

# Generalized The General Relativity Using Generalized Lorentz Transformation

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## Abstract

Generalized non linear Lorentz transformation is utilized to derive modified special relativistic space – time equations . The equations are found for particles moving in a potential field . The transformation is based on the usual Newtonian relation displacement in terms of initial velocity for constant acceleration . The displacement in all frames are expressed in terms of spatial coordinate time and potential per unit mass . The expressions for Lorentz transformation parameter , space and time reduces to that of ordinary special relativity in the absence of field . The energy relation reduces to special relativity for no field and to Newtonian one for low velocity .

**Key words : generalized Lorentz transformation , generalized special relativity , relativistic energy , Newton energy .**

## 1- Introduction

Einstein special theory of relativity SR is one of the most important physical theories , since it make radical modification to the concept of space and time . According to Newton laws of motion , the concepts of space , time and mass are absolute in the sense that these quantities have the same value in all frame of references. Unfortunately this concept is violated by Michelson – Morley experiment . In this experiment it was shown that the speed of light in vacuum is constant and is independent of the motion of the observer or the source .

This motivates Einstein to propose his relativity theory which is known as special relativity (SR) . In (SR) theory the space and time depends on the relative motion of the frame of references which moves with constant velocity to each other . Einstein (SR) succeeded in explaining the constancy of light speed

The SR also explains the time dilation for fast moving decaying meson, it also explains successfully pair production and annihilation phenomena .

However SR suffers from noticeable setbacks . First of all it does not satisfy correspondence principle . This is because in the Newtonian limit, its energy

expression does not give a potential term . The lack of potential term , in SR energy relation is in direct conflict with common sense and physical theories .

For instance , if one have an electron moving in free space with a certain velocity and another one with the same velocity moving in an electromagnetic field , SR energy for both is the same . This does not agree with common sense and quantum mechanics which predicts different energy values. The SR theory cannot explain also time dilation by gravitation beside the photon red shift due to the gravity field .

This draw backs motivates some physicists to search for an alternative theory that keeps the same framework of SR but accounts for the effect of potential . These attempts succeeded in curing SR defects but some of them deals with the weak field , while others looks complex .

This motivates us to search for simple alternative that keeps SR framework and holds for all fields . This is done in section (2) . sections (3) and (4) are devoted for discussion and conclusion .

## 2- New derivation of general special relativity

According to Newton's second law of motion the force  $F$  can be expressed in terms of the mass  $m$  and acceleration  $a$  as

$$F = ma \tag{1}$$

Thus the potential  $V$  is given by

$$V = m\Phi = \int F \cdot dx = \int madx = ma \int dx = ma x$$

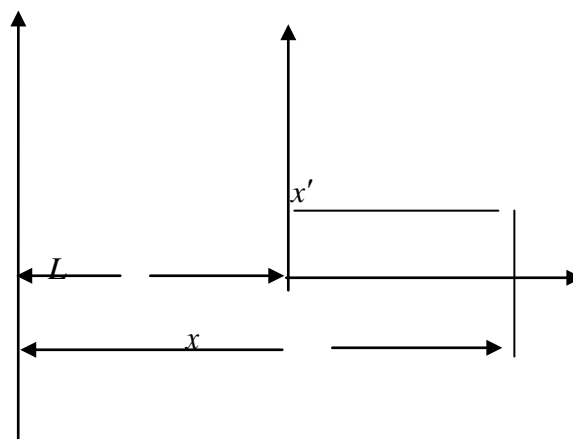
Where  $\Phi$  is defined as the potential per unit mass . Thus


$$m\Phi = ma x$$

Hence

$$\Phi = a x \tag{2}$$

Let two reference frames  $(x, t)$  and  $(x', t')$  moves with initial velocity  $v_0$  and constant acceleration  $a$  with respect to each others . Thus the



distance between their origin 

at any time  $t$  is given by

$$L = v_0 t + \frac{1}{2} a t^2$$

i-e

$$L = v_0 t + \frac{1}{2} \frac{a x}{x} t^2 \tag{3}$$

Using equation (3) one can rewrite equation (4) as

$$L = v_0 t + \frac{\Phi}{2x} t^2 \tag{4}$$

This represent the length as measured by the observer  $O$  .assuming  $v_0$  and  $\Phi$  to be the same for all observers , the length for observer  $O'$  is given by

$$L' = v_0 t' + \frac{1}{2} a t'^2 = v_0 t' + \frac{1}{2} a \frac{x'}{x'} t'^2$$

$$L' = v_0 t' + \frac{1}{2} \frac{a \Phi}{x'} t'^2 \tag{5}$$

The space – time coordinate in two frames can be described by Lorentz transformation . According to Lorentz transformation

$$x' = \gamma (x + L) = \gamma (x + v_0 t + \frac{\Phi}{2x} t^2) \tag{6}$$

$$x = \gamma (x' + L') = \gamma (x' + v_0 t' + \frac{\Phi}{2x'} t'^2) \tag{7}$$

Consider now a source of light that emits light pulse when the two frames origin coincide , i.e

$$t = t' = 0$$

The light pulse which are emitted travels distances  $x'$  and  $x$  respectively , where

