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# Natural Frequency & Mode Shape of Composite Drive Shaft

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**Abstract:**

In the recent days, the automotive industry is exploiting light weight material for structural components construction in order to obtain the reduction of the weight without decrease in vehicle quality and reliability. In this study, the different types of material like steel, Boron resin, E-glass /Epoxy taken for analysis. The advanced composite materials such as Boron, Graphite, Carbon, Kevlar and Glass with suitable resins are widely used nowadays for automotive and other industrial applications especially for rotor applications because of their high specific strength (strength/density) and high specific modulus (modulus/density). Present work is conducted to study modal analysis of composite drive shaft which is used for four wheel rear drive passenger cars. The Numerical study performed by Finite element analysis software ANSYS workbench. The shaft modeled in CATIA V5 & imported in ANSYS for analysis. The result obtained from theoretical, experimental & numerical study compared .

**Keywords:-** Composite Material ,Finite Element Analysis, Modal Analysis, Drive Shaft etc.

## I. INTRODUCTION

A large variety of fibers are available as reinforcement for composites. The desirable characteristics of most fibers are high strength, high stiffness, and relatively low density. Glass fibers are the most commonly used ones in low to medium performance composites because of their high tensile strength and low cost. In addition to the advantages of high strength (as well as high stiffness) and light weight, another advantage of the laminated composite plate is the controllability of the structural properties through changing the fiber orientation angles and the number of plies and selecting proper composite materials. In order to achieve the right combination of material properties and service performance, the dynamic behavior is the main point to be considered. To avoid the typical problems caused by vibrations, it is important to determine: a) natural frequency of the structure; b)the modal shapes to reinforce the most flexible regions or to locate the right positions where weight should be reduced or damping should be increased and c) the damping factors.In structural acoustics, recent work in sound transmission through laminated structures has shown that the fundamental frequency is a key parameter. The natural frequencies are sensitive to the orthotropic properties of composite plates and design-tailoring tools may help in controlling this fundamental frequency. The understanding of prediction models facilitates the development of such tools. Due to the advancement in computer aided data acquisition systems and instrumentation, Experimental Modal Analysis has become an extremely important tool in the hands of an experimentalist.This work presents an experimental study of modal testing of two different shafts steel & Kevlar composite . A program based on FEM is developed.. The experimental results have been compared with that obtained from the finite element analysis..

Variation of natural frequency with different parameter is studied.

Due to the requirement of high performance material in aerospace and marine structures, the prospect of future research of composite material, such as FRP (Fibre Reinforced Plastic) is very bright. Analysis of natural frequency and properties of composite components has started from 40 years ago. The natural frequencies and mode shapes of a number of Graphite/ Epoxy and Graphite/Epoxy-Aluminum plates and shells were experimentally determined by Crawly [1] (1979). Natural frequency and mode shape results compared with finite element method. Alam and Asani[2] (1986) studied the governing equations of motion for a laminated plate consisting of an arbitrary number of fiber-reinforced composite material layers have been derived using the variation principles. Each layer has been considered to be of a special orthotropic material with its directional elastic properties depending on the fiber orientation. A solution for simply supported rectangular plate is obtained in series summation form and the damping analysis is carried out by an application of the correspondence principle of linear viscoelasticity. Narita and Leissa[3] (1991) presented an analytical approach and accurate numerical results for the free vibration of cantilevered, symmetrically laminated rectangular plates. The natural frequencies are calculated for a wide range of parameters: e.g., composite material constants, fiber angles and stacking sequences.Qatu and Leissa[4] (1991) analyzed free vibrations of thin cantilevered laminated plates and shallow shells by Ritz method. Convergence studies are made for spherical circular cylindrical, hyperbolic, paraboloid shallow shells and for plates. Results are compared with experimental value and FEM. The effect of various parameters (material number of layers, fiber orientation, curvature) upon the frequencies is studied. A combined experimental and numerical study of the free vibration of composite GFRP plates has been carried out by Chakraborty, Mukhopadhyay and Mohanty[5] (2000). Modal testing has been conducted using impact excitation to determine the respective frequency response functions. FEM results, NISA package results compared with experimental results. Frequency response of composite shell-plate combinations was studied by G. Karthikeyan and A. Joseph Stanley[6] (2008).Natural Frequencies of fiber-reinforced composite circular plate and cylindrical shell are determined separately as well as for the shell-plate combination with clamped-clamped and simply supported boundary conditions by performing modal analysis on an eight noded, linear layered SHELL 99 element with six degrees of freedom per node Analysis of mechanical properties and free vibration response of composite laminates was done by Mr.M.Prabhakaran (2011). Yogesh Singh (2012) performed free vibration analysis of the laminated composite beam with various boundary conditions. This work presents an experimental study of testing of two different woven Composite laminates i) Glass/Epoxy and ii) Basalt/Epoxy composite plates using various boundary conditions. The experimental results have been compared with that obtained from the Composite pro software.

