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Solving Fuzzy Game of order 3 x 3 using Octagonal Fuzzy Numbers

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Abstract

In this paper, we consider a two persons zero sum game of order 3x3 with imprecise values in payoff matrix. All the imprecise values are assumed to be octagonal fuzzy numbers. The solution of such fuzzy games with pure strategies by maximin-minimax principle is discussed.

Keywords: Fuzzy numbers, Octagonal fuzzy numbers, Ranking of fuzzy numbers

1.Indroduction

Game theory is a decision theory applicable to competitive situations. It is usually used when two or more individuals or organisations with conflicting objectives try to make decisions. In such situations , decision made by one decision maker affects the decision made by one or more of the remaining decision makers. Game theory is based on the minimax principle which states that each competitor will act so as to minimise his maximum loss. (or maximise his minimum gain). Game theory is applicable to situations such as two players struggling to win at chess, candidates fighting an election, firms struggling to maintain their market shares, etc..

Following are the characteristics of competitive games: There are finite number of competitors. If the number of competitors is two, the game is called two person game. For numbers greater than two, it is called as n person game. The number of possible courses of action for each participants is finite. Every participant should know all the courses of action available to others but should know which of these will be chosen. A play of the game is said to occur when each player chooses one of his courses of action simultaneously. After all participants have chosen a course of action, their respective gains are finite. The gain of the participants depends upon his own actions as well as those of others.

1.1. Fuzzy set

Let X be a non empty set. A fuzzy set A in X is characterized by its membership function $A \rightarrow [0,1]$ and A(x) is interpreted as the degree of membership of element x in fuzzy A for each $x \in X$.

The Value zero is used to represent complete non-membership; the value one is used to represent complete membership and values in between are used to represent intermediate degrees of membership. The mapping A is also called the membership function of fuzzy set A.

1.2.Crisp set

A crisp set is a special case of a fuzzy set, in which the membership function only takes two values, commonly defined as 0 and 1.

1.3. Fuzzy number

A fuzzy number $\tilde{A}$ is a fuzzy set on the real line R, must satisfy the following conditions.

(i) There exist atleast one $x_0 \in R$ with $\mu_{\tilde{A}}(x_0)=1$.

(ii) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

(iii) $\tilde{A}$ must be normal and convex.

2. Octagonal fuzzy numbers:

Definition 2.1.

An octagonal fuzzy number denoted by $\tilde{A}_w$ is defined to be the ordered quadruple

$$\tilde{A}_w = (l_1(r), s_1(t), s_2(t), l_2(r)) \text{ for } r\in[0,k] \text{ and } t\in[k,w] \text{ where}$$

1. $l_1(r)$ is a bounded left continuous non-decreasing function over $[0,w_1]$, $[0\leq w_1\leq k]$
2. $s_1(t)$ is a bounded left continuous non-decreasing function over $[k,w_2]$, $[k\leq w_2\leq w]$
3. $s_2(t)$ is a bounded left continuous non-increasing function over $[k,w_2]$, $[k\leq w_2\leq w]$
4. $l_2(r)$ is a bounded left continuous non-increasing function over $[0,w_1]$, $[0\leq w_1\leq k]$

Definition 2.2.

A fuzzy number $\tilde{A}$ is a normal octagonal fuzzy number denoted by $(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8)$ where $a_1\leq a_2\leq a_3\leq a_4\leq a_5\leq a_6\leq a_7\leq a_8$ are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given below

$$\mu_{\tilde{A}}(x)=\begin{cases} 0 & for\ x < a_1 \\ k\left(\frac{x-a_1}{a_2-a_1}\right) & for\ a_1 \le x \le a_2 \\ k & for\ a_2 \le x \le a_3 \\ k+(1-k)\left(\frac{x-a_3}{a_4-a_3}\right) & for\ a_3 \le x \le a_4 \\ 1 & for\ a_4 \le x \le a_5 \\ k+(1-k)\left(\frac{a_6-x}{a_6-a_5}\right) & for\ a_5 \le x \le a_6 \\ k & for\ a_6 \le x \le a_7 \\ k\left(\frac{a_8-x}{a_8-a_7}\right) & for\ a_7 \le x \le a_8 \\ 0 & for\ x \ge a_8 \end{cases}$$

