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# Transient Stability Assessment of Multi-Machine Power System Using Cusp Catastrophe Theory

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**Abstract-**This manuscript presents a new technique to derive an accurate Transient Stability Boundary which can be readily used in multiple applications related to ensuring dynamic security of a power system. The intended applications of the proposed boundary are: (a) using it as a constraint in a dynamic security constrained optimal power flow program, (b) fast transient security assessment and control, and (c) redispatch of generation for preventive control. The proposed method approximates the ideal transient Stability boundary with an accuracy specified by the user. As the transient stability boundary is not perfect, some stable operating points may be classified as unstable and some unstable operating points may be classified as stable. For the applications stated above, the misclassification of operating points must be minimized and any misclassified cases must be confined to marginal cases. Further, the estimated Transient Stability Boundary must be expressed in terms of the pre-contingency operating point. In this manuscript catastrophe theory is used to determine the transient stability regions. Taylor series expansion is used to find the energy balance equation in terms of clearing time and system transient parameters. Numerical example of sugar 7machine power systems shows a very good agreement with the time solution in the practical range of first swing stability analysis. The method presented in this manuscript fulfills all the requirements for on-line assessment of transient stability of power systems.

**Keywords:** — Cusp Catastrophe Theory, Critical Machine, Distributed Machine, Large Power System, Transient Stability Assessment.

## 1. INTRODUCTION

Power systems are subjected to major disturbances such as: faults, loss or acquisition of generation, or large blocks of loads. After a disturbance, if the power system cannot reach a steady state operating condition, it is considered unstable. The purpose of a transient stability detection algorithm or device is to detect transient stability swings, and classify them as being stable or unstable. If the system is classified as Unstable, preventive or corrective control actions or system separations have to be done to restore system stability. Time-domain integration of power system differential equations is the most accurate method for transient stability assessment. However, it has a heave computational burden, which restricts it from being used for online applications. All other on-line transient stability assessment methods make some compromise between accuracy and speed. Earlier methods used to detect the transient stability swings were based on the out-of-step relays (OSR) using the apparent impedance concept [1,2]. Another method of OSR is based on the direct method of Lyapunov to determine the stability boundary for an equivalent two-machine system [3,4]. In other methods, the out-of-step (OS) prediction method is based on the generator's angular velocity [5], phase angle difference Among several large generator groups [6], and modeling the OS condition as a periodic motion represented by a Fourier series [7]. All these OSR schemes assume specific operating conditions in power systems. If these conditions drastically change, these relays misoperate. This led to the adaptive OSR method based on phasor measurement techniques [8,9]. However, this method is based on two-machine equivalent systems. Another method proposed a stability detection index based on the measured generator phase angles, angular velocity and their rate of change [10]. In this method, the instability is detected based on checking the characteristics of a hyper-surface on which post-fault trajectory lies, identifying it as being concave or convex. This method is applied to multimachine power system, and it is based on the classical model of the power system. Another approach for dynamic security assessment uses trajectory sensitivities [11]. It uses the sensitivity of the state variable of the system to the parameter variable and initial conditions. The values of these sensitivities are used as a measure of power system stability. It can be applied to multi-machine systems. However, it has limitations to power system modeling complexity. An on-line method was suggested for transient stability calculation based data taken from the state estimator and preliminary contingency Screening to identify suspected cases using defined indices [12]. This scheme assumes special communication system arrangement with high speed communication channels. Other methods for transient stability detection include the use o f Decision Tree (DT) methods using the angle threshold values [13], or Critical Clearing Time (CCT) [14]. One may expect better results from the pattern recognition method because of the fact that it is independent of the model used to simulate the power system. This method can be used for an on-line assessment of stability even for individual machines, but it cannot be generalized because each decision function is good only for the system trained for. The motivation for this work is to search for a method which is suitable for an on-line assessment of transient stability that can be used for every individual generator in the power system. If such a method can be found then the problem of transient stability can be dealt with on-line stability assessment. The most suitable method (from among the existing methods) for such an application is pattern recognition because of the fact that it is independent of the systems equations which couple the generators together. This brought up the idea of using catastrophe theory to define the stability region [1]