## II .THEORY AND FORMULATION:

### 2.1 Modal Analysis

Modal testing is the most widely used method. Modal testing is an experimental procedure in which the natural frequencies of a structure are determined by vibrating the structure with a known excitation. While it vibrates, the structure will behave in such a way that some of the frequencies will not respond at all or be highly attenuated, and some frequencies will be amplified in such a way that the only limiting factor is the energy available to sustain the vibration. These frequencies, where the structure resonates, are the natural frequencies of the structure.

By using Euler's Bernoulli beam theory,

$$\frac{\partial^2 w}{\partial t^2} + \frac{EI}{\rho}\frac{\partial^4 w}{\partial x^4} = 0 \qquad (1)$$

To find the response of the system one may use the variable separation method by using the following equation.

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$$w(x,t) = \varphi(x)q(t) \quad (2)$$

$\varphi(x)$ is known as the mode shape of the system and $q(t)$ is known as the time modulation. Now equation (1) reduces to

$$\varphi(x)\frac{\partial^2 q(t)}{\partial t^2} + \frac{EI}{\rho}\frac{\partial^4 \varphi(x)}{\partial x^4}q(t) \quad (3)$$

or

$$-\frac{EI}{\rho}\left(\frac{1}{\varphi(x)}\right)\left(\frac{\partial^4 \varphi(x)}{\partial x^4}\right) = \frac{1}{q}\left(\frac{\partial^2 q}{\partial t^2}\right) \quad (4)$$

Since the left side of equation (4) is independent of time t and the right side is independent of x the equality holds for all values of t and x. Hence each side must be a constant. As the right side term equals to a constant implies that the acceleration $\left(\frac{\partial^2 q}{\partial t^2}\right)$ is Proportional to displacement $q(t)$, one may take the proportionality constant equal to $-\omega^2$ to have simple harmonic motion in the system. If one takes a positive constant, the response will grow exponentially and make the system unstable. Hence one may write equation (4) as,

$$-\frac{EI}{\rho}\left(\frac{1}{\varphi(x)}\right)\left(\frac{\partial^4 \varphi(x)}{\partial x^4}\right) = \frac{1}{q}\left(\frac{\partial^2 q}{\partial t^2}\right) = -\omega^2 \quad (5)$$

Hence,

$$\frac{d^2 q}{dt^2} + \omega^2 q = 0 \quad (6)$$

And

$$\frac{\partial^4 \varphi(x)}{\partial x^4} - \frac{\rho\omega^2}{EI}\varphi(x) = 0 \quad (7)$$

Taking, $\beta^4 = \frac{\rho\omega^2}{EI}$

The above equation can be written as

$$\frac{\partial^4 \varphi(x)}{\partial x^4} - \beta^4\, \varphi(x) = 0 \quad (8)$$

The solution of equation (4.16) and (4.18) can be given by

$$q(t) = C_1 \sin \omega t + C_2 \cos \omega t \quad (9)$$

$$\varphi(x) = A\sinh(\beta x) + B\cosh(\beta x) + C\sin(\beta x) + D\cos(\beta x)$$

Hence,

$$w(x,t) = (A\sinh\beta x + B\cosh\beta x + C\sin\beta x + D\cos\beta x)(C_1 \sin\omega t + C_2 \cos\omega t)$$

Here constants $C_1$ and $C_2$ can be obtained from the initial conditions and constants A,B,C,D can be obtained from the boundary conditions. Let us now determine the mode Shape of cantilever beam.