where $0 < k < 1$

2.3 α-cut of an octagonal fuzzy number

The α-cut of an octagonal fuzzy number $\tilde{A} = (a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8)$ is

$$[\tilde{A}]_\alpha = \begin{cases} \left[a_1+\left(\frac{\alpha}{k}\right)(a_2-a_1), a_8-\left(\frac{\alpha}{k}\right)(a_8-a_7)\right] & for\ \alpha \in [0,k] \\ \left[a_3+\left(\frac{\alpha-k}{1-k}\right)(a_4-a_3), a_6-\left(\frac{\alpha-k}{1-k}\right)(a_6-a_5)\right] & for\ \alpha \in [k,1] \end{cases}$$

2.4 Ranking of octagonal fuzzy numbers:

A measure of fuzzy number $\tilde{A}_w$ is a function $M_\alpha: R_\omega(I) \rightarrow R^+$ which assigns a non negative real number $M_\alpha(\tilde{A}_w)$ that expresses the measure of $\tilde{A}_w$.

$$M_0^{oct}(\tilde{A}_w) = \frac{1}{2}\int_\alpha^k (l_1(r)+l_2(r))dr + \frac{1}{2}\int_k^\omega (s_1(t)+s_2(t))dt \quad \text{where } 0 \le \alpha \le 1$$

Definition 3.1.

The measure of an octagonal fuzzy number is obtained by the average of the two fuzzy side areas, left side area and right side area, from membership function to α-axis.

Definition 3.2.

Let Ã be a normal octagonal fuzzy number. The value $M_0^{oct}(\tilde{A})$, called the measure of Ã is calculated as follows:

$$M_0^{oct}(\tilde{A})=\frac{1}{2}\int_{\propto}^{k}(l_1(r)+l_2(r))dr+\frac{1}{2}\int_{k}^{\omega}(s_1(t)+s_2(t))dt \text{ where } 0\leq k\leq 1.$$

$$=\frac{1}{4}[(a_1+a_2+a_7+a_8)k+(a_3+a_4+a_5+a_6)(1-k)] \text{ where } 0\leq k\leq 1$$

Definition 3.3. Pure strategy.

Pure strategy is a decision making rule in which one particular course of action is selected.

For fuzzy games the min-max principle is described by Nishizaki[10]. The course of the fuzzy game is determined by the desire of A to maximize his gain and that of restrict his loss to a minimum.

Definition 3.4. Saddle point.

If the maxmin value equals the minimax value, then the game is said to have a saddle point and the corresponding strategies which give the saddle point are called optimal strategies. The amount of payoff at an equilibrium point is
called the crisp game value of the game matrix.

3.5. Solution of all 3x3 matrix game

Consider the general 3x3 game matrix $A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

To solve this game we proceed as follows:

(i) Test for a saddle point.

(ii) If there is no saddle point, solve by finding equalizing strategies.

The Optimal mixed strategies for player A = (p_1,p_2) and for player B = (q_1,q_2)

where $p_1 = \frac{a_{22}-a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$; $p_2=1-p_1$

$q_1 = \frac{a_{22}-a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$; $q_2=1-q_1$ and

Value of the game $V = \frac{a_{11}a_{22}-a_{12}a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$

3.6 NUMERICAL EXAMPLES

Example 1:

Consider the following fuzzy game problem.

