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Some more Results on Total Neighbourhood Cordial and Total Neighbourhood Product Cordial Labeling of Graphs

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Abstract

In this paper, total neighbourhood cordial labeling $P_{n+1}\cup B_{n,n}$, $C_n\cup B_{m,m}$, $K_{1,n}\cup B_{n,n}$, $(P_n\odot K_1) \cup C_m$ under some conditions and total neighbourhood product cordial labeling $P_n\cup K_{1,m,m}$, $C_n \cup K_{1,m,m}$, $K_{1,n} \cup K_{1,m,m}$, $P_{n+1} \cup B_{n,n}$, $C_n \cup B_{m,m}$, $K_{1,n} \cup B_{n,n}$, $(P_n\odot K_1)\cup C_m$ under some conditions are presented

Keywords: *Neighbourhood cordial graph, Total neighbourhood cordial graph, Neighbourhood product cordial graph, Total neighbourhood product cordial graph.*

1. Introduction

By a graph, we mean a finite, disconnected, undirected graph without loops and multiple edges, for terms not defined here, we refer to Harary [3]. For standard terminology and notations related to graph labeling, we refer to Gallian [2]. The concept of cordial labeling of a graph was introduced by Cahit [1]. In [6], Sundaram et al introduced the concept of product cordial labeling of a graph. The concept of total product cordial labeling was introduced by Sundaram et al [7]. In [4], Muthaiyan et. al., introduced the concept of neighbourhood cordial labeling, total neighbourhood cordial labeling, neighbourhood product cordial labeling and total neighbourhood product cordial labeling and they presented the neighbourhood cordial labeling of kP_2, kP_n, $K_{1,n}\cup P_n$, $K_{1,n}\cup C_n$, $C_n\cup P_n\cup K_{1,n,n}$, $C_n\cup K_{1,n}\cup K_{1,n,n}$, and $P_n\cup K_{1,n}\cup K_{1,n,n}$, the total neighbourhood cordial labeling of $P_n\cup K_{1,n,n}$, $C_n\cup K_{1,n,n}$, $K_{1,n}\cup K_{1,n,n}$ and total neighbourhood product cordial labeling of $K_{1,n}$. In [5], Muthaiyan et. al., prove that the graphs $C_n\cup P_m\cup K_{1,r,r}$, $C_n\cup K_{1,m}\cup K_{1,r,r}$ and $P_n\cup K_{1,m}\cup K_{1,r,r}$ are neighbourhood cordial graphs under some conditions, $(P_n\odot K_1) \cup P_m$, $(P_n\odot K_1) \cup K_{1,m}$, $(C_n\odot K_1) \cup P_m$, $(C_n\odot K_1) \cup C_m$, $(C_n\odot K_1) \cup K_{1,m}$, are total neighbourhood cordial graphs under some conditions and $(P_n\odot K_1) \cup P_m$, $(P_n\odot K_1) \cup K_{1,m}$, $(C_n\odot K_1) \cup P_m$, $(C_n\odot K_1) \cup C_m$, $(C_n\odot K_1) \cup K_{1,m}$, are total neighbourhood product cordial graphs under some conditions. In this paper total neighbourhood cordial labeling $P_{n+1} \cup B_{n,n}$, $C_n \cup B_{m,m}$, $K_{1,n}\cup B_{n,n}$, $(P_n\odot K_1) \cup C_m$ under some conditions and total neighbourhood product cordial labeling $P_n \cup K_{1,m,m}$, $C_n \cup K_{1,m,m}$, $K_{1,n} \cup K_{1,m,m}$, $P_{n+1} \cup B_{n,n}$, $C_n \cup B_{m,m}$, $K_{1,n}\cup B_{n,n}$, $(P_n\odot K_1) \cup C_m$ under some conditions are presented. The brief summaries of definition which are necessary for the present investigation are provided below.

Definition :1.1

A mapping $f : V(G)\rightarrow\{0,1\}$ is called binary vertex labeling of G and f(v) is called the label of the vertex v of G under f. If for an edge e = uv, the induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ is given by $f^*(e) = | f(u) - f(v) |$. Then $v_f(i)$ = number of vertices of having label i under f and $e_f(i)$ = number of edges of having label i under f*.

Definition :1.2

A binary vertex labeling f of a graph G is called a cordial labeling if $| v_f(0) – v_f(1) | \leq 1$ and $| e_f(0) – e_f(1) | \leq 1$. A graph G is cordial if it admits cordial labeling.

Definition :1.3

Let G be a simple graph and $f : V(G) \rightarrow \{0,1\}$ be a vertex labeling. For each edge uv, assign the label f(u)f(v). The labeling f is called a product cordial labeling of G if $| v_f(0) – v_f(1) | \leq 1$ and $| e_f(0) – e_f(1) | \leq 1$, where $v_f(i)$ and $e_f(i)$ denote the number of vertices and the number of edges respectively labeled with i (i = 0,1). A graph with a product cordial labeling is called a product cordial graph.