In case of cantilever beam the boundary conditions are : At left end i.e.,

$$at\ x = 0 \quad w(x,t) = 0 \quad (displacement = 0)$$

$$\frac{\partial w(x,t)}{\partial x} = 0 \quad (Slope = 0)$$

At the free end i.e.,

$$at\ x = L \quad \frac{\partial^2 w(x,t)}{\partial x^2} = 0 \quad (Bending\ Moment = 0)$$

$$\frac{\partial^3 w(x,t)}{\partial x^3} = 0 \quad (Shear\ Force = 0)$$

$$\text{At } x = 0 \quad \varphi(x) = 0 \text{ and } \frac{\partial \varphi(x)}{\partial x} = 0 \quad (11)$$

$$\text{At } x = L \quad \frac{\partial^2 \varphi(x)}{\partial x^2} = 0 \text{ and } \frac{\partial^3 \varphi(x,t)}{\partial x^3} = 0$$

(12)

Substituting these boundary conditions in the general solution,

$$\phi(x) = A\cosh\beta x + B\sinh\beta x + C\cos\beta x + D\sin\beta x \quad (13)$$

$$A = -C \text{ and } B = -D \quad (14)$$

$$\phi(x) = A(\cosh\beta x - \cos\beta x) + B(\sinh\beta x - \sin\beta x) \quad (15)$$

one may have,

$$\frac{A}{B} = -\frac{\sinh\beta l + \sin\beta l}{\cosh\beta l + \cos\beta l} = -\frac{\cosh\beta l + \cos\beta l}{\sinh\beta l - \sin\beta l} \quad (16)$$

or,

$$\cos\beta l \cosh\beta l = -1 \quad (17)$$

and the root of the equation is, $\beta l = \frac{(2n-1)\pi}{2}$

Hence one may solve the frequency equation $\cos\beta l \cosh\beta l = -1$ to obtain frequencies of different modes. For the first four modes the values of $\beta l$ are calculated as 1.875, 4.694, 7.85 and 10.99. For a simple elastic beam problem with uniform cross-sectional area, a well-known natural frequency can be calculated by

$$\omega = (\beta l)^2 \sqrt{E_b I / \rho A L_b{}^4}$$

in rad/sec2 (18)

While natural frequency in Hz,

$$f_n = \frac{\omega}{2\pi} \text{ in Hz} \quad (19)$$

Where, A and $L_b$ are the area of cross-section and the length of the flexible beam, respectively, $E_b$ is the Young's Modulus and I is the moment of inertia of beam.

## III. METHODOLOGY

Experimental nodal analysis of shaft is carried out by using FFT analyzer and natural frequencies of intermediate shaft were found out by using frequency domain and time domain analysis.

Finite Element Method is used to do analysis of the intermediate shaft. It is an approximation method where the object is divided into number of elements and the stiffness matrix is prepared. These matrices are solved to get results. In the next phase results obtained from experimental analysis and FEM analyses are verified.

From the comparison, significant difference between FEM results and results of experimental analysis is acceptable. With this procedure FEM analysis of intermediate shaft is carried out with change in geometrical and material parameters.

3.1. Steps involved in Finite Element Method as follows:

Step 1: Choose the Element

Step 2: Define Shape Function Matrix

Step 3: Derive Element stiffness Matrix

Step 4: Assemble Element stiffness Matrices.

Step 5: Set up Mass Matrix and Eigen Value Problem.