## 2. CATASTROPHE THEORY

Catastrophe theory is a nonlinear conceptual model developed to advance scientific explanations previously restricted to Newtonian order and predictability. Although credit is routinely given to Rene' Thom for the original genesis, the work can be traced to the French mathematical genius and savant, Jules Henri Poincare'. Poincare' developed, among many other famous tenets, an initial study examining how the qualitative properties of a system change as the parameters describing that system change. From this work, Thom developed a new form of topological mathematics that allowed for the analysis of discontinuous changes in complex systems (Thom, 1975) It was classified as a nonlinear theory and extolled as a major breakthrough in modeling. The name itself, catastrophe, is derived from the unusual hill and valley shape of parts of the model, and comes from the Greek derivation ‘kata’ meaning ‘down, and ‘strophe’ meaning ‘turning.” Although Thom’s work and subsequent publications appeared in the 1960s, English translations (from the French) were not published until 1975

This work generated great confusion and controversy. Thom (1975) developed seven representative models of the theory, one of which is the cusp catastrophe model that is used in this study. However, according to Poston and Stewart (1978) and Gilmore (1981), caution in extending these catastrophe models to realms where the applicability is not guaranteed by the appropriate mathematical formalism was not always exercised. Bogus derivative applications of the theory were postulated for everything from a dog’s barking response to why revolutions occur, to why a person has a nervous breakdown. Thus, to be more confident that the theory is being applied and interpreted correctly, an

Initial examination of the mathematical basis of the application is essential Comprehensive descriptions of the mathematical basis of catastrophe theory are found in

Brown (1995), Gilmore (1981), Poston and Stewart (1978), Thom (1975), and Zeeman (1976) and referenced periodically in this chapter. However, the recent work of van der Maas, Kolstein, and the van der Plight (2003) is particularly illuminating and will be used as a guideline in developing the rigor of the model for the conceptual framework of this study. To examine a cusp catastrophe model, a review of the essentials of the theory is Summarized in the following section paraphrased from Wagner and Huber (2003).

### 2.1. Catastrophe Theory And Bifurcation Analysis

Catastrophe theory can be briefly described as follows. Consider a system whose. The behavior is usually smooth but which exhibits some discontinuities. Suppose the system has a smooth potential function to describe the system dynamics and has “n” state variables and “m” control parameters. Given such a system, catastrophe theory tells us the following: The number of qualitatively different ‘configurations of discontinuities that can occur depends not on the number of state variables but on the number of control parameters. Specifically, if the number of the control parameters is not greater than four, there are seven basic or elementary catastrophes, and in none of these are more than

Two state variables involved [10].

Consider a continuous potential function *V (Y, C)* which represents the system behavior, where *Y* are the state variables and *C* are the control parameters. The potential function *V(Y,C)* can be mapped in terms of its control variables *C* to define the continuous region. Let the potential function be represented by:

$$V(Y,C): M \otimes R \tag{1}$$

Where *M, C* are manifolds in the state space $R^n$ and the control space $R^r$ respectively.

Now we define the catastrophe manifold $M$ as the equilibrium surface that represents all critical points of *V(Y,C)*. It is the subset $R^n \times R^r$ defined by:

$$\nabla_Y V_{c(Y)} = 0 \tag{2}$$

Where $V_{c(\mathrm{Y})}$ = $V(Y,C)$ and $\nabla$y is the partial derivative with respect to *Y*. Equation is the set of all critical points of the function $V(Y,C)$.Next we find the singularity set, $S$, which is the subset of *M* that focus on all degenerate critical points of $V$ . It is defined by:

$$\nabla_Y V_{c(Y)} = 0$$

And

$$\nabla_Y^2 V_{c(Y)} = 0 \tag{3}$$

The singularity set, $S$ , is then projected down onto the control space $R^r$ by eliminating the state variables $x$ using [3] and [4], to obtain the bifurcation set, B. The bifurcation set provides a projection of the stability region of the function $V(Y,C)$, i.e. it contains all non-degenerate critical points of the function $V$ bounded by the degenerate critical point at

