ON FUZZY G-DOMAINS

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Abstract. The fuzzy algebraic structures for the first time was studied in 1982 by Liu[16], so the subject of fuzzy rings presented and organazied in 1992 by Dixit[8,9]. In this paper on the first step we have pointed to the general properties of fuzzy rings by applying of the papers Swamy[21], Kumbhojkar[15], Zahedi[23], Mukherjee[19] and zhang Yue[24]. In addition it was produced some results and theorems related to the concepts of domains. Finally by expressing of the classic concept of G-domains from Kaplansky[12], we have a new definition of the fuzzy G-Domains and their related properties which will give us new resolutions.

1. Elementary Concepts of Fuzzy algebraic Structures

In this section, we first introduce some operations on fuzzy susets of a commutative ring R and The set of all fuzzy subsets on R denoted by $F(R)$.

Definition 1.1. Let μ, $\nu \in F(R)$. Define $\mu+\nu$, $-\mu$, $\mu-\nu$, $\mu o\nu \in F(R)$ as follows:
1) $(\mu+\nu) = sup\{inf\{\mu(y), \nu(z)\}|y, z \in R, y+z = x\} = \vee_{(y+z=x)}(\mu(y) \wedge \nu(z))$
2) $(-\mu)(x) = \mu(-x)$
3) $(\mu-\nu)(x) = sup\{inf\{\mu(y), \nu(z)\}|y, z \in R, y-z = x\} = \vee_{(y-z=x)}(\mu(y) \wedge \nu(z))$
4) $(\mu o\nu)(x) = sup\{inf\{\mu(y), \nu(z)\}|y, z \in R, yz = x\} = \vee_{(yz=x)}(\mu(y) \wedge \nu(z))$
$\forall x \in R$. $\mu+\nu$, $\mu-\nu$ and $\mu o\nu$ are called the sum, difference and product of μ and ν respectively and $-\mu$ is called the negative of μ.

Since R is commutative, $\mu\nu = \nu\mu$, $\forall \mu, \nu \in F(R)$.

Theorem 1.1 (18)**.** *Let μ, ν and $\xi \in F(R)$. Then the following assertions hold.*
1) $\mu o\nu \subseteq \mu\nu$.
2) $\nu \subseteq \xi \Longrightarrow \mu\nu \subseteq \mu\xi$.
3) $(\mu\nu)\xi = \mu(\nu\xi)$.
4) $(\mu\nu)(x+y) \geq inf\{(\mu\nu)(x), (\mu\nu)(y)\} = (\mu\nu(x) \wedge \mu\nu(y))$, $\forall x, y \in R$.
5) If R has an identity 1 and $1_{\{1\}} \subseteq \nu$, then $\mu \subseteq \mu\nu$.
6) $1_R o\mu \subseteq \mu$.

Definition 1.2. Let $\mu \in F(R)$. Define μ^n and $\mu^{(n)} \in (F(R)$ as follows, where $n \in \mathbb{N}$, $n > 1$:
$$\mu^1 = \mu \quad , \quad \mu^n = \mu^1 o\mu^{n-1}$$
and
$$\mu^{(1)} = \mu \quad , \quad \mu^{(n)} = \mu^{(1)}\mu^{(n-1)}$$

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Definition 1.3. [1] Let R is a ring and $A \in F(R)$, we say A is a fuzzy subring of R if for all $a, b \in R$:
i) $A(a-b) \geq inf\{A(a), A(b)\} = (A(a) \wedge A(b))$.
ii) $A(ab) \geq inf\{A(a), A(b)\} = (A(a) \wedge A(b))$.

Definition 1.4. The $A \in F(R)$ is called an fuzzy subdomain of R if:
i) A is an fuzzy subring of R.
ii) For all $x, y \in R$ if $A(xy) = 0$ then; $A(x) = 0$ or $A(y) = 0$.
It is obvious that $A(xy) \geq inf\{A(x), A(y)\}$ (Since A is an fuzzy subring of R).

Definition 1.5. The set of all fuzzy subdomain of D is called by $F(D)$, and for each $A, B \in F(D)$:
i) $(A \cap B)(x) = inf\{A(x), B(x)\} = (A(x) \wedge B(x))$.
ii) $(A \cup B)(x) = sup\{A(x), B(x)\} = (A(x) \vee B(x))$.
iii) $(A.B)(x) = A(x)B(x)$.
iv) $(A + B)(x) = A(x) + B(x) - A(x)B(x)$.
v) $(A \oplus B)(x) = inf\{1, A(x) + B(x)\}$.
vi) $(A \ominus B)(x) = sup\{0, A(x) + B(x) - 1\}$.