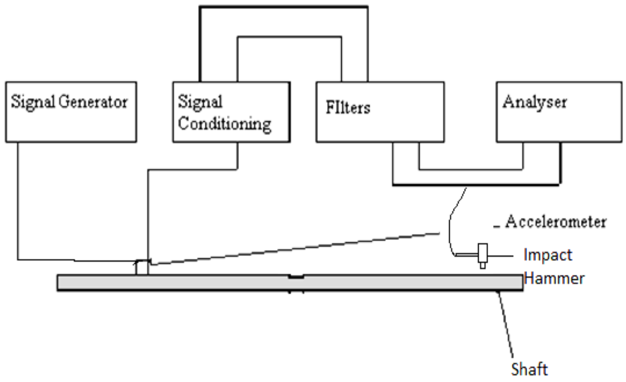


Fig.1 Setup of Model testing.:

3.2. Setup and Procedure of Modal Analysis

The connections of FFT analyzer, laptop, transducers, modal hammer, and cables to the system were done as per the guidance manual. The pulse lab shop version-9.0 software key was inserted to the port of laptop. The shaft was excited in a selected point by means of a small impact hammer (Model 2302-5), preferably at the fixed end. The input signals captured by a force transducer, fixed on the hammer. The resulting vibrations of the shaft in a select point are measured by an accelerometer. The accelerometer (B&K, Type 4507) was mounted on the shaft to the free end by means of bees wax. The signal was then subsequently input to the second channel of the analyzer, where its frequency spectrum was also obtained. The response point was kept fixed at a particular point and the location of excitation was varied throughout the shaft. Both input and output signals are investigated by means of spectrum-analyzer (Bruel & kjaer) and resulting frequency response functions are transmitted to a computer for modal parameter extraction. The output from the analyzer was displayed on the analyzer screen by using pulse software. Various forms of Frequency Response Functions (FRF) are directly measured.

However, the present work represents only the natural frequencies and mode shape of shaft. The spectrum analyzer provided facilities to record all the data displayed on the screen to a personal computer's hard disk or laptop and the necessary software. Normally in order to determine the natural frequencies of a system, recording the response spectrum for an excitation, where the excitation level is constant over the frequency band under consideration will suffice. However, it was observed, from the auto-spectrum of the excitation force, that it was not possible to maintain such uniform excitation in case of composite shaft.So, test should be within linear range. The hammer excitation method is fast and simple method. A sharp impact pulse corresponds to a large frequency domain. Unfortunately, since the energy of the force pulse is limited, the method has poor signal to noise characteristics, but the noise can be minimized by using an adequate weighting function. Nevertheless, the composite shaft showed very rapidly to have frequencies above 2000Hz, which are difficult to excite with enough energy by means of a hammer.

Fig.2. Exciting Hammer

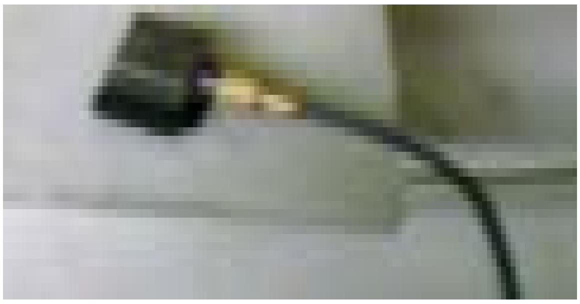

Fig.3. Accelerometer

Fig.4 Bruel & Kajer FFT Analyzer

Fig.5. Vibration testing Setup Connected to Laptop

The natural frequency in Hz is obtained theoretically by calculating the stiffness of the specimen and comparisons of results between the theoretical, experimental and software result at various boundary conditions are tabulated. Due to limitations (voltage limit) in the experimental set up the results of higher modes could not be interpreted. In experimental result, natural mode of frequency varies within a range with respect to software result. Un-damped natural frequency is considered in the program and damping was present in the system. So, the natural frequency from the experiment should less than the actual value. But the difference between both the results is reasonable.

3.3. Assumptions to perform an experimental modal analysis.

1. The structure is assumed to be linear i.e. the response of the structure to any combination of forces, simultaneously applied, is the sum of the individual responses to each of the forces acting alone.

2. The structure is time invariant, i.e. the parameters that are to be determined are constants. In general, a system which is time invariant has components that's mass, stiffness, or damping depend on factors that are not measured or not included in the model.

3. The structure obeys Maxwell's reciprocity, i. e. a force applied at degree of freedom p causes a response at degree of freedom q that is the same as the response at degree of freedom p caused by the same force applied at degree of freedom q.

4. The structure is observable; i.e. the input output measurements that are made enough information to generate an adequate behavioral model of the structure.

