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Similarity Solution for MHD Flow of Non-Newtonian Fluids

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Abstract

Similarity analysis is made of Magnetohydrodynamics (MHD) boundary layer flow of non-Newtonian fluid past infinite surface. The deductive group theoretic method is use to derive the similarity solution. The resulting similarity equation is solved using MATALAB ODE solver. It is observed that magnetic field strength having impact on fluid flow under consideration. Further, the well- known solutions of Newtonian and non-Newtonian Power –law fluids are found to be limiting cases of the present solutions.

Keywords:-Sisko fluid, Similarity Solution, Group Theoretic method, MHD, non-Newtonian fluids.

1 Introduction

Fluids are encounter at every stage of life. Fluids are of three types:-Liquid, Semi liquid and Gaseous. The fluid that obeys the Newtonian law of viscosity is called Newtonian fluids and in such case the shear stress is found to be a linear function of rate of strain. The situation is bit different when a fluid fails to obey the Newton's law of viscosity. Such fluids are Known as non-Newtonian fluids and in these type of fluids strain –stress relationship is non linear.

Usually flow of Newtonian fluids is governed by Navier –stokes equation where as flow of non-Newtonian fluids is governed by modified Navier-stokes equation, Biological fluids, Liquids crystals, Lubricating oils, Starch solutions, Rubber solutions, Super glue, Toothpaste etc. are the examples of non-Newtonian fluids. However a good number of fluid rehology are already in existence, particularly for most of those fluids used as lubricants and having non-Newtonian behavior, the flow can be analyzed with the help of a generalized Ostwald-de-wale power –law fluids known as Sisko fluids. It is worth to note that power -law fluid model characterizes both pseudoplastic and dilatant fluids –two important classes of non-Newtonian fluids and again can characterize Newtonian fluid as a special case. It is because of such wide coverage in the analysis of lubricants together with its mathematical simplicity that the Sisko fluid model [Sisko (1958)] has been preferred for application in the present problem. Some recent studies dealing with the flows of non-Newtonian fluids are mentioned by Hayat et al. (2004), Fetecau and Fetecau (2003a, b,) (2005); Asghar et al. (2002); Hayat (2005); Hayat and Kara (2006); Chen et al.(2003, 2004).

The present work concentrates on the similarity solutions for MHD flow of a non-Newtonian fluid to study the flow of an electrically conducting Sisko fluid. The similarity analysis of governing equation is discussed using the deductive group theoretic method which is already been successfully applied to several non-linear problems [Abd-el-Malek et al (2002); Parmar and Timol (2011)].

2. Governing equation

The fundamental equations governing the motion of steady incompressible boundary layer flow of electrically conducting fluid through porous medium are given by:

$$div\boldsymbol{V} = 0 \tag{1}$$

$$\rho \frac{DV}{Dt} -= -\nabla p + div\tau - \sigma B_0^2 V \tag{2}$$

where $\boldsymbol{V}$ is the velocity field, ρ is density, p the pressure, τ the extra stress tensor, σ the electrical conductivity, B_o the applied magnetic field. In this paper we consider a Sisko fluid whose stress-strain relationship is given by

$$\tau = \left[a + b \left| \sqrt{\frac{1}{2} tr\left(A_1^2\right)} \right|^{n-1} \right] A_1 \tag{3}$$

Where a, b and n are constants defined differently for different fluids and A_1 the rate of deformation tensor defined by

$$A_1 = L + L^T; \quad \text{Where } L = grad\boldsymbol{V} \tag{4}$$

For the problem considered here we define the velocity and the stress fields of the following form

$$\boldsymbol{V} = \left[u\left(y,t\right),0,0\right], \qquad \tau = \tau\left(y,t\right) \tag{5}$$

Under (5), the continuity equation (1) is satisfied identically and from equations (2), (3), we have

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left[\left(a + b\left|\frac{\partial u}{\partial y}\right|^{n-1}\right)\frac{\partial u}{\partial y}\right] - \sigma B_0^2 u \tag{6}$$

It is interesting to observe that for a=0 (b $\neq$ 0, n$\neq$ 1), the above equation will be equation of power-law fluids and for b=0 (a $\neq$ 0, and n=1) it will be equation of Newtonian fluid