Definition :1.4

Let G be a simple graph and $f : V(G) \rightarrow \{0,1\}$ be a vertex labeling. For each edge uv, assign the label f(u)f(v). The labeling f is called a total product cordial labeling of G if $| f(0) – f(1)| \leq 1$, where f(i) denotes sum of the number of vertices and the number of edges labeled with i (i = 0,1). A graph with a total product cordial labeling is called a total product cordial graph.

Definition :1.5

A binary vertex labeling f of a graph G is called a neighbourhood cordial labeling if for every vertex $v \in V(G)$, then all the vertices adjacent to the vertex v have the same label, $| v_f(0) - v_f(1) | \leq 1$ and $| e_f(0) - e_f(1) | \leq 1$. A graph G is neighbourhood cordial if it admits neighbourhood cordial labeling.

Definition :1.6

A binary vertex labeling f of a graph G is called a total neighbourhood cordial labeling, if for every vertex $v \in V(G)$, then all the vertices adjacent to the vertex v have the same label and $| f(0) - f(1) | \leq 1$, where f(i) denotes sum of the number of vertices and the number of edges labeled with i (i = 0,1). A graph G is total neighbourhood cordial if it admits total neighbourhood cordial labeling.

Definition :1.7

Let G = (V,E) be a simple graph and $f : V(G) \rightarrow \{0,1\}$ be a vertex labeling. For each edge uv, assign the label f(u)f(v). The labeling f is called a neighbourhood product cordial labeling of G, if for every vertex $v \in V(G)$, then all the vertices adjacent to the vertex v have the same label, $| v_f(0) - v_f(1) | \leq 1$ and $| e_f(0) - e_f(1) | \leq 1$, where $v_f(i)$ and $e_f(i)$ denote the number of vertices and the number of edges respectively labeled with i (i = 0,1). A graph with a neighbourhood product cordial labeling is called a neighbourhood product cordial graph.

Definition :1.8

Let G = (V,E) be a simple graph and $f : V(G) \rightarrow \{0,1\}$ be a vertex labeling. For each edge uv, assign the label f(u)f(v). The labeling f is called a total neighbourhood product cordial labeling of G, if for every vertex $v \in V(G)$, then all the vertices adjacent to the vertex v have the same label and $| f(0) - f(1)| \leq 1$, where f(i) denotes sum of the number of vertices and the number of edges labeled with i (i = 0,1). A graph with a total neighbourhood product cordial labeling is called a total neighbourhood product cordial graph.

Definition :1.9

A complete biparitite graph $K_{1,n}$ is called a star and it has n+1 vertices and n edges. $K_{1,n,n}$ is the graph obtained by the subdivision of the edges of the star $K_{1,n}$.

Definition :1.10

The corona $G_1 \odot G_2$ of two graphs $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ is defined as the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to all the vertices in the i^{th} copy of G_2.

2. Main Results

Theorem 2.1

The disconnected graph $P_{n+1} \cup B_{n,n}$ is total neighbourhood cordial graph, where $n \geq 1$.

Proof.

Let G be the disconnected graph $P_{n+1} \cup B_{n,n}$.
Let $u_1, u_2, ..., u_{n+1}$ and $v, w, v_1, v_2, ..., v_n, w_1, w_2, ..., w_n$ be the vertices of P_{n+1} and $B_{n,n}$ respectively.
Then $|V(G)| = 3n+3$ and $|E(G)| = 3n+1$.
Define vertex labeling $f : V(G) \rightarrow \{0,1\}$ as follows.

$f(u_i) = 0$ for $1 \leq i \leq n+1$,
$f(v) = 0$,
$f(w) = 1$,
$f(v_i) = 1$ for $1 \leq i \leq n$,
$f(w_i) = 0$ for $n+1 \leq i \leq 2n$,

In view of the above labeling pattern we have, $v_f(0) = 2n+2$, $v_f(1) = n+1$, $e_f(1) = 2n+1$ and $e_f(0) = n$.
Here, $f(0) = 3n+2$ and $f(1) = 3n+2$.
Therefore, $| f(0) - f(1) | = 0$.
Hence, the disconnected graph $P_{n+1} \cup B_{n,n}$ is total neighbourhood cordial graph, where $n \geq 1$.

Example 2.1

The graph $P_6 \cup B_{5,5}$ and its total neighbourhood cordial labeling is given in Figure 2.1.

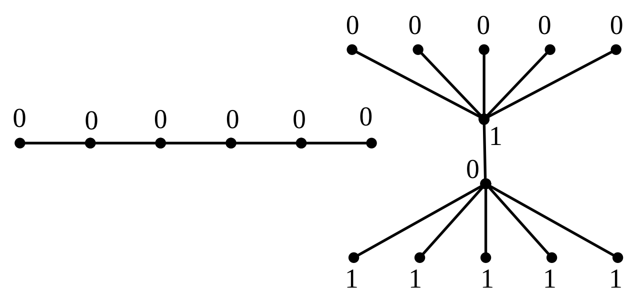

Figure 2.1

Theorem 2.2

The disconnected graph $C_n \cup B_{m,m}$ is total neighbourhood cordial graph for m = n–1 and n, where $n \geq 3$ and $m \geq 2$.