## IV. ANALYSIS METHODOLOGY

To meet the stringent design requirement, a shaft has to be design. In this work, we will compare the conventional steel shaft with the composite shaft with various material Kevlar analyzed at +/- 45 degrees ply orientation. The material properties of all material considered from design consideration. The analysis carried out using ANSYS 11.0 and work bench. The first Steel (SM45C) which is to be used for reference purpose.

Composite material: Kevlar

The methodology as per follows:

The specification of shaft for its loading and operating conditions. Obtaining 2D Drawing and loading conditions from design specifications. Obtaining boundary conditions required for analysis. Preparation 3D Finite element model Using HYPERMESH. Hyper mesh model analyses in ANSYS 11.

When an elastic system free from external forces is disturbed from its equilibrium position it vibrates under the influence of inherent forces and is said to be in the state of free vibration. It will vibrate at its natural frequency and its amplitude will gradually become smaller with time due to energy being dissipated by motion. The main parameters of interest in free vibration are natural frequency and the mode shapes. The natural frequencies and the mode shapes are important parameters in the design of a structure for dynamic loading conditions.

Modal analysis is used to determine the vibration characteristics such as natural frequencies and mode shapes of a structure or a machine component while it is being designed. It can also be a starting point for another more detailed analysis such as a transient dynamic analysis, a harmonic response analysis or a spectrum analysis. The natural frequency depends on the diameter of the shaft, thickness of the hollow shaft, specific stiffness and the length.

### 4.1 Modeling in CATIA:

FE Model Creation the 3D FE model for drive shaft was created by using FE modeling software CATIA V5.

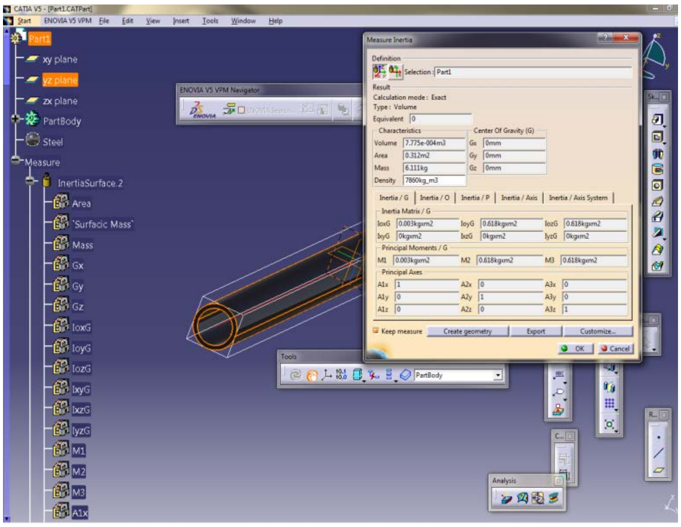


Fig .6 3D Model in CATIA V5

### 4.2 Define Element Type and Material/Geometric Properties

Solid 90 is the element type used in the FE analysis. It is mainly used for layered applications of a structural shell model. Solid90 allows up to 250 layers. If more than 250 layers are required, a user-input constitutive matrix is available. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes.