3. Formulation of the problem:

We consider a Cartesian coordinate system with y-axis in the vertical upward direction and x-axis parallel to the rigid plate at y = 0. The flow of an incompressible and electrically conducting Sisko fluid is bounded by an infinite rigid plate. The Sisko fluid occupies the porous half-space $y > 0$. The flow is produced by the motion of the plate with the time dependent velocity $U_0 V\left(t\right)$. For zero pressure gradient, the resulting problem from (6), yields

$$\rho\frac{\partial u}{\partial t} = \frac{\partial}{\partial y}\left[\left(a + b\left|\frac{\partial u}{\partial y}\right|^{n-1}\right)\frac{\partial u}{\partial y}\right] - \sigma B_0^2 u \tag{7}$$

$$\left.\begin{aligned} u\left(0,t\right) &= U_0 V\left(t\right), & t > 0 \\ u\left(\infty,t\right) &= 0, & t > 0 \\ u\left(y,o\right) &= g\left(y\right), & y > 0 \end{aligned}\right\} \tag{8}$$

in which U_0 is the characteristic velocity.

The above equations can be made dimensionless using the following variables,

$$u^* = \frac{u}{U_0}, \qquad y^* = \frac{yU_0}{v}, \qquad t^* = \frac{tU_0^2}{v}, \tag{9}$$

$$b^* = \frac{b}{a}\left|\frac{U_0^2}{v}\right|, \ N^2 = \frac{\sigma B_0^2 u}{\rho U_0^2} \tag{10}$$

Accordingly the above boundary value problem after dropping the asterisks (for simplicity) becomes

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial y}\left[\left(1 + b\left|\frac{\partial u}{\partial y}\right|^{n-1}\right)\frac{\partial u}{\partial y}\right] - N^2 u, \tag{11}$$

$u(0,x) = V(t), \quad \text{for } t > 0$
$u(\infty,t) = 0, \quad \text{for } t > 0$

$$u(y,0) = G(y), \quad \text{for } y > 0$$

Where $G(y) = g(y)U_0^{-1}$ (12)

Without loss of generality we assume that the velocity gradient is positive then equation (11) can be written as

$$u_t = u_{yy} + nb(u_y)^{n-1}u_{yy} - N^2u \quad (13)$$

Where suffices refer to partial derivatives.

4. Methodology and Solution of the problem:

Our method of solution depends on the application of a one-parameter deductive group of transformation to the partial differential equation (13). Under this transformation the two independent variables will be reduced by one and the differential equation (13) will transforms into an ordinary differential equation.

4.1The group systematic formulation:

Equation (13) contains two independent variables and one dependent variable and hence further procedure is initiated with the group G, a class of transformation of one-parameter a of the form:

$$G:\begin{cases}\bar{y} = A^y(a)y + S^y(a)\\ \bar{t} = A^z(a)z + S^z(a)\\ \bar{u} = A^u(a)u + S^u(a)\end{cases} \quad (14)$$

Where A's and S's are real-valued and at least differentiable in the real argument 'a'.

Equation (13) is said to be invariantly transformed, for some function $A(\varepsilon)$ whenever:

$$\bar{u}_{\bar{t}} - \bar{u}_{\bar{y}\bar{y}} - nb(\bar{u}_{\bar{y}})^{n-1}\bar{u}_{\bar{y}\bar{y}} + N^2\bar{u} = K(\varepsilon)[u_t - u_{yy} - nb(u_y)^{n-1}u_{yy} + Nu]$$

Substituting the values from the equation (14) in above equation (13), yields

$$\frac{A^u}{A}\bar{u}_{\bar{t}} - \frac{A^u}{(A^y)^2}\bar{u}_{\bar{y}\bar{y}} - nb\frac{(\bar{u}A^u)^n}{(A^y)^{n+1}}(\bar{u}_{\bar{y}})^{n-1}\bar{u}_{\bar{y}\bar{y}} + N^2(c^u u + h^u) = K(\varepsilon)[u_t - u_{yy} - nb(u_y)^{n-1}u_{yy} + N^2u] \quad (15)$$

The invariance of equation (15), implies that

$$S^u = 0 \qquad \text{and} \qquad \frac{A^u}{A^t} = \frac{A^u}{(A^y)^2} = \frac{(A^u)^n}{(A^y)^{n+1}} = K(\varepsilon) \quad (16)$$

Also for the absolute invariance of the auxiliary conditions, implies that $h^y = 0$

These yields,

$$A^y = A^u; \qquad A^t = (A^u)^2; \quad K(\varepsilon) = (A^u)^{-1}$$

Finally, we get the one-parameter sub group G, which transforms invariantly the differential equation (13) and the auxiliary conditions (12).