Note 1.1. For every fuzzy subdomain A of R and for each $t \in [0, 1]$, A_t is called "$t - cut$" or "$t - level\ set$" of A.

Theorem 1.2. *A is a fuzzy subdomain of R if and only if for all $t \in [0, 1]$ the t-level set of A_t is a domain.*

Proof. Suppose A is a fuzzy subdomain of R. Let $a \in A(R)$. Since $A(0) \geq A(x)$, $\forall x \in R$, $0 \in A_a$. Thus $A_a \neq \emptyset$. Let $x, y \in A_a$. Then $A(x) \geq a$ and $A(y) \geq a$. Since A is a fuzzy subdomain, $A(xy) \geq inf\{A(x), A(y)\} = (A(x) \wedge A(y)) \geq a \wedge a = a$. Hence $xy \in A_a$ and so A_a is a subdomain of R. Similarly if $a \leq A(0)$, then A_a is a subdomain of R.
Conversely, suppose A_a is a subdomain of R , $\forall a \in A(R) \cup \{b \in [0, 1] | b \leq A(0)\}$. Since $0 \in A_a$, $\forall a \in A(R)$, $A(0) \geq a$, $\forall a \in A(R)$. Let $x, y \in R$ and let $A(x) = a$ and $A(y) = b$. Let $m = a \wedge b$, then $x, y \in A_m$ and $m \leq A(0)$. Thus A_m is a subdomain of R and hence $xy \in A_m$. Hence $A(xy) \geq m = a \wedge b = A(x) \wedge A(y)$. Thus A is a fuzzy subdomain of R. □

Example 1.1. *If $A(x)$ is defined as the following:*

$$A(x) = \begin{cases} 1 & x = 0 \\ 1/2 & x \in 30\mathbb{Z} - \{0\} \\ 1/3 & x \in 6\mathbb{Z} - 30\mathbb{Z} \\ 1/5 & x \in 2\mathbb{Z} - 6\mathbb{Z} \\ 0 & x \in \mathbb{Z} - 2\mathbb{Z} \end{cases}$$

Since $A_0 = \mathbb{Z}$, $A_{\frac{1}{5}} = 2\mathbb{Z}$, $A_{\frac{1}{3}} = 6\mathbb{Z}$, $A_{\frac{1}{2}} = 30\mathbb{Z}$ and $A_1 =< 0 >$, then; $A(x)$ is a fuzzy subdomain of R.

The following theorem is an easy consequence of Definition "1.5" and Theorem "1.2".

Theorem 1.3. *If $A, B \in F(D)$ then:*
i) $(A \cap B) \in F(D)$
ii) $(AB) \in F(D)$
iii) $(A + B) \in F(D)$

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iv) $(A \oplus B) \in F(D)$
v) $(A \ominus B) \in F(D)$.

Proof. We prove "ii" and the other assertions are similar.
Clearly, $(AB)(-x) = (AB)(x), \quad \forall x \in D$, and

$$(AB)(x+y) \geq (AB)(x) \wedge (AB)(y), \quad \forall x, y \in D$$

Let $x, y \in D$ and concider $(AB)(xy)$. If $x = \sum_{i=1}^{m} x_i x_i'$ and $y = \sum_{j=1}^{n} y_j y_j'$, where $x_i, x_i' \in D$ for $1 \leq i \leq m$ and $y_j, y_j' \in D$ for $1 \leq j \leq n$, then:

$$xy = (\sum_{i=1}^{m} x_i x_i')(\sum_{j=1}^{n} y_j y_j') = \sum_{i=1}^{m}\sum_{j=1}^{n}(x_i y_j)(x_i' y_j')$$

By the definition of AB, it follows that:

$$(AB)(xy) \geq \wedge_{i=1}^{m} \wedge_{j=1}^{n} (A(x_i y_j) \wedge B(x_i' y_j')$$
$$\geq \wedge_{i=1}^{m} \wedge_{j=1}^{n} ((A(x_i) \wedge A(y_j)) \wedge (B(x_i') \wedge B(y_j')))$$
$$= \wedge_{i=1}^{m} \wedge_{j=1}^{n} ((A(x_i) \wedge B(x_i')) \wedge (A(y_j) \wedge B(y_j')))$$
$$= (\wedge_{i=1}^{m}(A(x_i) \wedge B(x_i'))) \wedge (\wedge_{j=1}^{n}(A(y_j) \wedge B(y_j'))).$$