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which the system exhibits sudden changes when it is subject to small changes.The seven elementary catastrophes of $r < 4$ are listed in[5].The geometric analysis of the seven elementary catastrophes is given in [7][6].

## 3. Application To The Transient Stability Problem

The application of the catastrophe theory to the steady state stability problem is fairly straightforward as shown [8][9]. However in the case of transient stability problem the situation is almost different because there are two switchings (discontinuities) during the transient period, one at fault occurrence and the other at fault clearance.

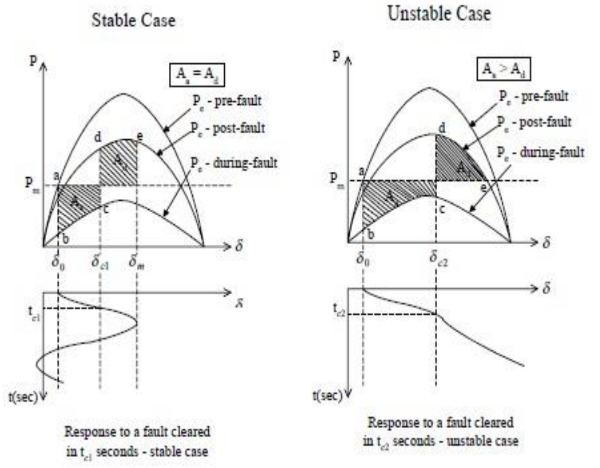


Figure (1 ) First Swing Analysis[10]

In power system, if a fault occurs on one of the lines near the machine bus, the rotor will accelerate and gain kinetic energy. If the fault is cleared at the critical clearing time, the generated kinetic energy due to the fault occurrence will be absorbed by the system and the gained energy at the end of the transient period will be exactly zero; the system is considered to be critically stable.A typical energy curve for a fault cleared at different clearing times is shown in Figure (1 ) First Swing Analysis. In terms of catastrophe theory, of Figure (1) can be considered as the energy equilibrium surface or catastrophe manifold at which the kinetic energy equals the potential energy of the system. All non-degenerate critical points lie on the energy equilibrium surface which corresponds to critical clearing times.The degenerate critical points which are defined by the bifurcation set are the points at which correspond to the transient stability limits of the power system at which any small disturbance to the power system will drive the system unstable regardless the clearing time.
The swing equation[11] representing the system behaviour might a given as:

$$M\frac{d^2\delta}{dt^2} = P_i - P_e = P_{ai} \qquad (4)$$

$$P_{ei} = \sum_{j=1}^{n} E_i E_j (g_{ij}\cos\delta_{ij} + b_{ij}\sin\delta_{ij}) \qquad (5)$$

Where

$P_{ei}$ : Electrical power output of machines

$P_{mi}$ : Mechanical power input

$M_i$ : Inertia constant

$\delta_i$ : Rotor angle

$w_i$ : Speed deviation

$E_i$ : Internal voltage

$g_{ij}$ :Transfer conductance

$b_{ij}$ : Transfer susceptance

$\delta_{ij} = \delta_i - \delta_j$

When a disturbance occurs in a large system, only a few machines are affected and these tend to oscillate against the rest of the system[12]. These machines are called critical machines; the other machines are non-critical.
The group of critical machines, j = 1, 2, 3,… k may be represented by a single equivalent machine with an inertia constant and rotor angle, respectively, of:

