

# Low Cost Algorithm to Estimate Received Bit using Maximum Likelihood Approach

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## Abstract

Maximum Likelihood (ML) Approach is motivating application for pulse detection purposes. Compared to many other methods, it can be considered simple and easy to implement. In this study, Maximum Likelihood is used to estimate received noisy pulse (bit). The novelty of the algorithm comes from its low cost and obvious robustness, as it uses mathematically derived probability density function (pdf) of multiple terms. Each term expresses one of the pulse’s possibilities. The algorithm substitutes each possibility in its own term. Therefore, the case that causes maximum value is chosen to be the estimated binary value of the received pulse. The algorithm is tested under the effect of the normal distributed noise (Gaussian), with two different pulse mapping schemes, and provided zero bit-error rate at relatively low SNR values.

**Keywords:** Maximum-likelihood estimation, noise, bit error, Gaussian, white noise.

## 1. Introduction

In many industrial applications, such as temperature threshold measurements, gas and/or liquid effective pressure monitoring, and alarm systems, the design depends on some sensors readings [1]. The reading usually is an indication for exceeding a certain threshold, where the sensor’s output produces a positive pulse (logic one) whenever the measurement exceeds some threshold, and negative (or zero) pulse when the output measurement goes below it [2] Sensor’s output is sent via media which could be a wireless channel, where some type of noise is added and finally the sum is received by PC station, or FC (Fusion Center), Figure1 shows general block diagram. This research proposes an algorithm can be installed in the PC station and estimates the received binary word. The algorithm is based on a derived pdf function [3] for the received random variable at the channel output, where two values are calculated, one value is determined by substituting the positive peak that represents the logic one in the pdf, while the other value is obtained, by substituting the negative peak that represents the logic zero. The algorithm simply compares between the two values, and chooses the bit, which makes the greater pdf. The results show that the algorithm is reliable and could be

used with low bit error estimation for different noise parameters, and under relatively low Signal to Noise Ratio (SNR).

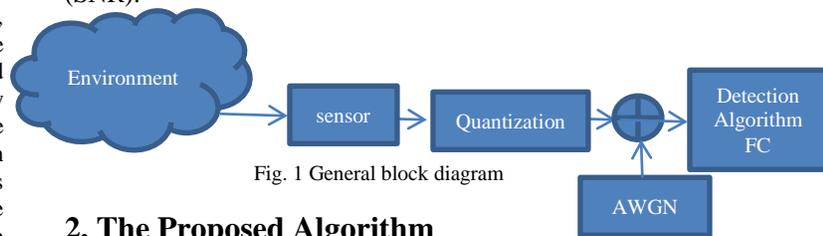


Fig. 1 General block diagram

## 2. The Proposed Algorithm

Consider a network of a sensor takes measurement of two probabilities (P1, P2), and quantizes them as logic 1 (positive peak) and logic 0 (negative peak). The sensor’s output is the input of channel characterized by adding white Gaussian noise. The channel delivers the result to Fusion Center, where the received pulse is to be estimated. The pdf of the channel input is defined as [3]:

$$f_{\alpha}(\alpha) = \sum_{j=1}^2 P_j \delta(\alpha - \lambda_j) = P_1 \delta(\alpha - \lambda_1) + P_2 \delta(\alpha - \lambda_2) \quad (1)$$

Where  $\alpha$  is a random variable of the generalized pdf  $f_{\alpha}(\alpha)$ , and  $\lambda_j$  is a reproduction point of the quantization level. P1 and P2 values depend on the measurement environment conditions, in this study each one is assumed equal to 1/2. Therefore equation (1) becomes [3]:

$$f_{\alpha}(\alpha) = \frac{1}{2} \delta(\alpha - \lambda_1) + \frac{1}{2} \delta(\alpha - \lambda_2) \quad (2)$$

The channel noise is Gaussian random variable  $v$ , and its pdf is [3]:

$$f_v(v) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(v-m)^2}{2\sigma^2}} \quad (3)$$

As mentioned above the random variable  $v$  is added to the random variable  $\alpha$ . Denote the result of the addition by  $z$ , we have:

$$z = \alpha + v \quad (4)$$

The problem is to estimate the received variable  $z$ .  $\alpha$  is sent over the channel as a positive or negative pulse to represent the logic zero or the logic one. Generalized case is considered here by representing more than two bits "cases" by using different pulse amplitudes.

### 3. Proposed Solution

This work adopts Maximum Likelihood (ML) approach to solve the problem of estimating the received bit via media channel.

The proposed solution starts with finding a pdf for the received variable at the channel's output  $z$ .  $f_z(z)$  can be obtained as follows [3]:

$$f_z(z) = f_v(z) * f_\alpha(z) \quad (5)$$

Where \* stands for convolution [3].