Proof.

Let G be the disconnected graph $C_n \cup B_{m,m}$.
Let $u_1, u_2, ..., u_n$ and $v, w, v_1, v_2, ..., v_m, w_1, w_2, ..., w_m$ be the vertices of P_n and $B_{m,m}$ respectively.
Then $|V(G)| = n+2m+2$ and $|E(G)| = n+2m+1$.
Define vertex labeling $f : V(G) \rightarrow \{0,1\}$ as follows.

Case (i) : m = n–1

$f(u_i) = 0$ for $1 \leq i \leq n$,
$f(v) = 0$,

$f(w) = 1$,
$f(v_i) = 1$ for $1 \le i \le n-1$,
$f(w_i) = 0$ for $n \le i \le 2n-2$,

In view of the above labeling pattern we have, $v_f(0) = 2n$, $v_f(1) = n$, $e_f(1) = 2n-1$ and $e_f(0) = n$.
Here, $f(0) = 3n$ and $f(1) = 3n-1$.
Then, $|f(0) - f(1)| = 1$.
Therefore, the disconnected graph $C_n \cup B_{m,m}$ is total neighbourhood cordial graph for $m = n-1$.

Case (ii) : $m = n$

$f(u_i) = 0$ for $1 \le i \le n$,
$f(v) = 0$,
$f(w) = 1$,
$f(v_i) = 1$ for $1 \le i \le n$,
$f(w_i) = 0$ for $n+1 \le i \le 2n$,

In view of the above labeling pattern we have, $v_f(0) = 2n+1$, $v_f(1) = n+1$, $e_f(1) = 2n+1$ and $e_f(0) = n$.
Here, $f(0) = 3n+1$ and $f(1) = 3n+2$.
Then, $|f(0) - f(1)| = 1$.
Therefore, the disconnected graph $C_n \cup B_{m,m}$ is total neighbourhood cordial graph for $m = n$.
Hence, the disconnected graph $C_n \cup B_{m,m}$ is total neighbourhood cordial graph for $m = n-1$ and n, where $n \ge 3$ and $m \ge 2$.

Example 2.2

The graph $C_5 \cup B_{5,5}$ and its total neighbourhood cordial labeling is given in Figure 2.2.

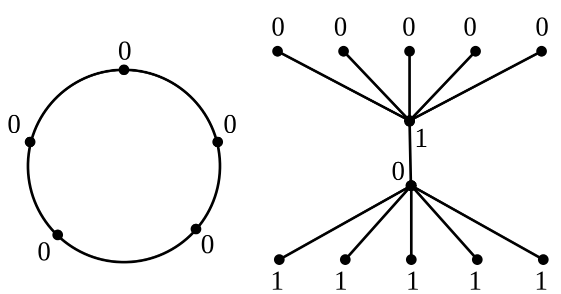

Figure 2.2

Theorem 2.3

The disconnected graph $K_{1,n} \cup B_{n,n}$ is total neighbourhood cordial graph, where $n \ge 2$.

Proof.

Let G be the disconnected graph $K_{1,n} \cup B_{n,n}$.
Let $u, u_1, u_2, ..., u_n$ and $v, w, v_1, v_2, ..., v_n, w_1, w_2, ..., w_n$ be the vertices of $K_{1,n}$ and $B_{m,m}$ respectively.
Then $|V(G)| = 3n+2$ and $|E(G)| = 3n$.
Define vertex labeling $f : V(G) \to \{0,1\}$ as follows.

$f(u) = 0$,
$f(u_i) = 0$ for $1 \le i \le n$,
$f(v) = 0$,
$f(w) = 1$,
$f(v_i) = 1$ for $1 \le i \le n$,
$f(w_i) = 0$ for $n+1 \le i \le 2n$,

In view of the above labeling pattern we have, $v_f(0) = 2n+2$, $v_f(1) = n+1$, $e_f(0) = n$ and $e_f(1) = 2n+1$.
Here, $f(0) = 3n+2$ and $f(1) = 3n+2$.
Then, $|f(0) - f(1)| = 0$.
Therefore, the disconnected graph $K_{1,n} \cup B_{n,n}$ is total neighbourhood cordial graph for $m = n$.

Example 2.3

The graph $K_{1,5} \cup B_{5,5}$ and its total neighbourhood product cordial labeling is given in Figure 2.12.

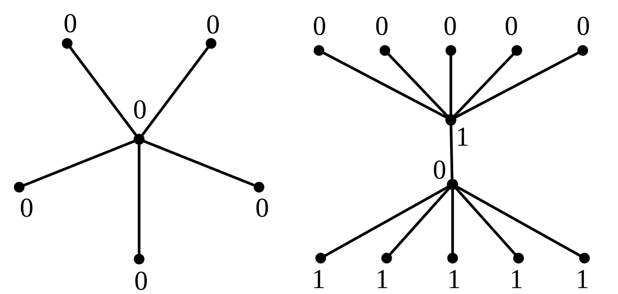

Figure 2.3

Theorem 2.4

The disconnected graph $(P_n \odot K_1) \cup C_m$ is total neighbourhood cordial graph for $m = n-1$ and n, where $n \ge 2$ and $m \ge 3$.