$$M_k = \sum_{j=1}^{k} M_j \qquad (6)$$

$$\delta_k = \frac{1}{M_k}\sum_{j=1}^{k} M_i\delta_i \qquad (7)$$

Similarly, the group of non-critical machines, j = k+1, k+2, k+3, ..n may be represented by another single equivalent machine with an inertia constant and rotor angle, respectively, of:

$$M_\circ = \sum_{j=k+1}^{n} M_j \qquad (8)$$

$$\delta_\circ = \frac{1}{M_\circ}\sum_{j=k+1}^{n} Mj\delta j \qquad (9)$$

By suitable algebraic manipulation, the swing equation for the group of critical machines can be put in the form of:

$$M\ddot{\theta}_k = P_m - P_c - T_k\sin(\theta_k + \alpha_k) \qquad (10)$$

Where

$$M = \frac{M_{\circ} M_k}{M_{\circ} + M_k} \tag{11}$$

$$\mu = \frac{M_{\circ} - M_k}{M_{\circ} + M_k} \tag{12}$$

$$\theta_k = \delta_k - \delta_{\circ} \tag{13}$$

$$P_m = \frac{M_{\circ}}{M_{\circ} + M_k} \sum_{j=1}^{k} P_{mj} - \frac{M_k}{M_{\circ} + M_k} \sum_{j=k+1}^{n} P_{mj} \tag{14}$$

$$P_c = [\frac{M_{\circ}}{M_{\circ} + M_k} \sum_{i=1}^{k} \sum_{j=1}^{k} E_i E_j (g_{ij} \cos\delta_{ij} + b_{ij} \sin\delta_{ij})]$$
$$-[\frac{M_k}{M_{\circ} + M_k} \sum_{i=k+1}^{n} \sum_{j=k+1}^{n} E_i E_j (g_{ij} \cos\delta_{ij} + b_{ij} \sin\delta \mathrm{ij})] \tag{15}$$

$$T_k = \sqrt{A_k^2 + B_k^2} \tag{16}$$

$$A_k = \sum_{i=1}^{k} \sum_{j=k+1}^{n} [E_i E_j (\mu g_{ij} \cos(\delta_{ij} - \theta_k)$$
$$+ b_{ij} \sin(\delta_{ij} - \theta_k))] \tag{17}$$

$$B_k = \sum_{i=1}^{k} \sum_{j=k+1}^{n} [E_i E_j (b_{ij} \cos(\delta_{ij} - \theta_k)$$
$$- \mu g_{ij} \sin(\delta_{ij} - \theta_k) \tag{18}$$

$$\alpha_k = \tan^{-1}(\frac{A_k}{B_k}) \tag{19}$$

With reference to Figure (1) and using the extended equal area stability criterion (EEAC) of Equation (10), we have:

$$\int_{\theta_k^{\circ}}^{\theta_k^c} [P_m^n - P_c^f - T_k^f \sin(\theta_k + \alpha_k^f)] \partial\theta_k$$
$$+ \int_{\theta_k^c}^{\theta_k^u} P_m^n - P_c^p - T_k^p \sin(\theta_k + \alpha_k^p)] \partial\theta_k = 0 \tag{20}$$

By substitution of $\theta_k^u = \theta_k^c + \theta_k$, and with some algebraic manipulation, Equation (20) may be put in the form of:

$$k_1 \Delta\theta_k + k_2 \cos(\Delta\theta_k) + k_3 \sin(\Delta\theta_k) + k_4 = 0 \tag{21}$$

Where

$$\theta_k^{\circ} = \sin^{-1} \frac{P_m^n - P_c^n}{T_k^n} - \alpha_k^n \tag{22}$$

$$k_1 = P_m^n - P_c^n \tag{23}$$

$$k_2 = T_k^n \cos(\theta_k^c + \alpha_k^n) \tag{24}$$

$$k_3 = -T_k^n \sin(\theta_k^c + \alpha_k^n) \tag{25}$$

$$k_4 = (P_m^n - P_c^f)(\theta_k^c + \alpha_k^{\circ})$$
$$+ T_k^f \cos(\theta_k^c + \alpha_k^f) - T_k^f \cos(\theta_k^{\circ} + \alpha_k^f)$$
$$- T_k^p \cos(\theta_k^{\circ} + \alpha_k^p) \tag{26}$$