The group G is of the form:

$$G: \ \bar{t} = [A^u(\varepsilon)]^2 t + S^t(\varepsilon), \ \bar{y} = A^u(\varepsilon)y, \ \ \bar{u} = A^u(\varepsilon)u \quad (17)$$

4.2The complete set of absolute invariants:

Our aim is to make use of group methods to represent the problem in the form of an ordinary differential equation. Then we have to proceed in our analysis to obtain a complete set of absolute invariants. If $\eta = \eta(y,t)$ is the absolute invariant of the independent variables then,

$$g(y,t,u) = F(\eta) \quad (18)$$

is the absolute invariant for the dependent variable $u(y,t)$
The application of a basic theorem in group theory, [Moran and Gaggioli (1968)], states that:
A function $g(y,t,u)$ is an absolute invariant of a one-parameter group if it satisfies the following first-order linear partial differential equation,

$$\sum_{i=1}^{3}(\alpha_i P_i + \beta_i)\frac{\partial g}{\partial P_i} = 0, \qquad P_i = t, y, u \tag{19}$$

Where

$$\alpha_i = \left.\frac{\partial A^i}{\partial \varepsilon}\right|_{\varepsilon=\varepsilon_0} \quad \text{and} \quad \beta_i = \left.\frac{\partial S^i}{\partial \varepsilon}\right|_{\varepsilon=\varepsilon_0} \qquad i = 1,2,3 \tag{20}$$

and 'ε_0'denotes the value of 'ε' which yields the identity element of the group G.
Since $h^y = h^u = 0$ implies that $\beta_2 = \beta_3 = 0$ and from (20) we get following relation between $\alpha' s$:
$\alpha_1 = 2\alpha_2 = 2\alpha_3.$
The equation (19) reduces to

$$(\alpha_1 t + \beta_1)\frac{\partial g}{\partial t} + (\alpha_2 y)\frac{\partial g}{\partial y} + (\alpha_3 u)\frac{\partial g}{\partial u} = 0. \tag{21}$$

The absolute invariant of independent variable owing the equation (19) is $\eta = \eta(y,t)$ if it will satisfies the first order linear partial differential equation

$$(2t + \beta)\frac{\partial \eta}{\partial t} + y\frac{\partial \eta}{\partial y} + u\frac{\partial \eta}{\partial u} = 0, \quad \text{where } \beta = \frac{\beta_1}{\alpha_1} \tag{22}$$

Applying the variable separable method one can obtain

$$\eta(y,t) = y(2t+\beta)^{-\frac{1}{2}} \tag{23}$$

Further the absolute invariant of dependent variable owing the equation (19) is followed by

$$g(y,t,u) = (2t+\beta)^{-\frac{1}{2}} u(y,t) \tag{24}$$

Hence from (18), (23), (24) implies that,

$$u(y,t) = (2t+\beta)^{\frac{1}{2}} F(\eta); \qquad \text{Where } \eta = y(2t+\beta)^{-\frac{1}{2}} \tag{25}$$

4.3The reduction to an ordinary differential equation:
Using the similarity transformation (25) in equation (13), yields to following non-linear ordinary differential equation

$$\left\{1 + \mathrm{nb}(\mathrm{F}^{|})^{\mathrm{n}-1}\right\}\mathrm{F}^{||} + \eta\mathrm{F}^{|} + (\mathrm{N}^2 - 1)\mathrm{F} = 0 \tag{26}$$

Subject to the reduced auxiliary conditions (12) to,

$$F(0) = L; \qquad F(\infty) = 0; \qquad F(y) = G(y)$$

Where primes denote the ordinary derivative with respect to η.

5. Result and Discussion:

The differential equation (26) is highly non-linear differential equation and hence it is bit difficult to find its analytic solution. The numerical solution of equation (26) is obtained for particular values of the parameters by using MATLAB ode solver and its graphical representation is given in below diagrams which shows the behavior of the normalized velocity

$$F(\eta) = (2t+\beta)^{-\frac{1}{2}} u(y,t) \quad (27)$$

The various normalized velocity profiles for the different parameters with specific values are generated, as show in Figures 1.1 & 1.2

- In figure 1.1, controlling the parameters n and b it is observe that as the magnetic field increase, the normalized velocity approaches to its final value fast. In other words, in presence of magnetic field the velocity increase depends upon the strength of applied magnetic field.
- Figure 1.2shows that increase in flow index n, reduces the normalized velocity corresponding to same applied magnetic field. That is as known Non-Newtonian behavior of fluid increase, the velocity decrease.