Proof.

Let G be the disconnected graph $(P_n \odot K_1) \cup C_m$.
Let $u_1, u_2, ..., u_m$ and $v_1, v_2, ..., v_n, w_1, w_2, ..., w_n$ be the vertices of C_m and $(P_n \odot K_1)$ respectively.
Then $|V(G)| = 2n+m$ and $|E(G)| = 2n+m-1$.
Define vertex labeling $f : V(G) \to \{0,1\}$ as follows.

Case (i) : $m = n-1$

$f(u_i) = 0$ for $1 \le i \le n-1$,
$f(v_{2i-1}) = 1$ for $1 \le i \le \left\lceil \frac{n}{2} \right\rceil$,
$f(v_{2i}) = 0$ for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$,
$f(w_{2i-1}) = 0$ for $1 \le i \le \left\lceil \frac{n}{2} \right\rceil$,
$f(w_{2i}) = 1$ for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$,

In view of the above labeling pattern we have, $v_f(0) = 2n-1$, $v_f(1) = n$, $e_f(1) = 2n-1$ and $e_f(0) = n-1$.
Here, $f(0) = 3n-2$and $f(1) =3n-1$.

Then, $| f(0) - f(1) | = 1$.
Therefore, the disconnected graph $(P_n \odot K_1) \cup C_m$ is total neighbourhood cordial graph for $m = n-1$.

Case (ii) : $m = n$

$f(u_i) = 0$ for $1 \le i \le n$,

$f(v_{2i-1}) = 1$ for $1 \le i \le \left\lceil \frac{n}{2} \right\rceil$,

$f(v_{2i}) = 0$ for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$,

$f(w_{2i-1}) = 0$ for $1 \le i \le \left\lceil \frac{n}{2} \right\rceil$,

$f(w_{2i}) = 1$ for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$,

In view of the above labeling pattern we have, $v_f(0) = 2n$, $v_f(1) = n$, $e_f(1) = 2n-1$ and $e_f(0) = n$.
Here, $f(0) = 3n$ and $f(1) = 3n-1$.
Then, $| f(0) - f(1) | = 1$.
Therefore, the disconnected graph $(P_n \odot K_1) \cup C_m$ is total neighbourhood cordial graph for $m = n$.
Hence, the disconnected graph $(P_n \odot K_1) \cup C_m$ is total neighbourhood cordial graph for $m = n-1$ and n, where $n \ge 2$ and $m \ge 3$.

Example 2.4

The graph $(P_5 \odot K_1) \cup C_5$ and its total neighbourhood cordial labeling is given in Figure 2.4.

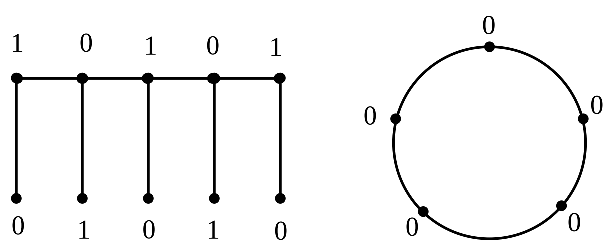

Figure 2.4

Theorem 2.5

The disconnected graph $P_n \cup K_{1,m,m}$ is total neighbourhood product cordial graph for $m = n$ and $m = n-1$, where $n, m \ge 2$.

Proof.
Let G be the disconnected graph $P_n \cup K_{1,m,m}$.
Let $u_1, u_2, ..., u_n$ and $v, v_1, v_2, ..., v_m, v_{m+1}, v_{m+2}, ..., v_{2m}$ be the vertices of P_n and $K_{1,m,m}$ respectively.
Then $|V(G)| = n+2m+1$ and $|E(G)| = n+2m-1$.
Define vertex labeling $f : V(G) \to \{0,1\}$ as follows.

Case (i) : $m = n$

$f(u_i) = 1$ for $1 \le i \le n$,

$f(v) = 1$,

$f(v_i) = 0$ for $1 \le i \le n$,

$f(v_i) = 1$ for $n+1 \le i \le 2n$,

In view of the above labeling pattern we have, $v_f(0) = n$, $v_f(1) = 2n+1$, $e_f(1) = n-1$ and $e_f(0) = 2n$.
Here, $f(0) = 3n$ and $f(1) = 3n$.
Then, $| f(0) - f(1) | = 0$.
Therefore, $P_n \cup K_{1,m,m}$ is total neighbourhood product cordial graph for $m = n$,

Case (ii) : $m = n-1$

$f(u_i) = 1$ for $1 \le i \le n$,

$f(v) = 0$,

$f(v_i) = 1$ for $1 \le i \le n-1$,

$f(v_i) = 0$ for $n \le i \le 2n-2$,

In view of the above labeling pattern we have, $v_f(0) = n$, $v_f(1) = 2n-1$, $e_f(1) = n-1$ and $e_f(0) = 2n-2$.
Here, $f(0) = 3n-2$ and $f(1) = 3n-2$.
Then, $| f(0) - f(1) | = 0$.
Therefore, $P_n \cup K_{1,m,m}$ is total neighbourhood product cordial graph for $m = n-1$.
Hence, $P_n \cup K_{1,m,m}$ is total neighbourhood product cordial graph for $m = n$ and $m = n - 1$.

