)$$

Let us an electron move near at one ends of orbit($\dot{r} \to 0$) in θ- direction, by approximately uniform velocity$\dot{\theta}$, also one consider the motion of electron is parallel to the earth and almost parallel to azimuth coordinate, then $\dot{r} \approx \dot{\phi} \approx 0$ and Eqn. (9) become

$$E_{vis} = \int \beta(r\dot{\theta}\hat{\theta}).(dr\hat{r} \;+ rd\theta\;\hat{\theta} + rsin\theta\; d\phi\hat{\phi})$$

Or

$$E_{vis} = \int \beta\left(r^2\dot{\theta}\right)d\theta \quad (10)$$

The value of angular momentum is usually constant in electron orbit; by tacking above consideration one can be obtain it as

$$L = r \times \mu\,\upsilon = \begin{pmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ r & 0 & 0 \\ 0 & r\dot{\theta} & 0 \end{pmatrix} = \mu r^2\dot{\theta} \quad (11)$$

But if the particle move in a central potential$\dot{\theta}$ must be equal

$$\dot{\theta} = \frac{L}{\mu\, r^2} \quad (12)$$

Substitute Eqn. (12) in Eqn. (11) to get

$$E_{vis} = \int \beta(r^2.\frac{L}{r^2\mu})d\theta$$

Or

$$E_{vis} = \int \frac{\beta.L}{\mu}\, d\theta = \frac{\beta.L}{\mu}\int_0^{\pi}\, d\theta = \frac{\pi\beta.L}{\mu} \quad (13)$$

This is the electron energy loss on its orbit due to viscosity of fluid around nuclear of hydrogen atom work done of Stock force of electron hydrogen atom in terms of conserved angular momentum. For laminar flow can be obtained by substitute equation(2) in Eqn. (13) as

$$E_{vis} = \frac{6\pi^2\gamma\eta}{\mu}L = \frac{\pi\beta_0}{\mu}L \quad (14)$$

And the energy loss operator is

$$\hat{E}_{\mathrm{vis}} = \frac{6\pi^2\gamma\eta}{\mu}L = \frac{\pi\beta_0}{\mu}\hat{L} \quad (15)$$

Also the energy loses and its operator for non laminar flow is given by substitute Eqn. (4) in Eqn. (13)

$$E_{vis} = \frac{1}{4}\frac{\pi^2\eta\, N_g\gamma C_D}{\mu}L = \frac{\pi\beta_1}{\mu}L \quad (16)$$

$$\hat{E}_{\mathrm{vis}} = \frac{1}{4}\frac{\pi^2\eta\, N_g\gamma C_D}{\mu}L = \frac{\pi\beta_1}{\mu}\hat{L} \quad (17)$$

In general form the energy loss of electron due to viscous media around nuclear of hydrogen atom is given as

$$\hat{E}_{\mathrm{vis}} = \frac{\pi\beta}{\mu}\hat{L} \quad (18)$$

III. Effect of Viscosity on Total Energy of Electron in Hydrogen Atom

To obtain the effect of viscosity on that total energy of electron in the hydrogen atom, one can apply equation (13) in Schrodinger equation to get

$$\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2 sin\theta}\frac{\partial^2}{\partial\phi^2}\right]\psi(r,\theta,\phi) + \frac{2\mu}{\hbar^2}\left(E - V - \frac{\pi\beta}{\mu}\hat{L}\right)\psi(r,\theta,\phi) = 0 \quad (19)$$

Where β is either β_0 or β_1 according to the type of flow.

And the wave function can be separated as

$$\psi(r,\theta,\phi) = R(r)Y(\theta,\phi) \quad (20)$$

Using the fact that

$$\hat{L}Y(\theta,\phi) = \sqrt{\ell(\ell+1)}\,\hbar Y(\theta,\phi) \quad (21)$$

The result of applying energy loses operator Eqn. (18) on the wave function Eqn. (21), produces

$$\frac{\pi\beta}{\mu}\hat{L}\,\psi(r,\theta,\phi) = \frac{\pi\beta}{\mu}R\hat{L}\,Y(\theta,\phi) = \frac{\pi\beta}{\mu}R\sqrt{\ell(\ell+1)}\,\hbar Y(\theta,\phi)$$

$$= \frac{\pi\beta}{\mu}\sqrt{\ell(\ell+1)}\,\hbar\psi(r,\theta,\phi) \quad (22)$$

Substitute equation (22) in (19) to get

$$\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2 sin\theta}\frac{\partial^2}{\partial\phi^2}\right]\psi+$$

$$\frac{2\mu}{\hbar^2}\left(E-V-\frac{\pi\beta}{\mu}\sqrt{\ell(\ell+1)\hbar}\right)\psi=0 \qquad (23)$$