And the superscript:

n: Pre-fault value

f: During fault value

p: Post fault value

By using Taylor series expansion [13]of $\sin(\Delta\theta_k)$ and $\cos(\Delta\theta_k)$ ofequation (21) up to the fourth order, we obtain:

$$k_1 \Delta\theta_k + k_2 \{1 - \frac{\Delta\theta_k^2}{2!} + \frac{\Delta\theta_k^4}{4!}\} \tag{27}$$

Let:

$$\Delta\theta_k = y + \beta \tag{28}$$

So that

$$(\Delta\theta_k)^4 = (y + \beta)^4 = N$$
$$N = y^4 + 4\beta y^3 + 6\beta^2 y^2 + 4\beta^3 y + \beta^4 \tag{29}$$

We need to eliminate the third order term in order to have equation (29) in swallowtail catastrophe manifold form.

$$(\Delta\theta_k)^4 - 4\frac{k_3}{k_2}(\Delta\theta_k)^3 - 12(\Delta\theta_k)^2 + 24\frac{k_1+k_3}{k_2}(\Delta\theta_k) + 24\frac{k_2+k_4}{k_2} = 0 \qquad (30)$$

Substitute $\Delta\theta_k = y+\beta$, expand $(y+\beta)^n$ where n=1,2,…,4 and make the coefficient of $y^3$ zero so that:

$$\beta = \frac{k_3}{k_2} \qquad (31)$$

Hence we get the swallowtail manifold equation

$$y^4 + uy^2 + vy + w = 0 \qquad (32)$$

Where

$$u = -6(2+\beta^2) \qquad (33)$$

$$v = -8\beta^2 + 24\frac{k_1}{k_2} \qquad (34)$$

$$w = 3\beta^2(4-\beta^2) + 24\left(1+\frac{k_4+k_1\beta}{k_2}\right) \qquad (35)$$

## 4. NUMERICAL EXAMPLE

To prove the effectiveness of this method, the algorithm will be tested Transmission line parameters and loads are given in per unit on a 100MVA base. Generator data, bus data and line data are given in Tables in the appendix [A].