Example 2.5

The graph $P_4 \cup K_{1,3,3}$ and its total neighbourhood product cordial labeling is given in Figure 2.5

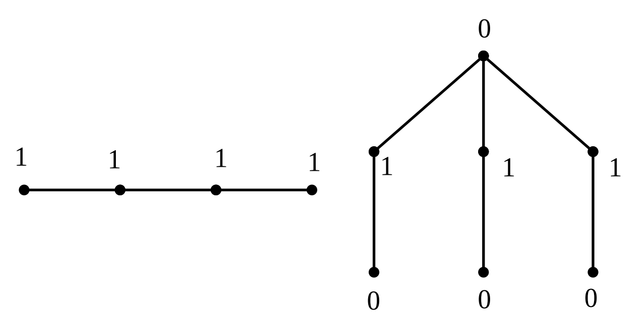

Figure 2.5

Theorem 2.6

The disconnected graph $C_n \cup K_{1,m,m}$ is total neighbourhood product cordial graph for $m = n-1$, n and $n+1$, where $n \ge 3$ and $m \ge 2$.

Proof.
Let G be the disconnected graph $C_n \cup K_{1,m,m}$.
Let $u_1, u_2, ..., u_n$ and $v, v_1, v_2, ..., v_m, v_{m+1}, v_{m+2}, ..., v_{2m}$ be the vertices of C_n and $K_{1,m,m}$ respectively.
Then $|V(G)| = n+2m+1$ and $|E(G)| = n+2m$.
Define vertex labeling $f : V(G) \to \{0,1\}$ as follows.

Case (i) : $m = n-1$

$f(u_i) = 1$ for $1 \le i \le n$,

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$f(v) = 0,$
$f(v_i) = 1$ for $1 \le i \le n-1$,
$f(v_i) = 0$ for $n \le i \le 2n-2$,

In view of the above labeling pattern we have, $v_f(0) = n$, $v_f(1) = 2n-1$, $e_f(1) = n$ and $e_f(0) = 2n-2$.
Here, $f(0) = 3n-2$ and $f(1) = 3n-1$.
Then, $| f(0) - f(1) | = 1$.
Therefore, the disconnected graph $C_n \cup K_{1,m,m}$ is total neighbourhood product cordial graph for $m = n-1$.

Case (ii) : $m = n$
$f(u_i) = 1$ for $1 \le i \le n$,
$f(v) = 0,$
$f(v_i) = 1$ for $1 \le i \le n$,
$f(v_i) = 0$ for $n+1 \le i \le 2n$,

In view of the above labeling pattern we have, $v_f(0) = n+1$, $v_f(1) = 2n$, $e_f(1) = n$ and $e_f(0) = 2n$.
Here, $f(0) = 3n+1$ and $f(1) = 3n$.
Then, $| f(0) - f(1) | = 1$.
Therefore, the disconnected graph $C_n \cup K_{1,m,m}$ is total neighbourhood product cordial graph for $m = n$.

Case (iii) : $m = n+1$
$f(u_i) = 1$ for $1 \le i \le n$,
$f(v) = 1,$
$f(v_i) = 0$ for $1 \le i \le n+1$,
$f(v_i) = 1$ for $n+2 \le i \le 2n+2$,

In view of the above labeling pattern we have, $v_f(0) = n+1$, $v_f(1) = 2n+2$, $e_f(1) = n$ and $e_f(0) = 2n+2$.
Here, $f(0) = 3n+3$ and $f(1) = 3n+2$.
Then, $| f(0) - f(1) | = 1$.
Therefore, the disconnected graph $C_n \cup K_{1,m,m}$ is total neighbourhood product cordial graph for $m = n$.
Hence, the disconnected graph $C_n \cup K_{1,m,m}$ is total neighbourhood product cordial graph for $m = n-1$, n and $n+1$, where $n \ge 3$ and $m \ge 2$.

Example 2.6

The graph $C_4 \cup K_{1,5,5}$ and its total neighbourhood product cordial labeling is given in Figure 2.6.