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{pmatrix} (5,6,7,8,9,10,11,12) & (-1,0,1,2,3,4,5,6) & (0,1,2,3,4,5,6,7) \\ (3,4,5,6,7,8,9,10) & (2,3,4,5,6,7,8,9) & (4,5,6,7,8,9,10,11) \\ (-11,-10,-9,-8,-7,-6,-5,-4) & (1,2,3,4,5,6,7,8) & (-2,-1,0,1,2,3,4,5) \end{pmatrix} \end{array}$$

Solution:

By definition of octagonal fuzzy number $\tilde{A}$ is calculated as

$$M_0^{oct}(\tilde{A}) = \frac{1}{2}\int_{\propto}^{k}(l_1\,(r) + l_2(r))dr + \frac{1}{2}\int_{k}^{\omega}(s_1\,(t) + s_2(t))dt \text{ where } 0 \le k \le 1.$$

$$= \frac{1}{4}[(a_1 + a_2 + a_7 + a_8)k + (a_3 + a_4 + a_5 + a_6)(1-k)] \text{ where } 0 \le k \le 1$$

Step 1:

Convert the given fuzzy problem into a crisp value problem.

This problem is done by taking the value of k as 0.4, we obtain the value of $M_0^{oct}(a_{ij})$

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a_{11}=(5,6,7,8,9,10,11,12)	$M_0{}^{oct}(a_{11})=\frac{1}{4}[(5+6+11+12)(0.4)+(7+8+9+10)(1-0.4)]$ $=\frac{1}{4}[(34)(0.4)+(34)(0.6)]$= 8.5
a_{12}=(-1,0,1,2,3,4,5,6)	$M_0{}^{oct}(a_{12})=\frac{1}{4}[(-1+0+5+6)(0.4)+(1+2+3+4)(1-0.4)]$ $=\frac{1}{4}[(10)(0.4)+(10)(0.6)]$= 2.5
a_{13}=(0,1,2,3,4,5,6,7)	$M_0{}^{oct}(a_{13})=\frac{1}{4}[(0+1+6+7)(0.4)+(2+3+4+5)(1-0.4)]$ $=\frac{1}{4}[(14)(0.4)+(14)(0.6)]$= 3.5
a_{21}=(3,4,5,6,7,8,9,10)	$M_0{}^{oct}(a_{21})=\frac{1}{4}[(3+4+9+10)(0.4)+(5+6+7+8)(1-0.4)]$ $=\frac{1}{4}[(26)(0.4)+(26)(0.6)]$= 6.5
a_{22}=(2,3,4,5,6,7,8,9)	$M_0{}^{oct}(a_{22})=\frac{1}{4}[(2+3+8+9)(0.4)+(4+5+6+7)(1-0.4)]$ $=\frac{1}{4}[(22)(0.4)+(22)(0.6)]$= 5.5
a_{23}=(4,5,6,7,8,9,10,11)	$M_0{}^{oct}(a_{23})=\frac{1}{4}[(4+5+10+11)(0.4)+(6+7+8+9)(1-0.4)]$ $=\frac{1}{4}[(30)(0.4)+(30)(0.6)]$= 7.5
a_{31}=(-11,-10,-9,-8,-7,-6,-5,-4)	$M_0{}^{oct}(a_{31})=\frac{1}{4}[(-11-10-5-4)(0.4)+(-9-8-7-6)(1-0.4)]$ $=\frac{1}{4}[(-30)(0.4)+(-30)(0.6)]$=- 7.5
a_{32}=(1,2,3,4,5,6,7,8)	$M_0{}^{oct}(a_{32})=\frac{1}{4}[(1+2+7+8)(0.4)+(3+4+5+6)(1-0.4)]$ $=\frac{1}{4}[(18)(0.4)+(18)(0.6)]$= 4.5
a_{33}=(-2,-1,0,1,2,3,4,5)	$M_0{}^{oct}(a_{33})=\frac{1}{4}[(-2-1+4+5)(0.4)+(0+1+2+3)(1-0.4)]$ $=\frac{1}{4}[(6)(0.4)+(6)(0.6)]$= 1.5

Since the condition $a_1+a_2+a_7+a_8=a_3+a_4+a_5+a_6$ is satisfied by all the octagonal numbers for any value of k. We will get the same matrix as below.