Therefore:

$$\begin{aligned} f_z(z) &= \frac{1}{2\sqrt{2\pi\sigma^2}} e^{-\frac{(z-m)^2}{2\sigma^2}} * [\delta(z - \lambda_1) + \delta(z - \lambda_2)] \\ &= \frac{1}{2\sqrt{2\pi\sigma^2}} \left[ e^{-\frac{(z-m-\lambda_1)^2}{2\sigma^2}} + e^{-\frac{(z-m-\lambda_2)^2}{2\sigma^2}} \right] \end{aligned} \quad (6)$$

Generalized pdf can be evaluated by assuming  $n$  reproduction quantization levels:

$$f_z(z) = \frac{1}{2\sqrt{2\pi\sigma^2}} \left[ e^{-\frac{(z-m-\lambda_1)^2}{2\sigma^2}} + e^{-\frac{(z-m-\lambda_2)^2}{2\sigma^2}} + \dots + e^{-\frac{(z-m-\lambda_n)^2}{2\sigma^2}} \right] \quad (7)$$

The proposal suggests to substitute  $z$  in each term individually and to choose the term which makes the maximum value.

#### 3.1 One Threshold Scheme

In this case, the algorithm substitutes the received value in the pdf formula to estimate the received bit, which is logic zero if  $f_z(z, \lambda_1) > f_z(z, \lambda_2)$  and logic one otherwise.

$$f_z(z, \lambda_1) = e^{-\frac{(z-m-\lambda_1)^2}{2\sigma^2}}, f_z(z, \lambda_2) = e^{-\frac{(z-m-\lambda_2)^2}{2\sigma^2}} \quad (8)$$

Fig. 1 presents the results.

#### 3.2 Three-Threshold Scheme

Here, we considered four binary words to be sent over the channel. Each word represents a reproduction point.

$$\begin{aligned} f_z(z, \lambda_1) &= e^{-\frac{(z-m-\lambda_1)^2}{2\sigma^2}}, f_z(z, \lambda_2) = e^{-\frac{(z-m-\lambda_2)^2}{2\sigma^2}} \\ , f_z(z, \lambda_3) &= e^{-\frac{(z-m-\lambda_3)^2}{2\sigma^2}}, f_z(z, \lambda_4) = e^{-\frac{(z-m-\lambda_4)^2}{2\sigma^2}} \end{aligned} \quad (9)$$

Two mapping methodologies are applied to examine the algorithm, first one uses only one pulse to modulate each binary word, therefore, four different pulses (in phase and amplitude) are used. While the second method uses two pulses (positive and negative) both are used to represent one binary word.

##### 3.2.1 One-Pulse Mapping

Four pulses with values of 5, -5, 2, and -2 have been used to simulate the algorithm. Million binary words are sent under various SNR values. The results are illustrated in Fig 2.

##### 3.2.2 Two-Pulse Mapping

Four vectors with values (4 4), (-4 4), (4 -4), and (-4 -4) are adopted to modulate the four binary words, each element in the vector is substituted in its term in the pdf function, the algorithm adds the results of each vector and chooses the vector of the maximum result as the estimated word. Million words have been examined, fig 3 demonstrates the results.

Attached appendix gives flow chart for each scheme.

### 4. Numerical Results

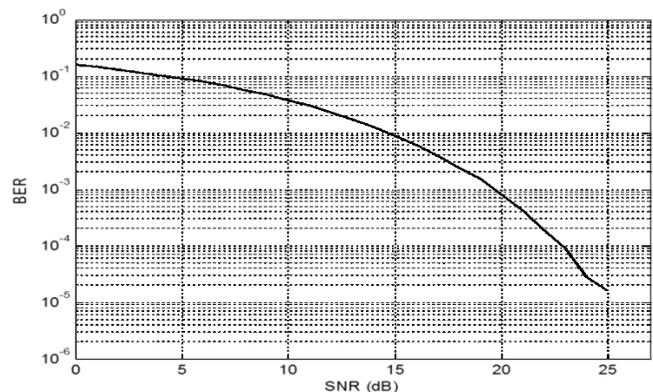


Fig. 1 One-threshold scheme.