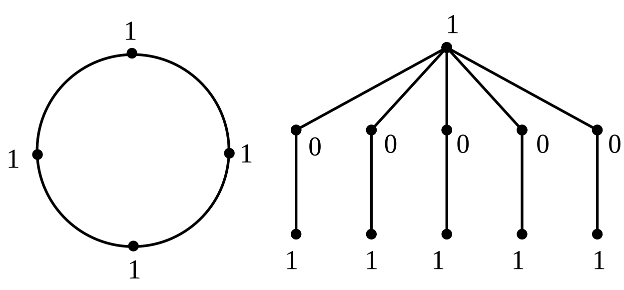

Figure 2.6

Theorem 2.7

The disconnected graph $K_{1,n} \cup K_{1,m,m}$ is total neighbourhood product cordial graph for $m = n$ and $n+1$, where $n, m \ge 2$.

Proof.
Let G be the disconnected graph $K_{1,n} \cup K_{1,m,m}$.
Let $u, u_1, u_2, ..., u_n$ and $v, v_1, v_2, ..., v_m, v_{m+1}, v_{m+2}, ..., v_{2m}$ be the vertices of $K_{1,n}$ and $K_{1,m,m}$ respectively.
Then $|V(G)| = n+2m+2$ and $|E(G)| = n+ 2m$.
Define vertex labeling $f : V(G) \to \{0,1\}$ as follows.

Case (i) : $m = n$
$f(u) = 1,$
$f(u_i) = 1$ for $1 \le i \le n$,
$f(v) = 0$
$f(v_i) = 1$ for $1 \le i \le n$,
$f(v_i) = 0$ for $n+1 \le i \le 2n$,

In view of the above labeling pattern we have, $v_f(0) = n+1$, $v_f(1) = 2n+1$, $e_f(0) = 2n$ and $e_f(1) = n$.
Here, $f(0) = 3n+1$ and $f(1) = 3n+1$.
Then, $| f(0) - f(1) | = 0$.
Therefore, the disconnected graph $C_n \cup K_{1,m,m}$ is total neighbourhood product cordial graph for $m = n$.

Case (ii) : $m = n+1$.
$f(u) = 1,$
$f(u_i) = 1$ for $1 \le i \le n$,
$f(v) = 1$
$f(v_i) = 0$ for $1 \le i \le n+1$,
$f(v_i) = 1$ for $n+2 \le i \le 2n+2$,

In view of the above labeling pattern we have, $v_f(0) = n+1$, $v_f(1) = 2n+3$, $e_f(0) = 2n+2$ and $e_f(1) = n$. Here, $f(0) = 3n+3$ and $f(1) = 3n+3$. Then, $| f(0) - f(1) | = 0$.
Therefore, the disconnected graph $C_n \cup K_{1,m,m}$ is total neighbourhood product cordial graph for $m = n+1$.
Hence, the disconnected graph $C_n \cup K_{1,m,m}$ is total neighbourhood product cordial graph for $m = n$ and $m = n+1$.

Example 2.7

The graph $K_{1,5} \cup K_{1,5,5}$ and its total neighbourhood product cordial labeling is given in Figure 2.7.

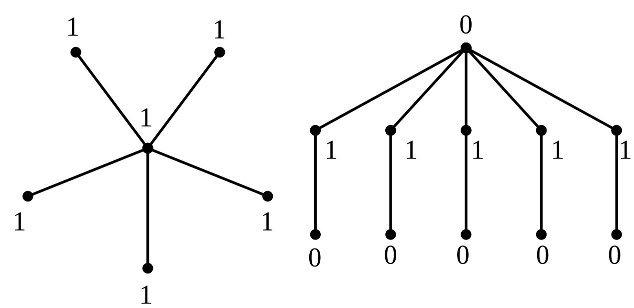

Figure 2.7

Theorem 2.8

The disconnected graph $P_{n+1} \cup B_{n,n}$ is total neighbourhood product cordial graph, where $n \geq 1$.

Proof.

Let G be the disconnected graph $P_{n+1} \cup B_{n,n}$.
Let $u_1, u_2, ..., u_{n+1}$ and $v, w, v_1, v_2, ..., v_n, w_1, w_2, ..., w_n$ be the vertices of P_{n+1} and $B_{n,n}$ respectively.
Then $|V(G)| = 3n+3$ and $|E(G)| = 3n+1$.
Define vertex labeling $f : V(G) \rightarrow \{0,1\}$ as follows.

$f(u_i) = 1$ for $1 \leq i \leq n+1$,
$f(v) = 0$,
$f(w) = 1$,
$f(v_i) = 1$ for $1 \leq i \leq n$,
$f(w_i) = 0$ for $n+1 \leq i \leq 2n$,

In view of the above labeling pattern we have, $v_f(0) = n+1$, $v_f(1) = 2n+2$, $e_f(1) = n$ and $e_f(0) = 2n+1$.
Here, $f(0) = 3n+2$ and $f(1) = 3n+2$.
Therefore, $| f(0) - f(1) | = 0$.
Hence, the disconnected graph $P_{n+1} \cup B_{n,n}$ is total neighbourhood product cordial graph, where $n \geq 1$.

Example 2.8

The graph $P_6 \cup B_{5,5}$ and its total neighbourhood product cordial labeling is given in Figure 2.8.