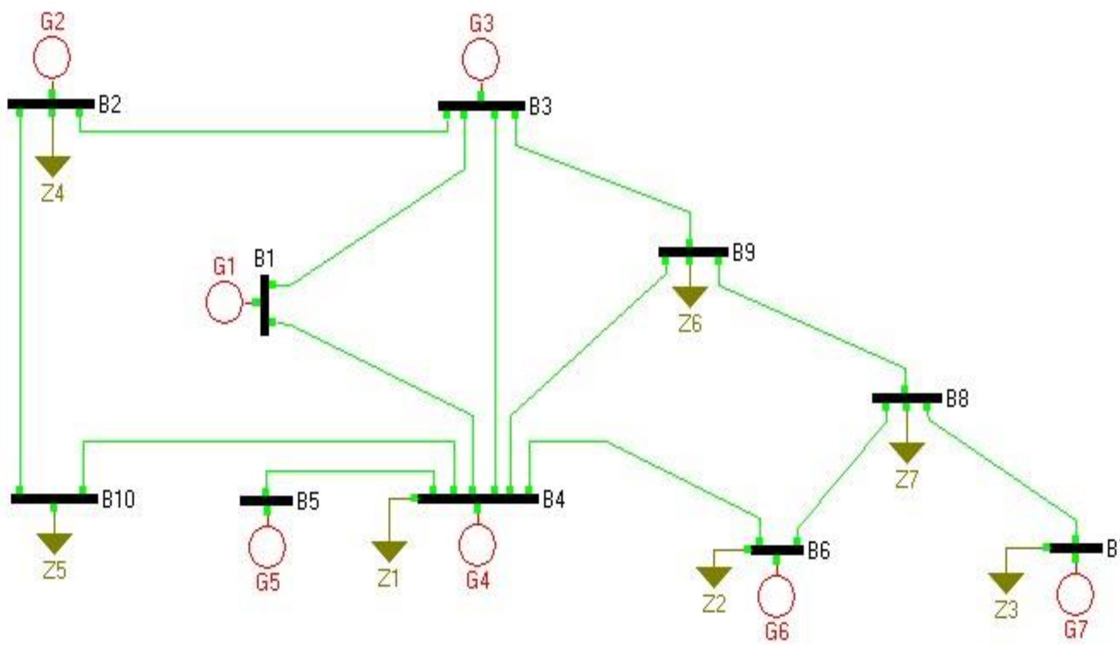


Figure (2).The Three-machine power system

**TABLE 1**
*(TCC) Estimation by Proposed Method*

| Fault at Bus | CCT simulation | CCT Proposed method |
|---|---|---|
| 1 | 0.36 | 0.36 |

| | | |
|---|---|---|
| 2 | 0.41 | 0.37 |
| 3 | 0.39 | 0.39 |
| 4 | 0.50 | 0.48 |
| 5 | 0.35 | 0.35 |
| 6 | 0.52 | 0.51 |
| 7 | 0.33 | 0.33 |
| 8 | 0.44 | 0.45 |

Three-phase short circuits are considered at different locations. The transient stability is evaluated for each fault by the Runge-Kutta time solution[15] and catastrophe theory methods. A comparison between the two methods is given in Table1 in terms of the critical clearing time. This method shows a very good agreement with the time solution.

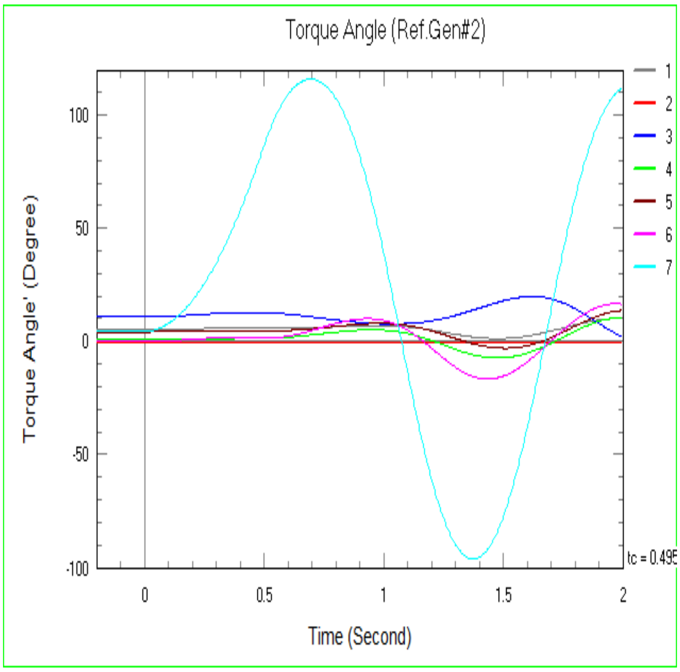


Figure (4)Generators curve during a disturbance at (CCT =0. 33).