But the terms E and $\frac{\pi\beta}{\mu}\sqrt{\ell(\ell+1)\hbar}$ are constants, thus one can consider

$$E_0=E-\frac{\pi\beta}{\mu}\sqrt{\ell(\ell+1)\hbar} \qquad (24)$$

Then equation (23) become

$$\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2 sin\theta}\frac{\partial^2}{\partial\phi^2}\right]\psi(r,\theta,\phi)+$$

$$\frac{2\mu}{\hbar^2}(E_0-V)\psi(r,\theta,\phi)=0 \qquad (25)$$

Where this equation is solved for of hydrogen atom similar to ordinary one, where the energy Eigen values takes the form

$$E_0=\frac{-1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{\mu e^4}{\hbar^2}\frac{Z^2}{n^2} \qquad (26)$$

By inserting equation (24) in (26), one gets

$$E-\frac{\pi\beta}{\mu}\sqrt{\ell(\ell+1)\hbar}=\frac{-1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{\mu e^4}{\hbar^2}\frac{Z^2}{n^2}$$

Thus the electron energy for Hydrogen atom for viscose medium, is given by

$$E=\frac{-1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{\mu e^4}{\hbar^2}\frac{Z^2}{n^2}+\frac{\pi\beta}{\mu}\sqrt{\ell(\ell+1)\hbar} \qquad (27)$$

For laminar flow one replaces β by β_0 [see equation. (1), (3), (5) and(6)]. Then Eqn. (27) becomes

$$E=\frac{-1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{\mu e^4}{\hbar^2}\frac{Z^2}{n^2}+\frac{6\pi^2\gamma\eta}{\mu}\sqrt{\ell(\ell+1)\hbar} \qquad (28)$$

The second term is the energy loss by the electron due to viscose media around nuclear of hydrogen, and can be calculated for $\ell=1$ and the practical coefficient of viscosity in Hydrogen atom approximately equal $9\times10^{-6}\ Pa.s$.

$$E_{vis}=\frac{6\times(3.14)^2\times2.8179\times10^{-15}\times9\times10^{-6}\times\sqrt{2}\times1.0545726663\times10^{-34}}{9.104431325951217\times10^{-31}}$$

$$E_{vis}=2.4576346247563871175818831209471\times10^{-22}\ Joule$$

The definition of stocks force in terms of energy in θ – spherical coordinate can be used to get

$$F_{stock}=\frac{E_{stock}}{ds}=\frac{6\pi^2\gamma\eta}{\mu}\sqrt{\ell(\ell+1)\hbar}\div\int_0^{\pi}r\,d\theta$$

$$F_{stock}=\frac{6\pi\gamma\eta}{r\,\mu}\sqrt{\ell(\ell+1)\hbar} \qquad (29)$$

Stock force can be calculated by substitute r equal Bohr radius as

$$F_{Stock}=\frac{6\times3.14\times2.8179\times10^{-15}\times9\times10^{-6}\times\sqrt{2}\times1.0545726663\times10^{-34}}{0.52917724924\times10^{-10}\times9.104431325951217\times10^{-31}}$$

$$F_{Stock}=1.4790624263462501601953965253334\times10^{-12}\ N$$

The pressure of fluid surround nuclear between can be found from Eqn. (29) beside the relation between pressure and force, to be

$$P = \frac{F_{stock}}{particle\ surface} = \frac{6\pi\gamma\eta}{r\,\mu}\sqrt{\ell(\ell+1)}\hbar \div \pi\gamma^2$$

$$P = \frac{6\eta}{r\,\gamma\,\mu}\sqrt{\ell(\ell+1)}\hbar \qquad (30)$$

$$P = \frac{6\times 9\times 10^{-6}\times\sqrt{2}\times 1.0545726663\times 10^{-34}}{0.52917724924\times 10^{-10}\times 2.8179\times 10^{-15}\times 9.104431325951217\times 10^{-31}}$$

$$P = 5.9320622847858325650289777375129\times 10^{16}\ N/m^2$$

Coefficient of viscosity of fluid around nuclear of hydrogen is

$$\eta = \left[E + \frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{\mu e^4}{\hbar^2}\frac{Z^2}{n^2}\right].\frac{\mu}{6\pi^2\gamma\sqrt{\ell(\ell+1)}\hbar} \qquad (31)$$

The first term E represent energy of electron move in viscosity energy level but the second term is the electron energy with non viscosity level of electron.