Consequently,

$$(AB)(xy) \geq (\vee\{\wedge_{i=1}^{m}(A(x_i) \wedge B(x_i'))|x_i, x_i' \in D, 1 \leq i \leq m, \sum_{i=1}^{m} x_i x_i' = x\})\wedge$$
$$(\vee\{\wedge_{j=1}^{n}(A(y_j) \wedge B(y_j'))|y_j, y_j' \in D, 1 \leq j \leq n, \sum_{j=1}^{n} y_j y_j' = y\})$$
$$= (AB)(x) \wedge (AB)(y).$$

Hence $AB \in F(D)$. □

Theorem 1.4. *Let A, B and $C \in F(D)$, then $AoB \subseteq C$ if and only if $AB \subseteq C$.*

Proof. Clearly $AoB \subseteq AB$. Thus if $AB \subseteq C$, then $AoB \subseteq C$. Conversely, suppose $AoB \subseteq C$. Let $x \in D$ and let $x = \sum_{i=1}^{n} y_i z_i$, $y_i, z_i \in D$, $i = 1, 2, ..., n$. Then

$$C(x) = C(\sum_{i=1}^{n} y_i z_i) \geq \wedge_{i=1}^{n}(y_i z_i).$$

Now

$$C(y_i z_i) \geq (AoB)(y_i z_i) \geq A(y_i) \wedge B(z_i) \quad , \quad \forall i = 1, 2, ..., n$$

Thus

$$\wedge_{i=1}^{n}\{A(y_i) \wedge B(z_i)\} \leq \wedge_{i=1}^{n}\{C(y_i z_i)\} \leq C(x)$$

Hence

$$\vee\{\wedge_{i=1}^{n}(A(y_i) \wedge B(z_i))|x = \sum_{i=1}^{n} y_i z_i, \quad n \in \mathbb{N}\} \leq C(x)$$

That is $AB(x) \leq C(x)$. Hence $AB \subseteq C$. □

Corollary 1.1. *For each $A, B \in F(D)$, we have:*

$$(A \ominus B) \subseteq (A.B) \subseteq (A \cap B) \subseteq (A + B) \subseteq (A \oplus B).$$

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Corollary 1.2. *For any arbitrary family of fuzzy subdomains* $\{D_\alpha\}_{\alpha\in\Lambda} \in F(D)$ *we have:*

$$D = \bigcap_{\alpha\in\Lambda} D_\alpha \in F(D).$$

Where $\bigcap_{\alpha\in\Lambda} D_\alpha(x) = inf\{D_\alpha(X)|x \in D\}$.

Theorem 1.5. *Let* A, B *and* $C \in F(D)$ *and* $B(0) = C(0)$*. Then*

$$A(B + C) = AB + AC$$

Proof. Since $B \subseteq B+C$ and $C \subseteq B+C$, $AB \subseteq A(B+C)$ and $AC \subseteq A(B+C)$. Hence $AB + AC \subseteq A(B + C)$. Let $x \in D$. Now

$$\begin{aligned}(A(B+C))(x) &= \vee\{\wedge_{i=1}^{n}(A(y_i)\wedge(B+C)(z_i))|y_i, z_i \in D, 1 \le i \le n, n \in \mathbb{N}, \sum_{i=1}^{n} y_i z_i = x\}\\
&= \vee\{\wedge_{i=1}^{n}(A(y_i) \wedge (\vee\{B(u_i) \wedge C(v_i)|u_i, v_i \in D, u_i + v_i = z_i\\
&\qquad \}))|y_i, z_i \in D, 1 \le i \le n, n \in \mathbb{N}, \sum_{i=1}^{n} y_i z_i = x\}\\
&= \vee\{\wedge_{i=1}^{n}(A(y_i) \wedge B(u_i) \wedge C(v_i))|u_i, v_i, y_i \in D, 1 \le i \le n,\\
&\qquad n \in \mathbb{N}, \sum_{i=1}^{n}(y_i u_i + y_i v_i) = x\}\\
&\le \vee\{(\wedge_{i=1}^{l}(A(y_i') \wedge B(u_i'))) \wedge (\wedge_{k=1}^{s}(A(y_k'') \wedge C(v_k'))|u_i', v_i', y_i', y_k'' \in D,\\
&\qquad 1 \le i \le l, 1 \le k \le s, s, n \in \mathbb{N}, \sum_{i=1}^{l}(y_i' u_i' + \sum_{k=1}^{s} y_k'' v_k') = x\}\\
&= \vee\{(AB)(a) \wedge (AC)(b)|a, b \in D, a + b = x\}\\
&\qquad ((AB) + (AC))(x).\end{aligned}$$