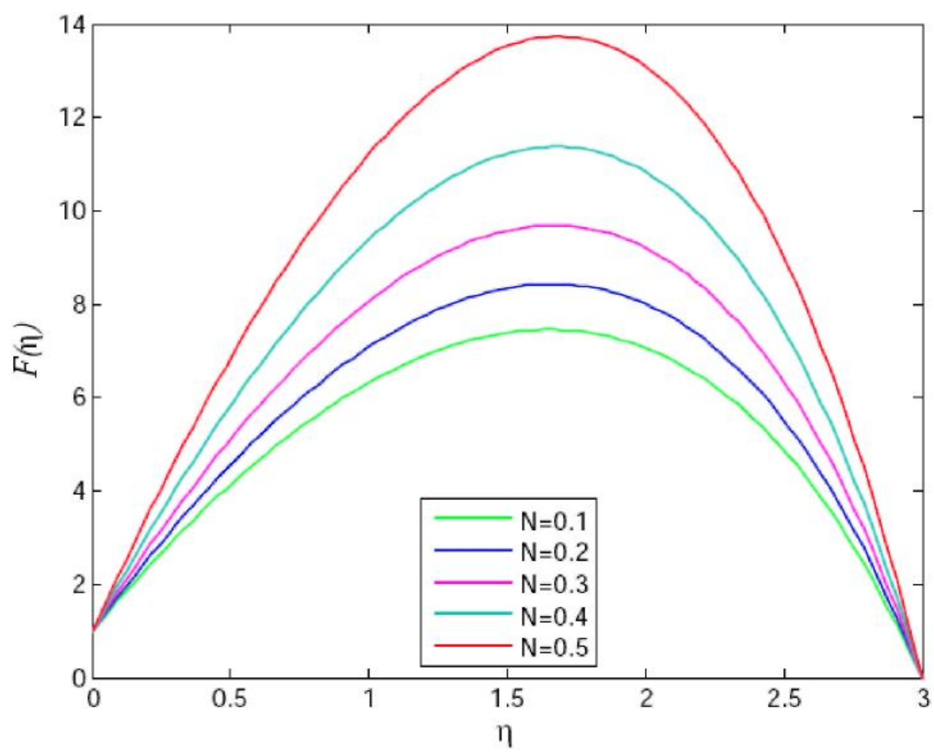

Figure 1.1: Effect of magnetic parameter on Newtonian fluid for *n=1 b=0.5*

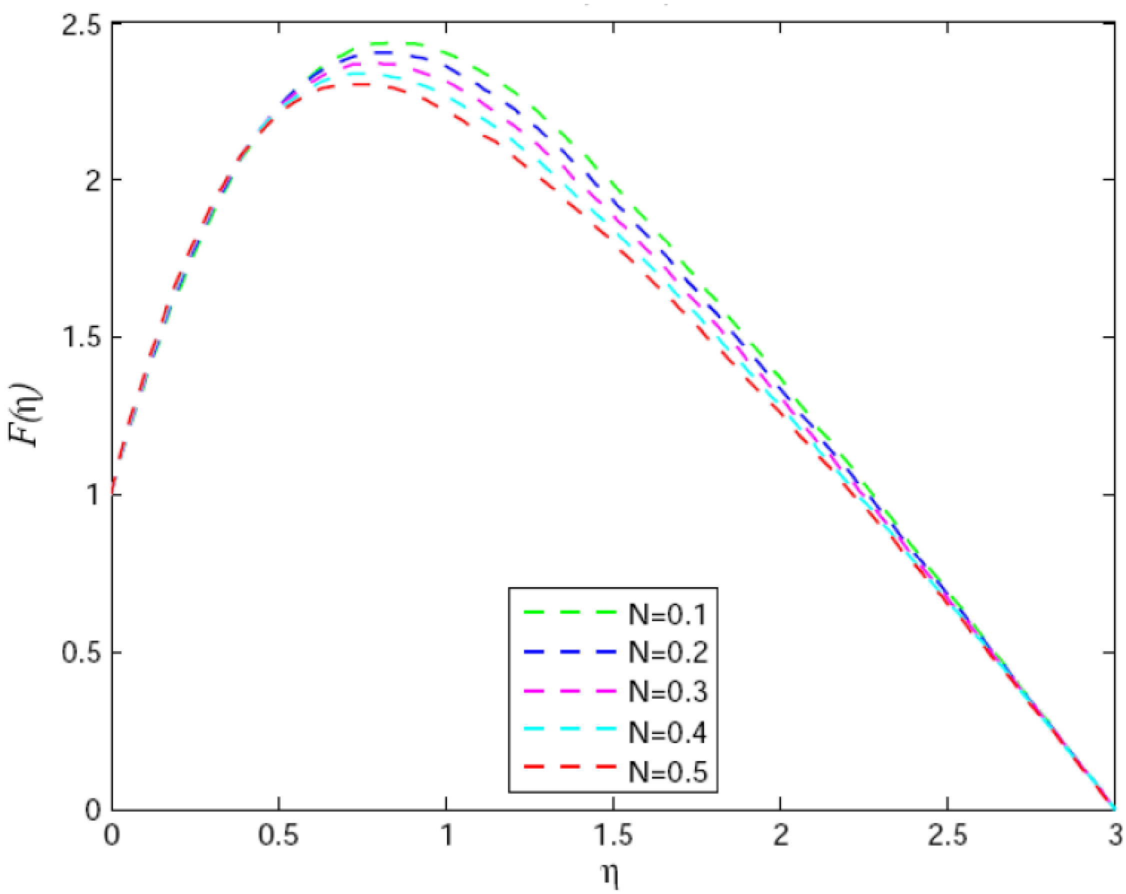

Figure 1. 2: Effect of magnetic parameter on Non-Newtonian fluid for $n=2, b=0.5$

6.Conclusion

In the present research paper we have successfully derive the conditions, through the deductive group invariant technique, under which similarity solution of the flow of MHD Sisko fluids exist. The Matlab ode solver results are presented in graphical form for both Newtonian and non-Newtonian Sisko MHD fluids.

References

[1] Abd-el-Malek MB, Badran NA, and Hassan HS (2002). Solution of the Rayleigh problem for a power law non-Newtonian conducting fluid via group method. Int. J. Eng. Sci. 40,1599-1609.

[2] Asghar S, Hayat T, and Siddiqui A. M (2002). Moving boundary in a non-Newtonian fluid. Int. J. Non-Linear Mech. 37, 75-80.

[3] Chen CI, Chen CK, and Yang YT (2003). Unsteady unidirectional flow of second grade fluid between the parallel plates with different given volume flow rate conditions. Appl. Math. Comp.137, 437-450.

[4] Chen CI, Chen CK, and Yang YT (2004). Unsteady unidirectional flow of an Oldroyd-B fluid in a circular duct with different given volume flow rate conditions. Heat Mass Transf. 40, 203-209.