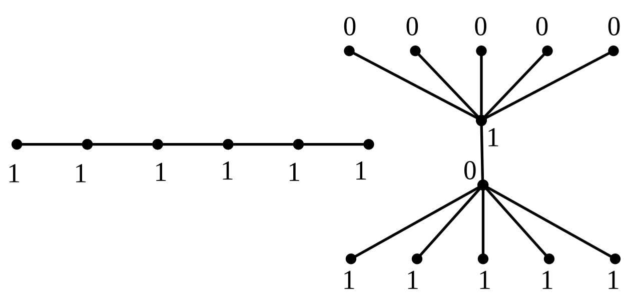

Figure 2.8

Theorem 2.9

The disconnected graph $C_n \cup B_{m,m}$ is total neighbourhood product cordial graph for $m = n-1$ and n, where $n \geq 3$ and $m \geq 2$.

Proof.

Let G be the disconnected graph $C_n \cup B_{m,m}$.
Let $u_1, u_2, ..., u_n$ and $v, w, v_1, v_2, ..., v_m, w_1, w_2, ..., w_m$ be the vertices of P_n and $B_{m,m}$ respectively.
Then $|V(G)| = n+2m+2$ and $|E(G)| = n+2m+1$.
Define vertex labeling $f : V(G) \rightarrow \{0,1\}$ as follows.

Case (i) : $m = n-1$

$f(u_i) = 1$ for $1 \leq i \leq n$,
$f(v) = 0$,
$f(w) = 1$,
$f(v_i) = 1$ for $1 \leq i \leq n-1$,
$f(w_i) = 0$ for $n \leq i \leq 2n-2$,

In view of the above labeling pattern we have, $v_f(0) = n$, $v_f(1) = 2n$, $e_f(1) = n$ and $e_f(0) = 2n-1$.
Here, $f(0) = 3n-1$ and $f(1) = 3n$.
Then, $| f(0) - f(1) | = 1$.
Therefore, the disconnected graph $C_n \cup B_{m,m}$ is total neighbourhood product cordial graph for $m = n-1$.

Case (ii) : $m = n$

$f(u_i) = 1$ for $1 \leq i \leq n$,
$f(v) = 0$,
$f(w) = 1$,
$f(v_i) = 1$ for $1 \leq i \leq n$,
$f(w_i) = 0$ for $n+1 \leq i \leq 2n$,

In view of the above labeling pattern we have, $v_f(0) = n+1$, $v_f(1) = 2n+1$, $e_f(1) = n$ and $e_f(0) = 2n+1$.
Here, $f(0) = 3n+2$ and $f(1) = 3n+1$.
Then, $| f(0) - f(1) | = 1$.
Therefore, the disconnected graph $C_n \cup B_{m,m}$ is total neighbourhood product cordial graph for $m = n$.
Hence, the disconnected graph $C_n \cup B_{m,m}$ is total neighbourhood product cordial graph for $m = n-1$ and n, where $n \geq 3$ and $m \geq 2$.

Example 2.9

The graph $C_5 \cup B_{4,4}$ and its total neighbourhood product cordial labeling is given in Figure 2.9.

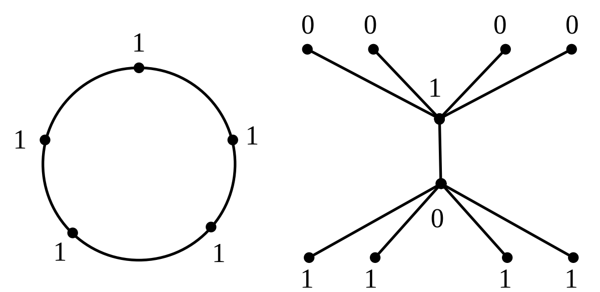

Figure 2.9

Theorem 2.10

The disconnected graph $K_{1,n} \cup B_{n,n}$ is total neighbourhood cordial graph, where $n \geq 2$.

Proof.

Let G be the disconnected graph $K_{1,n} \cup B_{n,n}$.
Let $u, u_1, u_2, ..., u_n$ and $v, w, v_1, v_2, ..., v_n, w_1, w_2, ..., w_n$ be the vertices of $K_{1,n}$ and $B_{m,m}$ respectively.
Then $|V(G)| = 3n+2$ and $|E(G)| = 3n$.
Define vertex labeling $f : V(G) \rightarrow \{0,1\}$ as follows.

$f(u) = 1$
$f(u_i) = 1$ for $1 \le i \le n$,
$f(v) = 0$,
$f(w) = 1$,
$f(v_i) = 1$ for $1 \le i \le n$,
$f(w_i) = 0$ for $n+1 \le i \le 2n$,

In view of the above labeling pattern we have, $v_f(0) = n+1$, $v_f(1) = 2n+2$, $e_f(0) = 2n+1$ and $e_f(1) = n$.
Here, $f(0) = 3n+2$ and $f(1) = 3n+2$.
Then, $| f(0) – f(1) | = 0$.
Therefore, the disconnected graph $K_{1,n} \cup B_{n,n}$ is total neighbourhood product cordial graph, .