□

Corollary 1.3. *If* D *is a commutative domain then for each* $A, B \in F(D)$*:*

$$(AB) = (BA)$$

Corollary 1.4. *For each* $A, B, C \in F(D)$

$$\textit{if} \quad A \subseteq B \quad \textit{then} \quad AC \subseteq BC$$

2. Fuzzy G-Subdomains

Remark 2.1 (kaplansky)**.** Let D be an integral domain with quotient field K, the following two statements are equivalent:
i) K is a finitely generated ring over D.
ii) K as a ring, can be generated over D by one element.

Definition 2.1. An integral domain satisfying either (hence both) of the statements in Theorem "2.1" is called a $G-domain$.

The name honors Oscar Goldman. His paper [11] appeared at virtually the same time as a similar paper br Krull [13]. Since Krull already has a class of rings named after him, it seems advisable not to attempt to honor Ktull in this connection. $G-domains$ were also considered by Artin and Tate in [4]. Further results concerning the material in this section appear in Gilmer's paper [10].

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Example 2.1. *1) A field is a* $G-domain$*.*
2) A pricipal ideal domain is a $G-domain$ *if and only if it has only a finite number of primes(up to units).*
3) A Noetherian domain is a $G-domain$ *if and only if it has only a finite number of non-zero prime ideals such that all of which are maximal.*

Definition 2.2. Let D be a $G-domain$, A is a fuzzy $G-subdomain$ of D if for all $x, y \in D$ we have:
i) A is a fuzzy subdomain of D(i.e. $A \in F(D)$).
ii) If K be the quotient field of D, then $K = D[u^{-1}]$, for some $0 \neq u \in D$.

Theorem 2.1. *Let* A *be a fuzzy* $G-subdomain$ *with quotient fuzzy subfield* K *and let* B *be a fuzzy subring lying between* A *and* K*, then* B *is a fuzzy* $G-subdomain$*.*

Proof. If $K = A[u^{-1}]$ be the quoitent fuzzy subfield of A, then $K = B[u^{-1}]$ is also the quotient fuzzy subfield of B, therefore B is a fuzzy $G-subdomain$. □

Corollary 2.1. *If* A *be a fuzzy* $G-subdomain$ *with it's quotient fuzzy subfield of* K*, then For* $0 \neq u \in A$ *we have:*

$$K = A[u^{-1}]$$

Theorem 2.2. *Let* $A, B \in F(D)$ *be arbitrary members, if* $A \subset B$ *and* B *is algebraic on* A *and* B *as a fuzzy subring above* A *is finitely generated, then:*
A *is a fuzzy* $G-subdomain$ *if and only if* B *is a fuzzy* $G-subdomain$*.*

Proof. Let K , L be the quotient fuzzy subfield of A , B. Suppose first that A is a *fuzzy* $G-subdomain$, say $K = A[u^{-1}]$. Then $B[u^{-1}]$ is a fuzzy subdomain algebraic over the fuzzy subfield of K, hence is itself a fuzzy subfield, necessarily equal to L. Thus B is a *fuzzy* $G-subdomain$.
Conversely, We assume that B is a *fuzzy* $G-subdomain$, $L = B[v^{-1}]$ and $B = A[w_1, w_2, ..., w_k]$. The elements $v^{-1}, w_1, w_2, ..., w_k$ are algebraic over A and consequently satisfy equations with coefficients in A which lead off, say

$$av^{-m} + ... = 0$$

$$b_i w_i^{n_i} + ... = 0 \qquad (i = 1, ..., k)$$

Adjoin $a^{-1}, b_1^{-1}, ..., b_k^{-1}$ to A, obtaining a fuzzy subring A_1 between A and K. The field L is generated over A by $w_1, ..., w_k, v^{-1}$. Of course these elements generate L over A_1. Now over A_1 we have arranged that $w_1, ..., w_k, v^{-1}$ are integral. Hence L is integral over A_1. Therefore, A_1 is a fuzzy subfield , necessarily K. So K is a finitely generated fuzzy subring over A and A is a *fuzzy* $G-subdomain$, as required. □

Corollary 2.2. *Let* A *be a fuzzy subdomain and* u *be an element located on a fuzzy subdomain contaning* A*, if* $A[u]$ *is a fuzzy* $G-subdomain$ *then:*
A *is fuzzy* $G-subdomain$ *and* u *is algebraic on* A *.*