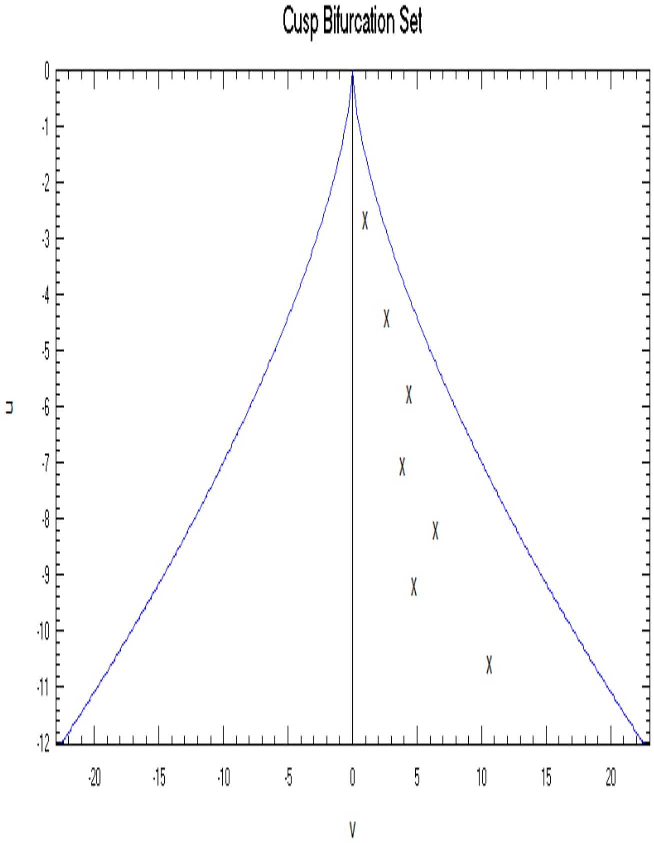


Figure (3) Bifurcation of cusp

The importance of the transient stability regions provided by the bifurcation set of the catastrophe manifold is not only in terms of speed and accuracy. It provides another important dimension to the transient stability problem that is the stability limits. The method checks for the violations of the transient stability limits of the post-fault network in terms of the system parameters. Of course, if the limits are exceeded the system is unstable. This important feature isnotpossible with existing direct methods, i.e., the existing direct methods cannot give any solution when the stability limits of the post-fault network are violated.To obtain good accuracy beyond this limit higher order terms have to be included in the computations. This will complicate the stability region and slow down the calculation procedure. In practice, however, this problem is very rare. Faults in large power systemsare usually tripped in a few cycles, typically in range 0.1-0.3 seconds. The concern isthe delivery of maximum power at low clearing time without risking the power system security. This practical consideration can easily be handled by the proposed method without loss of accuracy.

## 5. Conclusion

In this manuscript, the results obtained by the proposed method are in good agreement with those obtained by the time solution method. A large number of simulations are presently needed in system planning in order to determine the critical clearing time or stability limits. The proposed method gives directly the critical clearing time and stability limits with good accuracy and less computation. Therefore, they can be used to greatly reduce the large number of time simulations in the system planning stage. In system operations, the proposed method provides fast solutions to the transient stability problem with definite stability boundaries so that corrective action can be taken to prevent the acceptance of the instability.The study shows that catastrophe theory can be successfully used to define the transient stability regions and to determine the maximum mechanical power input of power systems before system instability occurs. Higher order catastrophes, in general, show better accuracy than that of lower order. The swallowtail catastrophe, however, provide adequate result without overshooting the stability boundary.

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APPENDIX-A

Table (A.1) Generator data of three machines system

| Gen | Bus | H (p.u) | D (p.u) | Xd' (p.u) | Xq' (p.u) | Td' (sec) | Tq' (sec) | \|Eg\| (p.u) | Delta degree | Ig (p.u) | Sg (p.u) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 9 | 3.24 | 0 | 0.05 | 0.05 | 8.96 | 0.8 | 1.029 | 1.8964 | 0.6813 + j0.221 | 0.7086 - j0.2047 |
| 2 | 1 | 5.56 | 0 | 0.045 | 0.04 | 6 | 0.8 | 0.998 | 9.2937 | 1.5284 + j0.798 | 1.6352 - j0.5406 |
| 3 | 4 | 1.96 | 0 | 0.12 | 0.12 | 5.89 | 0.8 | 0.967 | 10.498 | 0.7861 + j0.591 | 0.8522 - j0.4240 |

