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Modelling, Simulation & Control Design of Lateral & Longitudinal Wheel Reactobot

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Abstract

The aim of this research work is to utilize the design a controller that would work in theory and then figure out how to translate a mathematical representation of that controller into my current working model. After thinking for some days, It was decided the classic control problem of the one wheel balancing bot was the perfect problem to do this with. It was already dealt with this problem in, Feedback Control Systems, in two of the experiments, but the extent of the design was to come up with a mathematical model of a controller and then upload it into an existing system to test the controller. On the other hand, this project will be the complete system design including all of the mechanical, hardware, and software design that can be done at optimal cost.

Keywords: *Reactobot, Omnibot, Single Wheel Robot, One Wheel Robot*

1. Introduction

To make a self balancing bot, it is essential to solve the inverted pendulum problem or an inverted pendulum on cart. While the calculation and expressions are very tedious, the aim is quite simple. The aim of the project is to adjust the wheels' position so that the inclination angle remains stable within a pre-determined value (e.g. the angle when the robot is not outside the premeasured angel boundary). When the robot starts to fall in one direction, the wheels should move in the inclined direction with a speed proportional to angle and acceleration of falling to correct the inclination angle. So I get an idea that when the deviation from equilibrium is small, we should move "gently" and when the deviation is large we should move more quickly.

To simplify things a little bit, I take a simple assumption; the robot's movement should be confined on one axis (e.g. only move forward and backward) and thus both wheels will move at the same speed in the same direction. Under this assumption the mathematics become much simpler as we only need to worry about sensor readings on a single plane. If we want to allow the robot to move sidewise, then you will have to control each wheel independently. The general idea remains the same with a less complexity since the falling direction of the robot is still restricted to a single axis.

Design and control of dynamically stable single wheel robots is a developing area to study in research. The balancing robot which utilizes the inverted pendulum concept is being studied and researched intensively in order to develop effective control system that are able to control this naturally unstable system.

A significant but frequently overlooked problem is that statically stable single wheeled robot can be easily become dynamically unstable. Conventional statically-stable single wheeled robots if physically tall enough to interact with people, they must maximize their platform stability by lowering their centre of gravity. If the robot's center of gravity is too high, or the robot's motion accelerates rapidly, the robot can tip over. Therefore, a wide and heavy base with low accelerations motion is applied for statically stable robot to avoid the robot tipping over [2]. These conditions present a number of performance limitations and ill-suited for navigation in human environments.

To achieve effective interactions in human environments, drive & reaction wheel bot with human-like height, width and weight is the most suitable choice. The dynamic stability affords it advantages in maneuverability over statically stable robots and makes it a good candidate for operating in human environments. Besides, balancing on a single wheel allow the bot to move in any direction without turning. [3]

2. Problem defination:

It is virtually impossible to balance a wheel bot without applying some external force to the system.Our main problem is to stabilize the system such that the position of the drive wheel on the track is controlled quickly and accurately and that the body and reaction wheel is always maintained tightly during such movements.The problem involves a drive wheel, able to move backwards and forwards, and a body with reaction wheel, hinged to the DRIVE WHEEL at the bottom of its length such that the body can move in the same plane as the drive wheel.That is, the body mounted on the wheel is free to fall along the wheel's axis of motion. The system is to be controlled so that the body remains balanced and upright, and is resistant to a step disturbance

3. Methodology:

3.1 2D MATHEMATICAL MODELLING :

The nature of this robot is similar to that of a human being riding on a classic mono- cycle. This is an unstable system that analyzed separately considering two different dynamics: longitudinal (pitch) and lateral (roll). The robot in analysis is composed of a wheel, a body and an inertia disc. The wheel has the objective of balancing the system in the movements of pitch, while the disc allows to balance the movements roll. It then assumes that the yaw movement is prevented because of the structure itself of the system, the following:

3.2 PRIMARY CONCERNS:

For the study of the robot in question is used the mathematical model proposed in abstact. Such dynamic model is housed applying the Lagrange approach to the mechanical system, assuming that the only motions available are the longitudinal and lateral. During the entire analysis there is the possibility of decoupling between the two movements, which allows to study the behavior of the system in two different planes. Any coupling effects they are to be considered negligible.