[5] Fetecau C and Fetecau C (2003a). The Rayleigh–Stokes problem for a fluid of Maxwellian type. Int. J. Non-Linear Mech. 38, 603–607.

[6] Fetecau C and Fetecau C (2005). Decay of a potential vortex in a Maxwell fluid. Int. J. Non-Linear Starting solutions for some unsteady unidirectional flows of a second grade fluid. Int. J. Eng. Sci. 43, 781-789.

[7] Hayat T, Wang Y, and Hutter K (2004). Hall effects on the unsteady hydromagnetic oscillatory flow of a second grade fluid. Int. J. Non-Linear Mech. 39, 1027–1037.

[8] Hayat T (2005). Oscillatory solution in rotating flow of a Johnson-Segalman fluid. Z. Angew. Math. 85, 449–456.

[9] Hayat T and Kara AH(2006). A variational analysis of a non-Newtonian flow in a rotating system. Int. J.Comp. Fluid Dyn. 20, 157–162.

[10] Parmar H and Timol MG (2011). Deductive Group Technique for MHD coupled heat and mass transfer natural convection flow of non-Newtonian power law fluid over a vertical cone through porous medium, Int. J. of Appl. Math and Mech. 7 (2): 35-50.

[11] Sisko, AW (1958). The flow of lubricating greases. Ind. Eng. Chem. 50, 1789–1792.