### 4.3. 3D MODEL of Shaft Imported in ANSYS 11.

The 3D model created in CATIA V5 imported in ANSYS 11 for the analysis

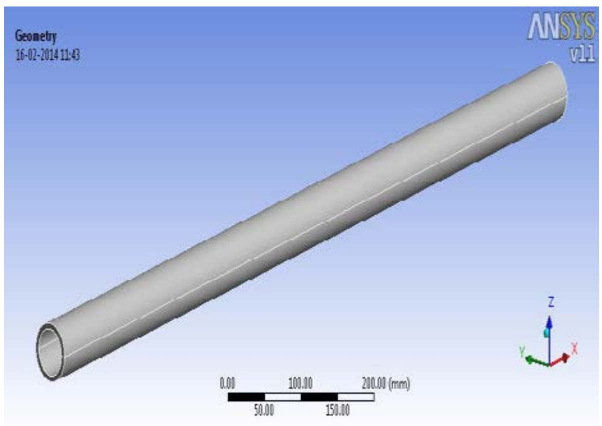


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Fig .7 3D Model in ANSYS 11

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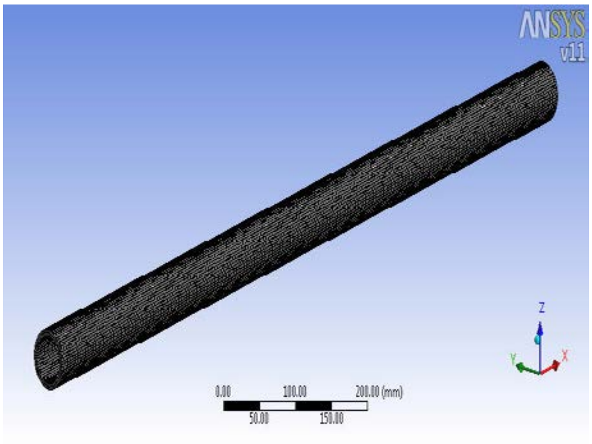