3.3 LONGITUDINAL EQUATIONS OF MOTION:

For the analysis of the robot in its longitudinal movement it is been taken into account the following reference diagram.

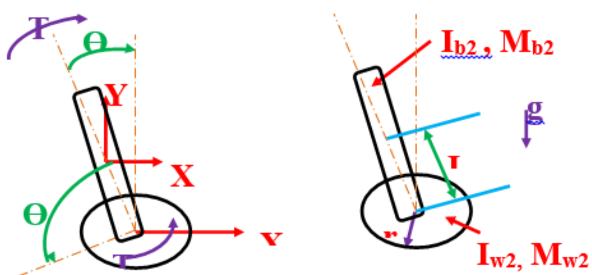

FIGURE 3.1 FBD For longitudinal equations

NOTATIONS:

Θ1 = Angle between wheel & body

Θ2 = Angle between body & vertical axis

T = Applied toque from motor to wheel

X1 = Horizontal position of wheel

X2 = Horizontal position of body

Y2 = Vertical position of body

MW1 = Mass of wheel

MB2 = Mass of body

IW1= M.I. of wheel w.r.t. CG

IB2 = M.I. of body w.r.t. CG

LB= Distance between center of wheel & CG of body

rw = Radius of wheel

g = Acceleration due to gravity.

FOR LONGITUDINAL SYSTEM,

$$\left(M_{B2} r_W^{\,2} + M_{W1} r_W^{\,2} + I_{W1}\right)\ddot{\theta}_1 + \left(M_{B2} r_W l_B \cos\theta_2 + M_{B2} r_W^{\,2} + M_{W1} r_W^{\,2} + I_{W1}\right) \ddot{\theta}_2 - M_{B2} r_W l_B \dot{\theta}_2^{\,2} \sin\theta_2 = T \qquad 1$$

$$\left(M_{B2} r_W l_B \cos\theta_2 + M_{B2} r_W^{\,2} + M_{W1} r_W^{\,2} + I_{W1}\right)\ddot{\theta}_1 + \left(2M_{B2} r_W l_B \cos\theta_2 + M_{B2} r_W^{\,2} + M_{B2} l_B^{\,2} + I_{B2} + M_{W1} r_W^{\,2} + I_{W1}\right)\ddot{\theta}_2 - M_{B2} r_W l_B \dot{\theta}_2^{\,2} \sin\theta_2 - M_{B2} g l_B \sin\theta_2 = 0 \qquad 2$$

3.4 LATERAL EQUATIONS OF MOTION

For the analysis of the robot in its lateral movement it is been taken into account the following reference diagram,

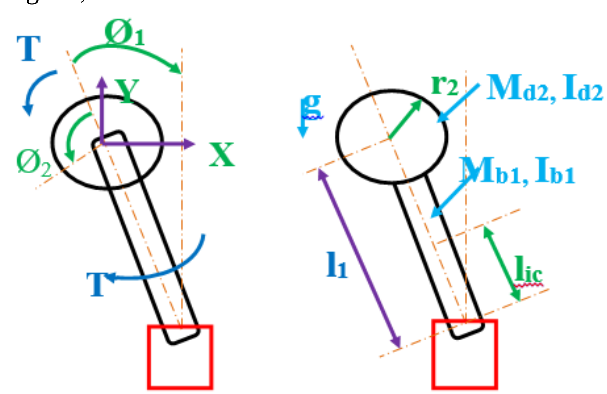

FIGURE4.2 FBD For lateral equations

NOTATIONS:

Ø1 = angle between body & vertical axis

Ø2 = angle between disc & body

T = applied torque from motor to axis of disc

XD2 = the horizontal position of disc

YD2 = the vertical position of disc

MB1 = Mass of body

MD2 = Mass of disc

IB1 = M.I. of body w.r.t. point A

ID2 = M.I. of disc w.r.t. its center

l1 = distance between center of disc & A

lic = distance between point A & CG of body

r2 = radius of disc

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g = acceleration due to gravity.