$$x = ct \quad \therefore \quad x' = ct' \tag{8}$$

Substituting (8) in (6) yields

$$ct' = \gamma (ct + v_0 t + \frac{\Phi}{2ct} t^2)$$

$$t' = \gamma \left[ \left(1 + \frac{v_0}{c}\right)t + \frac{\Phi}{2c^2}t \right]$$

$$t' = \gamma \left[ 1 + \frac{v_0}{c} + \frac{\Phi}{2c^2} \right] t \tag{9}$$

Inserting also (8) in (7) gives

$$ct = \gamma \left[ ct' - v_0 t - \frac{\Phi}{2ct} t'^2 \right]$$

$$t = \gamma \left[ 1 - \frac{v_0}{c} - \frac{\Phi}{2c^2} \right] t' \tag{10}$$

From (9) and (10)

$$t = \gamma^2 \left[ 1 + \frac{v_0}{c} + \frac{\Phi}{2c^2} \right] \left[ 1 - \frac{v_0}{c} - \frac{\Phi}{2c^2} \right] t$$

Therefore

$$\gamma = \frac{1}{\sqrt{\left(1 + \frac{v_0}{c} + \frac{\Phi}{2c^2}\right)\left(1 - \frac{v_0}{c} - \frac{\Phi}{2c^2}\right)}} \tag{11}$$

It is very interesting to note that when no field exists

$$\Phi = 0 \tag{12}$$

The factor  $\gamma$  in equation (11) reduces to

$$\gamma = \frac{1}{\sqrt{\left(1 + \frac{v_0}{c} + \frac{\Phi}{2c^2}\right)\left(1 - \frac{v_0}{c} - \frac{\Phi}{2c^2}\right)}} = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}} \tag{13}$$

Which is ordinary SR relation . A direct insertion of equation (11) in (6) and (7) yields

$$x' = \frac{\left(x + v_0 t + \frac{\Phi}{2c} t^2\right)}{\sqrt{\left(1 + \frac{v_0}{c} + \frac{\Phi}{2c^2}\right)\left(1 - \frac{v_0}{c} - \frac{\Phi}{2c^2}\right)}} \tag{14}$$

$$x = \frac{(x' - v_0 t' - \frac{\Phi}{2x'} t^2)}{\sqrt{(1 + \frac{v_0}{c} + \frac{\Phi}{2c^2})(1 - \frac{v_0}{c} - \frac{\Phi}{2c^2})}} \quad (15)$$

In the absence of fields again (14) and (15) reduces to that of SR .

The expression for energy is given by

$$E = mc^2 = \gamma m_0 c^2 \quad (16)$$

Inserting (11) in (16) yields

$$E = \frac{m_0 c^2}{\sqrt{(1 + \frac{v_0}{c} + \frac{\Phi}{2c^2})(1 - \frac{v_0}{c} - \frac{\Phi}{2c^2})}} \quad (17)$$

When no field exists the energy relation reduces to

$$E = \frac{m_0 c^2}{(1 - \frac{v_0^2}{c^2})} \quad (18)$$

Let now

$$x = \frac{v_0}{c} + \frac{\Phi}{2c^2} \quad (19)$$

Assuming

$$\frac{\Phi}{c^2} \ll \frac{v_0}{c} \quad (20)$$

Equation (17) becomes

$$E = m_0 c^2 (1 - \frac{v_0^2}{c^2})^{-\frac{1}{2}} \quad (21)$$

But for low speed

$$\frac{v_0}{c} \ll 1 \quad (22)$$

Thus

$$E = m_0 c^2 \left[ 1 + \frac{1}{2} \frac{v_0^2}{c^2} \right] \quad (23)$$

But according to Newton's laws

$$v^2 = v_0^2 + 2\Phi \quad (24)$$

Thus

$$E = m_0 c^2 \left[ 1 + \frac{1}{2} \frac{v^2}{c^2} - \frac{\Phi}{c^2} \right] = m_0 c^2 + \frac{1}{2} m_0 v^2 + m_0 \Phi - m_0 c^2 + T + V \quad (25)$$

Where

$$\begin{aligned} V &= -m_0 \Phi \\ T &= \frac{1}{2} m_0 v^2 \end{aligned} \quad (26)$$

Which is the usual Newton energy relation beside rest mass term .

### 3- Discussion

Non linear Generalized Lorentz transformation in equations (6) and (7) are used to find new generalized SR . This new transformation is non linear in t . This transformations deals with particles moving with constant initial velocity and constant acceleration under the action of a potential field . The spatial displacement is found in terms of initial velocity and potential per unit mass . By assuming the speed of light is constant in all frames moving in a potential field, the transformation coefficient is found to depend on as well as as shown by equation (11) . The new space and time transformations are shown in equations (6) and (7) . It is very interesting to note the expressions for , and reduces to that of SR when no field exists as shown by equations (13) , (6) , (7) and (18) respectively . Unlike SR , which recognize rest mass and kinetic energy only , the generalized energy expression (17) recognize potential and reduces to Newton energy relation , as equation (25) indicates , with kinetic and potential term , beside rest mass term .

### 4- Conclusion

The generalized Lorentz transformation and generalized SR can describe successfully the motion of particles in a field . It reduces to SR when no field exist . It also satisfy Newtonian limit by consisting of kinetic and potential term .

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