Fig.8 Meshing: 3D MODEL

## V. MODE SHAHPES OF STEEL & KEVLAR SHAFT:

5.1 ANSYS Result of Mode Shape of Steel Shaft

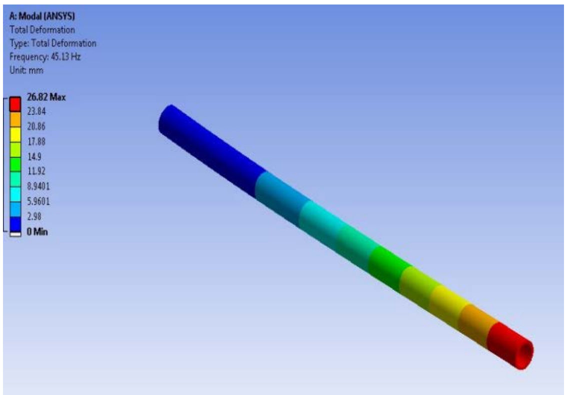


Fig .9 First Mode Shape of Steel Shaft

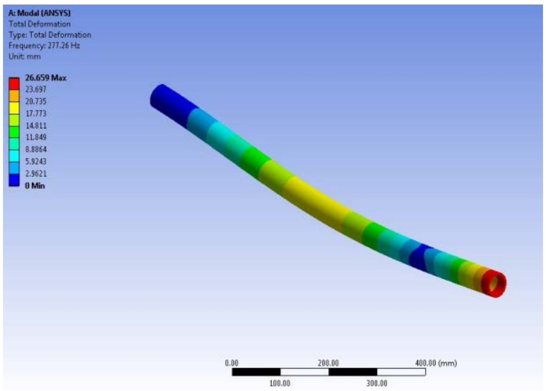


Fig .10 Second Mode Shape of Steel Shaft

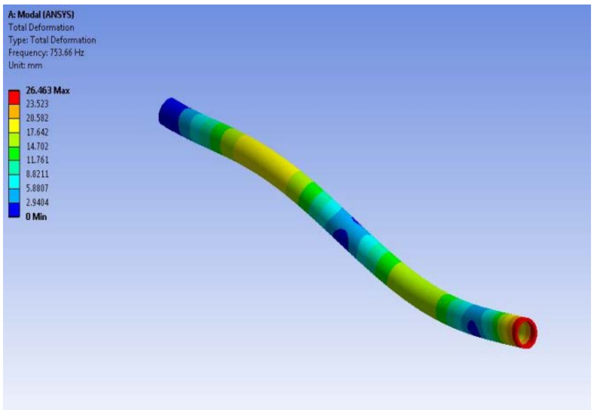


Fig .11 Third Mode Shape of Steel Shaft

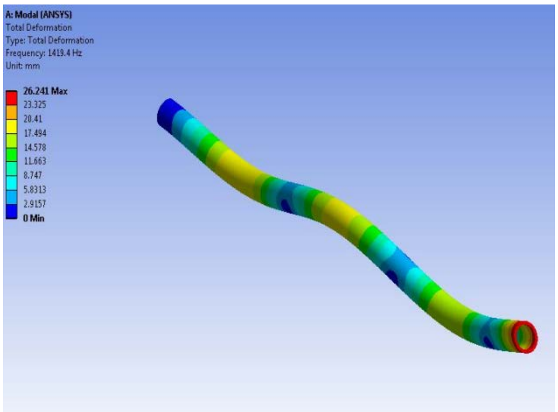


Fig .12 Fourth Mode Shape of Steel Shaft

5.2 ANSYS Result of Natural Frequency for Kevlar Shaft

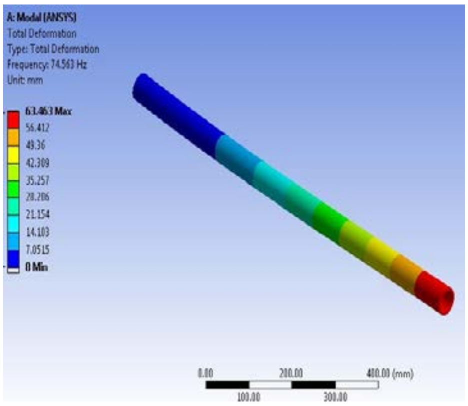


Fig .13. First Mode Shape of Kevlar Shaft

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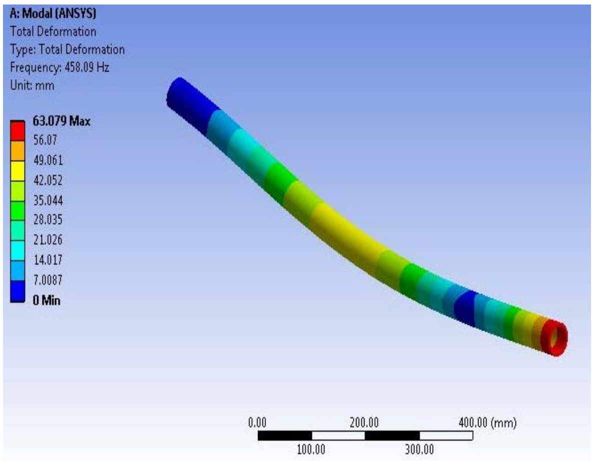


Fig .14 Second Mode Shape of Kevlar Shaft

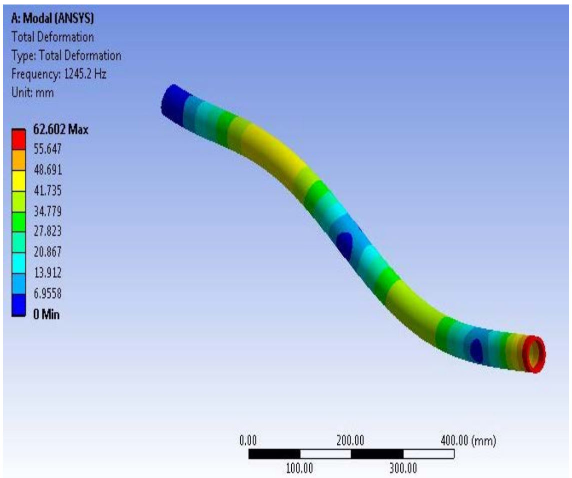


Fig .15 Third Mode Shape of Kevlar Shaft

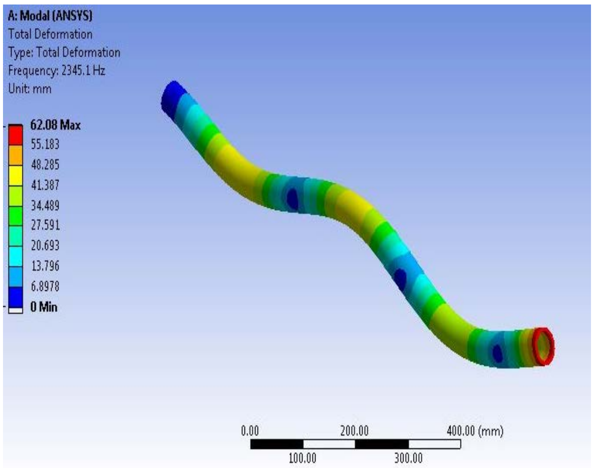


Fig .16 Fourth Mode Shape of Kevlar Shaft

## VI. RESULT & DISCUSSION

6.1 Result of Natural Frequency of Steel

Table 2. Result of Natural Frequency of First Four mode of Steel & Composite Material

| Mode | Natural Frequency in Hz | |
|---|---|---|
| | Steel | Kevlar |
| I | 45.83 | 75.48 |
| II | 288.29 | 474.78 |
| III | 807.66 | 1330.11 |
| IV | 1583.01 | 2607.00 |