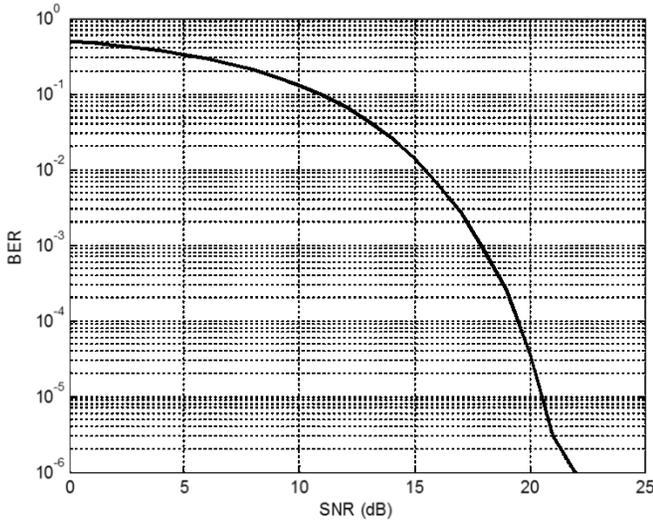


Fig. 2 Three-threshold scheme, one-pulse mapping.

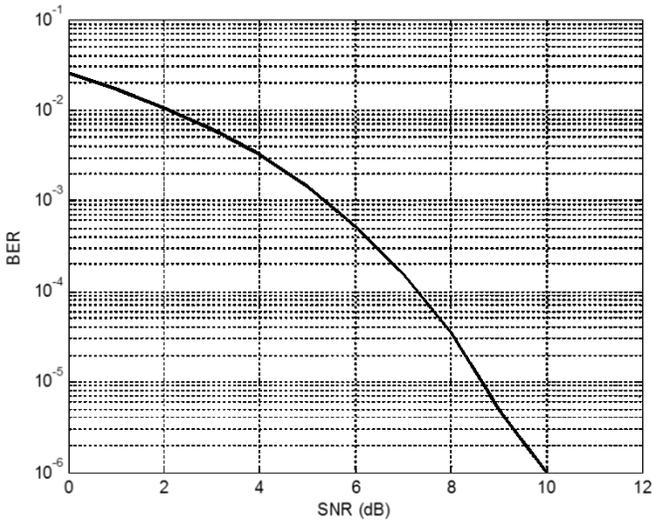


Fig. 3 Three-threshold scheme, two-pulse mapping.

### 4. Conclusions

Algorithm based on Maximum Likelihood Approach is built and simulated. Complex *pdf* function is derived and split into many terms; each term has been handled individually, as every single one is caused by one of the possibilities of the sent pulse. The results show that with simple structure, zero BER could be achieved under relative low SNR environments. Hardware structure could be easily implemented, as only one procedure is performed, which is the comparison between the terms of the already math derived pdf equation. The best result is documented for the “two-pulse mapping” scheme, which uses the orthogonal modulation.

### Appendix

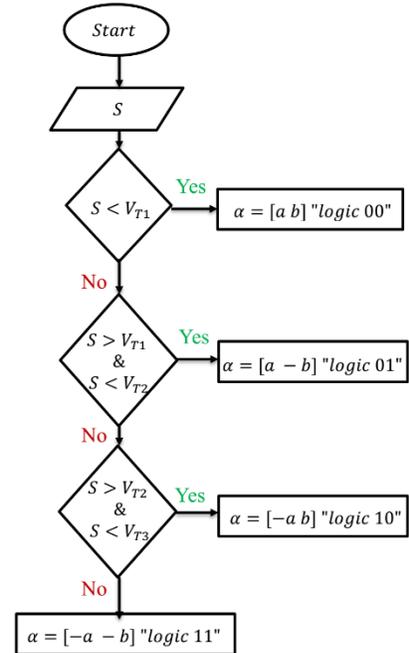


Fig.4 Two-bit scheme algorithm (two-dimension method), transmitter

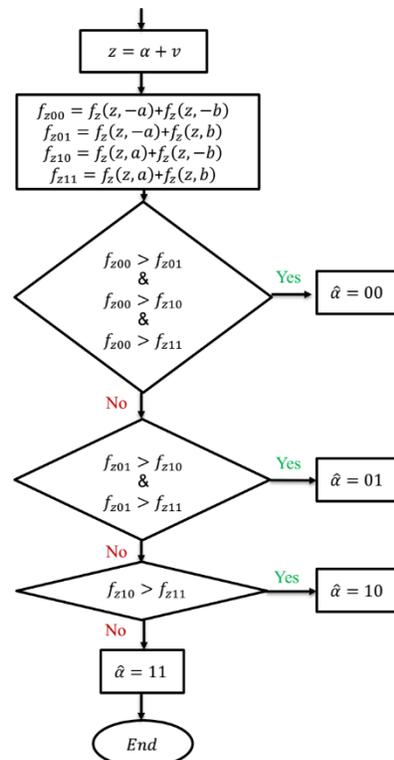


Fig.5 Two-bit scheme algorithm (two-dimension method), receiver

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