Then Eqn. (31) can be write as

$$\eta = \frac{\Delta E\,\mu}{6\pi^2\gamma\sqrt{\ell(\ell+1)}\hbar} \qquad (32)$$

For non laminar flow replace the value of βin Eqn. (27) by β_1 in Eqn. (3),

$$E = \frac{-1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{\mu e^4}{\hbar^2}\frac{Z^2}{n^2} + \frac{1}{4}\frac{\pi^2\eta\,N_g\gamma C_D}{\mu}\sqrt{\ell(\ell+1)}\hbar \qquad (33)$$

Can be calculated energy loss (the second term in above equation) by a given value of $\ell = 1, \eta =, C_D = \times 0.5, N_g$ is Reynolds number can be obtained from comparison calculated of all results $N_g = 47.9999999999$ the energy loss become

$$E_{vis} = \frac{(3.14)^2\times 9\times 10^{-6}\times 2.8179\times 10^{-15}\times 0.5\times\sqrt{2}\times 1.0545726663\times 10^{-34}\times 47.9999999999}{4\times 9.104431325951217\times 10^{-31}}$$

$$E_{vis} = 2.4576346247512670454469739811188\times 10^{-22}\ Joule$$

The stocks force in θ – spherical coordinate is

$$F_{stock} = \frac{E_{stock}}{ds} = \frac{1}{4}\frac{\pi^2\eta\,N_g\gamma C_D}{\mu}\sqrt{\ell(\ell+1)}\hbar \div \int_0^{\pi} r\,d\theta$$

$$F_{stock} = \frac{1}{4}\frac{\pi^2\eta\,N_g\gamma C_D}{\mu\pi r}\sqrt{\ell(\ell+1)}\hbar \qquad (34)$$

$$F_{Stock} = \frac{3.14\times 9\times 10^{-6}\times 2.8179\times 10^{-15}\times 0.5\times\sqrt{2}\times 1.0545726663\times 10^{-34}\times 47.9999999999}{4\times 9.104431325951217\times 10^{-31}\times 0.52917724924\times 10^{-10}}$$

$$F_{Stock} = 1.4790624263431687801405085041663\times 10^{-12}\ N$$

The pressure of fluid surround nuclear of a given values is

$$P = \frac{F_{stock}}{particle\ surface} = \frac{1}{4}\frac{\pi\,\eta\,N_g\gamma C_D}{\mu r}\sqrt{\ell(\ell+1)}\hbar \div \pi\gamma^2$$

$$P = \frac{1}{4}\frac{\eta\,N_g C_D}{\gamma\mu r}\sqrt{\ell(\ell+1)}\hbar \qquad (35)$$

$$P = \frac{9\times 10^{-6}\times 47.9999999999\times 0.5\times\sqrt{2}\times 1.0545726663\times 10^{-34}}{4\times 2.8179\times 10^{-15}\times 9.104431325951217\times 10^{-31}\times 0.52917724924\times 10^{-10}}$$

$$P = 5.9320622847734741019356739196691\times 10^{16}\ N/m^2$$

$$\rho = \frac{\eta\,N_g}{2\gamma v} \qquad (36)$$

$$\rho = \frac{9 \times 10^{-6} \times 47.9999999999}{2 \times 2.8179 \times 10^{-15} \times 2.1882661167883211678832116788321 \times 10^{6}}$$

$$\rho = 7.0058048227993549980989389693641 \times 10^{16} \ kg/m^3$$

The coefficient of viscosity from Eqn. (33) can be define as

$$\eta = \left[E + \frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{\mu e^4}{\hbar^2}\frac{Z^2}{n^2} \right] . \frac{4\,\mu}{\pi^2 \gamma N_g C_D \sqrt{\ell(\ell+1)}\hbar} \qquad (37)$$

V. Discussion

The quantum equation which describes particle motion in a viscous medium requires an expression for energy lost by viscous medium, this relation is shown by equation(13). It relates angular momentum to viscosity as shown by equation(14). The corresponding operator is exhibited in equation(15) and (18). The new Schrodinger equation accounts for the effect of viscosity. It was derived by replacing the viscous classical energy, which depends on L, by the corresponding operator after adding it to the energy part in conventional Schrodinger equation [see Eqn. (19). By separating variables to angular and radial parts, the viscous energy adds to Schrodinger equation additional terms in equation (22) and (28) which shows viscous energy quantization. This viscous energy is proportional to the viscosity coefficient η. It reduces to the ordinary energy of Hydrogen atom in the absence of viscosity. The Reynolds number obtained in equation (34) is consistent with the observed values.

IV. Conclusion

The modified Schrodinger equation obtained in this model accounts for the effect of viscosity in the presence of viscous medium. This equation reduces to ordinary Schrodinger equation in the absence of viscosity. The orbital electron energy and viscous energy are quantized. This energy reduces again to the conventional orbital energy in the absence of viscosity.

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