FOR LATERAL SYSTEM,

$$\left(I_{B1}+I_{D2}+M_{D2}l_1^{2}\right)\ddot{\phi}_1+I_{D2}\ddot{\phi}_2-\left(M_{B1}L_{ic}+M_{D2}L_1\right)g\sin\phi_1=0 \quad 3$$

$$I_{D2}\ddot{\phi}_1+I_{D2}\ddot{\phi}_2=T \quad 4$$

obtained to linearize this equations.

3.5 MOTOR DYNAMICS:

$$\left(M_{B2}r_W^{2}+M_{W1}r_W^{2}+I_{W1}\right)\ddot{\theta}_1+\left(M_{B2}r_Wl_B+M_{B2}r_W^{2}+M_{W1}r_W^{2}+I_{W1}\right)\ddot{\theta}_2+\frac{K^2K_MK_E\dot{\theta}_1}{R_M}=\frac{KK_MV_M}{R_M} \quad 5$$

$$\left(M_{B2}r_Wl_B+M_{B2}r_W^{2}+M_{W1}r_W^{2}+I_{W1}\right)\ddot{\theta}_1+\left(2M_{B2}r_Wl_B+M_{B2}r_W^{2}+M_{B2}l_B^{2}+I_{B2}+M_{W1}r_W^{2}+I_{W1}\right)\ddot{\theta}_2$$
$$-M_{B2}gl_B\theta_2=0 \quad 6$$

$$\left(I_{B1}+I_{D2}+M_{D2}l_1^{2}\right)\ddot{\phi}_1+I_{D2}\ddot{\phi}_2-\left(M_{B1}L_{ic}+M_{D2}L_1\right)g\,\phi_1=0 \quad 7$$

$$I_{D2}\ddot{\phi}_1+I_{D2}\ddot{\phi}_2+\frac{K^2K_MK_E\dot{\phi}_2}{R_M}=\frac{KK_MV_M}{R_M} \quad 8$$

3.63D MATHEMATICAL MODELLIG:

PRINCIPLE: D' Alembert's principle which states that the laws of static equilibrium apply to a dynamical system if the inertial forces, as weIl as the actual external forces, are considered as applied forces acting on the system.

METHOD

1. D alemberts torques acting on Wheel, Frame and R wheel.
2. D alemberts forces acting on Wheel,Frame and reaction wheel.
3. Gravitational forces acting on wheel,Frame and R wheel

STEPS.

- Set the two horizontal components of the moment about the ground contact point P equal to zero .

Set the vertical component of the moment about the P equal to zero for the whole unicycle and include the applied ground friction torque.

Set the moment about the axle of the wheel equal to zero for the frame plus r wheel and include the applied torque Qw and wheel drive friction torque.

Set the moment about the axle of the Rwheel equal to zero for the reaction wheel and include the applied torque QT and the reaction wheel drive friction torque.

LATERAL EQUATION OF MOTIONS:

$$0=(I_3^W\ddot{\psi}+I_2^W\Omega_0\dot{\phi})-I_3^F\ddot{\psi}+f_T\bar{\eta}-Q_T-f_G\dot{\psi}\theta \quad 9$$

$$0=-[I_2^W+(m_W+m_F+m_T)r_W^2]\Omega_0-(m_Fr_F-m_Tr_T)r_W\ddot{\theta}-f_W(\bar{\Omega}-\dot{\theta})+Q_W \quad 10$$

$$(I_3^W+I_3^F)\ddot{\psi}=-I_2^W\Omega_0\dot{\phi}-f_G\dot{\psi}+f_T\bar{\eta}-Q_T \quad 11$$

LONGITUDINAL EQUATIONS,

$$[I_1^W+I_1^F+I_1^T+m_Wr_W^2+m_F(r_W+r_F)^2+m_T(r_W+r_T)^2]\ddot{\phi}$$
$$=[I_2^W+m_Wr_W^2+m_Fr_W(r_W+r_F)+m_Tr_W(r_W+r_T)]\Omega_0\dot{\varphi} \quad 12$$
$$+[m_Wr_W+m_F(r_W+r_F)+m_T(r_W+r_T)]g\phi-I_3^T\eta_0\theta$$

$$I_3^T(\ddot{\psi}+\dot{\eta})=-f_T\bar{\eta}+Q_T \quad 13$$

3.7 STATES SPACE EQUATIONS

After obtaining the mathematical equations we can use this equation to calculate transfer function and finally the state space for a controller design.