Table 3. Result of Natural Frequency of Steel shaft First Four Modes

| Mode | Natural Frequency in Hz (Steel ) | | |
|---|---|---|---|
| | Theoretical | Numerical | Experimental |
| Ist | 45.83 | 45.5 | 43.55 |
| IInd | 288.29 | 277.26 | 274.98 |
| IIIrd | 807.66 | 753.66 | 788.05 |
| IVth | 1583.01 | 1419.4 | 1414.7 |

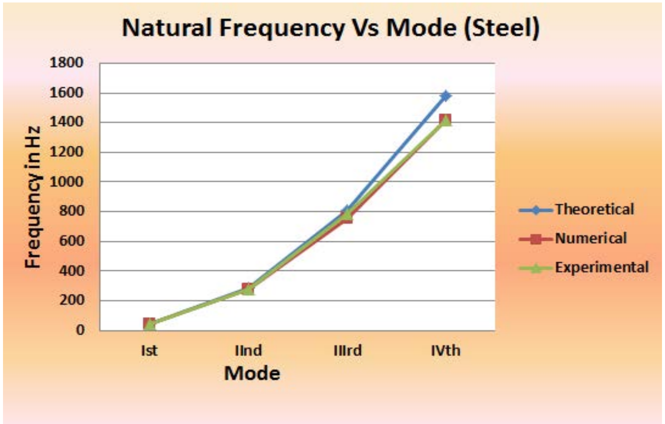


Fig.17. Graph of mode shape of steel

6.2 Result of Natural Frequency of Kevlar

Table 4. Result of Natural Frequency of Kevlar Material at First Four Modes

| Mode | Natural Frequency in Hz (Kevlar ) | | |
|---|---|---|---|
| | Theoretical | Numerical | Experimental |
| I st | 75.48 | 74.56 | 73.75 |
| II nd | 474.78 | 458.09 | 456.09 |
| III rd | 1330.11 | 1245.2 | 1240.01 |
| IV th | 2607 | 2345.01 | 2338.23 |

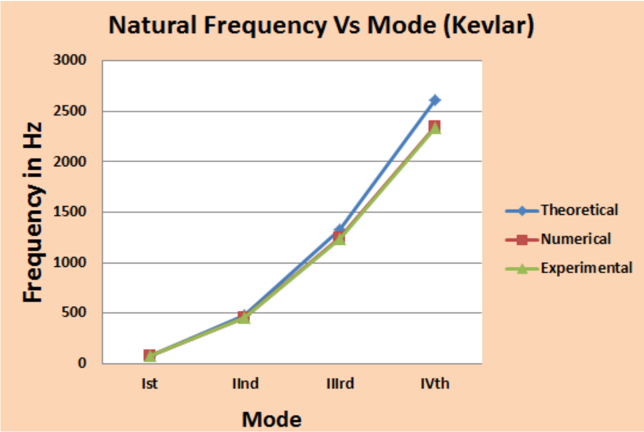


Fig18 Natural Frequency Vs Modes (Kevlar)

Fig.18 shows the nature of graph of natural frequency of Kevlar material .the plot is obtained by taking modes on X-axis and Natural frequency in Hz on Y axis. The results of natural frequency of Kevlar obtained from theoretical, numerical and experiment are compared at different modes and observed the behavior of material same in three method. It has been observed that Natural frequency by three methods is very close to each other.

## VII.CONCLUSION

The natural frequencies of steel shaft & composite shaft have been reported with boundary condition, It is found that natural frequency increases with increase in mode of vibration from software result. Natural Frequency of Kevlar composite shaft is more than that of steel material. In case of replacement of steel material, Kevlar composite is suitable for drive shaft.

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