Example 2.10

The graph $K_{1,5} \cup B_{5,5}$ and its total neighbourhood product cordial labeling is given in Figure 2.10.

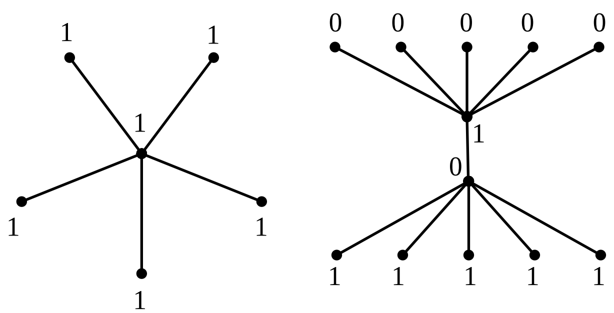

Figure 2.10

Theorem 2.11

The disconnected graph $(P_n \odot K_1) \cup C_m$ is total neighbourhood product cordial graph for $m = n–1$ and n, where $n \ge 2$ and $m \ge 3$.

Proof.
Let G be the disconnected graph $(P_n \odot K_1) \cup C_m$.
Let $u_1, u_2, ..., u_m$ and $v_1, v_2, ..., v_n, w_1, w_2, ..., w_n$ be the vertices of C_m and $(P_n \odot K_1)$ respectively.
Then $|V(G)| = 2n+m$ and $|E(G)| = 2n+m–1$.
Define vertex labeling $f : V(G) \to \{0,1\}$ as follows.

Case (i) : $m = n–1$

$f(u_i) = 1$ for $1 \le i \le n–1$,
$f(v_{2i-1}) = 1$ for $1 \le i \le \left\lceil \frac{n}{2} \right\rceil$,
$f(v_{2i}) = 0$ for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$,
$f(w_{2i-1}) = 0$ for $1 \le i \le \left\lceil \frac{n}{2} \right\rceil$,
$f(w_{2i}) = 1$ for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$,

In view of the above labeling pattern we have, $v_f(0) = n$, $v_f(1) = 2n–1$, $e_f(1) = n–1$ and $e_f(0) = 2n–1$.
Here, $f(0) = 3n–1$ and $f(1) = 3n–2$.
Then, $| f(0) – f(1) | = 1$.
Therefore, the disconnected graph $(P_n \odot K_1) \cup C_m$ is total neighbourhood product cordial graph for $m = n–1$.

Case (ii) : $m = n$

$f(u_i) = 1$ for $1 \le i \le n$,
$f(v_{2i-1}) = 1$ for $1 \le i \le \left\lceil \frac{n}{2} \right\rceil$,
$f(v_{2i}) = 0$ for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$,
$f(w_{2i-1}) = 0$ for $1 \le i \le \left\lceil \frac{n}{2} \right\rceil$,
$f(w_{2i}) = 1$ for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$,

In view of the above labeling pattern we have, $v_f(0) = n$, $v_f(1) = 2n$, $e_f(1) = n$ and $e_f(0) = 2n–1$.
Here, $f(0) = 3n–1$ and $f(1) = 3n$.
Then, $| f(0) – f(1) | = 1$.
Therefore, the disconnected graph $(P_n \odot K_1) \cup C_m$ is total neighbourhood product cordial graph for $m = n$.
Hence, the disconnected graph $(P_n \odot K_1) \cup C_m$ is total neighbourhood product cordial graph for $m = n–1$ and n, where $n \ge 2$ and $m \ge 3$.

Example 2.11

The graph $(P_5 \odot K_1) \cup C_4$ and its total neighbourhood product cordial labeling is given in Figure 2.11.

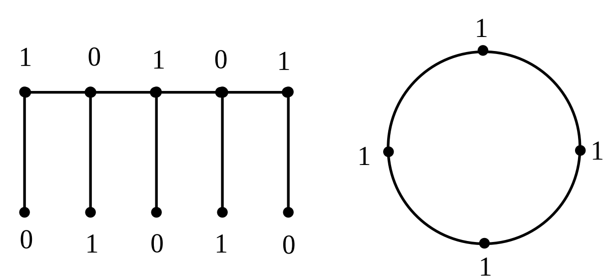

Figure 2.11

4. Conclusions

In this paper, we present total neighbourhood cordial labeling $P_{n+1} \cup B_{n,n}$, $C_n \cup B_{m,m}$, $K_{1,n} \cup B_{n,n}$, $(P_n \odot K_1) \cup C_m$ under some conditions and total neighbourhood product cordial labeling $P_n \cup K_{1,m,m}$, $C_n \cup K_{1,m,m}$,

$K_{1,n} \cup K_{1,m,m}$, $P_{n+1} \cup B_{n,n}$, $C_n \cup B_{m,m}$, $K_{1,n} \cup B_{n,n}$, $(P_n \odot K_1) \cup C_m$ under some conditions.

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