$$\begin{bmatrix} I_{11} & I_{12} & 0 & 0 & 0 & 0 & I_{17} & I_{18} & 0 & 0 & 0 \\ I_{21} & I_{22} & 0 & 0 & 0 & 0 & I_{27} & 0 & 0 & 0 & 0 \\ 0 & I_3^T & I_3^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I_{41} & I_{42} & 0 & I_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ I_{71} & I_{72} & 0 & 0 & 0 & 0 & I_{77} & I_{78} & 0 & 0 & 0 \\ 0 & I_{82} & 0 & 0 & 0 & 0 & I_{87} & I_{88} & 0 & 0 & 0 \\ 0 & I_{92} & 0 & 0 & 0 & 0 & I_{97} & I_{98} & I_{99} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{\phi} \\ \ddot{\varphi} \\ \dot{\eta} \\ \ddot{\sigma} \\ \dot{\phi} \\ \dot{\sigma} \\ \ddot{\theta} \\ \dot{\Omega} \\ \ddot{\rho} \\ \dot{\theta} \\ \dot{\rho} \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} & F_T & 0 & F_{15} & 0 & 0 & 0 & 0 & F_{110} & 0 \\ 0 & F_{22} & 0 & 0 & F_{25} & 0 & F_{27} & 0 & 0 & 0 & 0 \\ 0 & 0 & -F_T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & F_{42} & 0 & -F_P & F_{45} & F_{46} & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ F_{71} & 0 & 0 & 0 & 0 & 0 & -F_W & F_W & 0 & F_{710} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & F_W & -F_W & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & g & -F_P & F_{910} & F_{911} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\varphi} \\ \eta \\ \dot{\sigma} \\ \phi \\ \sigma \\ \dot{\theta} \\ \Omega \\ \dot{\rho} \\ \theta \\ \rho \end{bmatrix}$$

$$+\begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} Q_T \\ Q_W \end{bmatrix} + \begin{bmatrix} 0 & F_T & 0 \\ K_{21} & 0 & 0 \\ 0 & -F_T & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ K_{71} & 0 & F_W \\ 0 & 0 & -F_W \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g \\ \eta_0 \\ \Omega_0 \end{bmatrix}$$

3.8 SOLID MODELLING

A 3-Dimensional model is prepared using Solid works 2014. This Model represents an overview of the actual model of the Drive & Reaction Wheel Robot.

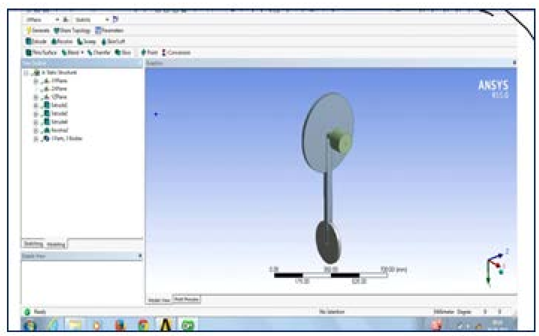

FIGURE 4.3 SOLID MODELLING

4. Results and Discussion

In this section after obtaining equations of motions and the state space and implement in comparison of all the controllers based on Simulation Results are discussed. Parameters compared are percentage peak overshoot and rise time as shown in Table

Controller	Peak overshoot (%)	Rise time (SEC)	Settling time (SEC)	Peak amplitude(RAD)
PID	2.1	0.007	0.997	0.058
LQR	4.1	0.006	2.1	0.035

TABLE2.2:Comparison of controllers based on Test results

From the above results it clear that the objectives kept for this design are achieved

5. Conclusions

To build a one wheel -balancing bot we first derived the system equation then check its real time response (both time and frequency). Then we designed a PID/LQR/FUZZY controller to control the close loop function. We checked the controllability and set the pole location. Then by choosing the appropriate components we analyze their simulation successfully and finally the bot is successfully balanced.

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