Table (A.2) Bus data on three machines system

| Bus | Type | \|V\| | Angle | PG (p.u) | QG (p.u) | PL (p.u) | QL (p.u) | surpluse current |
|---|---|---|---|---|---|---|---|---|
| 1 | PV | 1.0257 | 5.17441 | 1.6352 | -0.674449 | 0 | 0 | 1.52843 + j0.79865 |
| 2 | PQ | 1.03627 | 3.79701 | -0.00015439 | 0 | 0 | 0 | -0.000147695 - j2.36492e-05 |
| 3 | PQ | 1.05768 | 1.94569 | 0.000256891 | 0 | 0 | 0 | -0.000242033 - j2.9078e-05 |
| 4 | PV | 1.0257 | 4.5841 | 0.852204 | -0.540166 | 0 | 0 | 0.786104 + j0.59135 |
| 5 | PQ | 1.05349 | -2.13507 | -0.0001096 | 0 | 0 | 0 | -0.000104924 - j2.00322e-05 |
| 6 | PQ | 1.03323 | -3.85097 | 0 | 0 | 1.25 | 0.5 | -1.17456 + j0.564084 |
| 7 | PQ | 1.05758 | -3.64282 | 0 | 0 | 0.9 | 0.3 | -0.831251 + j0.337165 |
| 8 | PQ | 1.04104 | 0.794074 | 0 | 0 | 1 | 0.35 | -0.965138 + j0.322864 |
| 9 | SW | 1.04 | 0 | 0.70865 | -0.230436 | 0 | 0 | 0.681394 + j0.221573 |
| Total | | | | 3.19554 | -1.44501 | 3.15 | 1.15 | 0.360462 + j0.168422 |

Table (A.3) Line data on three machines system

| Line | From Bus | To Bus | R (p.u) | X (p.u) | BC /2 (p.u) | Tap | Line Losses | Line Current |
|---|---|---|---|---|---|---|---|---|
| 1 | 9 | 5 | 0 | 0.0576 | 0 | 1 | j0.0295714 | 0.681394 + j0.221573 |
| 2 | 1 | 2 | 0 | 0.015625 | 0 | 1 | j0.0464677 | 1.52843 + j0.79865 |
| 3 | 4 | 3 | 0 | 0.0586 | 0 | 1 | j0.0567045 | 0.786104 + j0.59135 |
| 4 | 2 | 8 | 0.017 | 0.144 | 0.0745 | 1 | 0.00241394 - j0.140296 | 0.360462 + j0.168422 |
| 5 | 8 | 3 | 0.0238 | 0.2016 | 0.1045 | 1 | 0.000416769 - j0.226627 | -0.11592 + j0.175271 |
| 6 | 2 | 6 | 0.032 | 0.161 | 0.306 | 1 | 0.0226338 - j0.541401 | 0.807356 + j0.461783 |
| 7 | 3 | 7 | 0.0396 | 0.17 | 0.358 | 1 | 0.0138204 - j0.741579 | 0.564543 + j0.503377 |
| 8 | 6 | 5 | 0.01 | 0.085 | 0.176 | 1 | 0.00189324 - j0.367132 | -0.367442 + j0.394008 |
| 9 | 7 | 5 | 0.017 | 0.092 | 0.158 | 1 | 0.0015309 - j0.343792 | -0.277908 + j0.0842601 |
| 10 | 2 | 8 | 0.017 | 0.144 | 0.0745 | 1 | 0.00241394 - j0.140296 | 0.360462 + j0.168422 |
| 11 | 8 | 3 | 0.0238 | 0.2016 | 0.1045 | 1 | 0.000416769 - j0.226627 | -0.11592 + j0.175271 |