Theorem 2.3. A *is a fuzzy* $G-subdomain$ *if and only if for each* $t \in [0, 1]$*,* A_t *is a* $G-domain$*.*

Proof. Let A be a fuzzy $G-subdomain$ of D and $t \in [0, 1]$, since $A(0) \geq A(x)$, $\forall x \in D$, then: $0 \in A_t$, therefore $A_t \neq \emptyset$. Now let $x, y \in A_t$, since $A(x) \geq t$, $A(y) \geq t$ and A is a fuzzy subdomain of D, then:

$$A(xy) \geq inf\{A(x), A(y)\} = A(x) \wedge A(y) \geq t \wedge t = t$$

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$$\Rightarrow A(xy) \geq t \Rightarrow xy \in A_t$$

Futhermore:

$$A(x-y) \geq inf\{A(x), A(y)\} = A(x) \wedge A(y) \geq t \wedge t = t$$

$$\Rightarrow A(x-y) \geq t \Rightarrow x - y \in A_t$$

On the other hand, for the quotient field of K related to D, we have $K = D[u^{-1}]$, for $0 \neq u \in K$.
Therefore $\forall t \in [0,1]$, A_t is a domain.
Now since A_t is a subdomain of D and for some $0 \neq u \in k$, $K = D[u^{-1}]$, so

$$A_t \leq D \Rightarrow A_t[u^{-1}] \leq D[u^{-1}] = K$$

Therefore by Corollary "2.2" u is algebraic on D and D is a $G-domain$ too.
Conversely, let A_t be a $G-subdomain$ of D, for each $t \in A(D) \cup \{b \in [0,1] | b \leq A(0)\}$.
Since $0 \in A_t$, $\forall t \in A(D)$, then $A(0) \geq t$, $\forall t \in A(D)$. Let $x, y \in D$ and let $A(x) = t_1$, $A(y) = t_2$ and let $t_3 = t_1 \wedge t_2$ then, $x, y \in A_{t_3}$, therefore $t_3 \leq A(0)$. Hence A_{t_3} is a subdomain of D and so $xy \in A_{t_3}$. Now we have:

$$A(xy) \geq t_3 = t_1 \wedge t_2 = A(x) \wedge A(y)$$

And since A has the $G-structure$, therefore A is a fuzzy subdomain of D.

□

Example 2.2. *Let $\mathbb{Q}$ be the Rational numbers, since for each prime number $2, 3, 5, 7, ...$ the extended fields $\mathbb{Q}[\sqrt{2}]$, $\mathbb{Q}[\sqrt{2}, \sqrt{3}]$, $\mathbb{Q}[\sqrt{2}, \sqrt{3}, \sqrt{5}]$ and $\mathbb{Q}[\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}]$ are $G-domains$ too. If we define the $A(x)$ as the following:*

$$A(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ 1/2 & x \in \mathbb{Q}[\sqrt{2}] - \mathbb{Q} \\ 1/3 & x \in \mathbb{Q}[\sqrt{2}, \sqrt{3}] - \mathbb{Q}[\sqrt{2}] \\ 1/5 & x \in \mathbb{Q}[\sqrt{2}, \sqrt{3}, \sqrt{5}] - \mathbb{Q}[\sqrt{2}, \sqrt{3}] \\ 1/7 & x \in \mathbb{Q}[\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}] - \mathbb{Q}[\sqrt{2}, \sqrt{3}, \sqrt{5}] \\ 0 & x \in \mathbb{Q}[\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, ..] = \mathbb{R} \end{cases}$$

Since for each $t \in [0,1]$, A_t is a $G-domain$ as the follows:
$A_0 = \mathbb{Q}[\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, ..] = \mathbb{R}$, $A_{\frac{1}{7}} = \mathbb{Q}[\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}]$, $A_{\frac{1}{5}} = \mathbb{Q}[\sqrt{2}, \sqrt{3}, \sqrt{5}]$, $A_{\frac{1}{3}} = \mathbb{Q}[\sqrt{2}, \sqrt{3}]$, $A_{\frac{1}{2}} = \mathbb{Q}[\sqrt{2}]$ and $A_1 = \mathbb{Q}$, therefore A is a fuzzy $G-subdomain$ of $\mathbb{R}$.

It is suggested that further research in this direction is likely going to reveal additional properties of fuzzy $G-ideals$ associated to fuzzy subdomains and thus contribute to our understanding of how such structures defines on the underlying fuzzy $G-subdomains$.

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