Step 2:

The pay-off matrix is

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{pmatrix} 8.5 & 2.5 & 3.5 \\ 6.5 & 5.5 & 7.5 \\ -7.5 & 4.5 & 1.5 \end{pmatrix} \end{array}$$

Minimum of 1st row = 2.5

Minimum of 2nd row = 5.5

Minimum of 3^{rd} row=-7.5

Maximum of 1st column = 8.5

Maximum of 2nd column = 5.5

Maximum of 3^{rd} column= 7.5

Max(min) =5.5; Mini(max) =5.5

It has saddle Point.

The Crisp solution to the problem is saddle point= (A2,B2), Value of the game=5.5.

Example 2:

Consider the following fuzzy game problem.

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{pmatrix} (1,2,3,4,5,6,7,8) & (0,1,2,3,4,5,6,7) & (2,3,4,5,6,7,8,9) \\ (8,9,10,11,12,13,14,15) & (9,10,11,12,13,14,15,16) & (6,7,8,9,10,11,12,13) \\ (-2,-1,0,1,2,3,4,5) & (-1,0,1,2,3,4,5,6) & (1,2,3,4,5,6,7,8) \end{pmatrix} \end{array}$$

Solution:

By definition of octagonal fuzzy number Ã is calculated as

$M_0^{oct}(\tilde{A})=\frac{1}{2}\int_{\propto}^{k}(l_1(r)+l_2(r))dr+\frac{1}{2}\int_{k}^{\omega}(s_1(t)+s_2(t))dt$ where $0\leq k\leq 1$.

$=\frac{1}{4}[(a_1+a_2+a_7+a_8)k+(a_3+a_4+a_5+a_6)(1-k)]$ where $0\leq k\leq 1$

Step 1:

Convert the given fuzzy problem into a crisp value problem.

This problem is done by taking the value of k as 0.4, we obtain the value of $M_0^{oct}(a_{ij})$

a_{11}=(1,2,3,4,5,6,7,8)	$M_0^{oct}(a_{11})=\frac{1}{4}[(1+2+7+8)(0.4)+(3+4+5+6)(1-0.4)]$ $=\frac{1}{4}[(18)(0.4)+(18)(0.6)]=4.5$
a_{12}=(0,1,2,3,4,5,6,7)	$M_0^{oct}(a_{12})=\frac{1}{4}[(0+1+6+7)(0.4)+(2+3+4+5)(1-0.4)]$ $=\frac{1}{4}[(14)(0.4)+(14)(0.6)]=3.5$
a_{13}=(2,3,4,5,6,7,8,9)	$M_0^{oct}(a_{13})=\frac{1}{4}[(2+3+8+9)(0.4)+(4+5+6+7)(1-0.4)]$ $=\frac{1}{4}[(22)(0.4)+(22)(0.6)]=5.5$
a_{21}=(8,9,10,11,12,13,14,15)	$M_0^{oct}(a_{21})=\frac{1}{4}[(8+9+14+15)(0.4)+(10+11+12+13)(1-0.4)]$ $=\frac{1}{4}[(46)(0.4)+(46)(0.6)]=11.5$
a_{22}=(9,10,11,12,13,14,15,16)	$M_0^{oct}(a_{22})=\frac{1}{4}[(9+10+15+16)(0.4)+(11+12+13+14)(1-0.4)]$ $=\frac{1}{4}[(50)(0.4)+(50)(0.6)]=12.5$
a_{23}=(6,7,8,9,10,11,12,13)	$M_0^{oct}(a_{23})=\frac{1}{4}[(6+7+12+13)(0.4)+(8+9+10+11)(1-0.4)]$ $=\frac{1}{4}[(38)(0.4)+(38)(0.6)]=9.5$
a_{31}=(-2,-1,0,1,2,3,4,5)	$M_0^{oct}(a_{31})=\frac{1}{4}[(-2-1+4+5)(0.4)+(0+1+2+3)(1-0.4)]$ $=\frac{1}{4}[(6)(0.4)+(6)(0.6)]=1.5$
a_{32}=(-1,0,1,2,3,4,5,6)	$M_0^{oct}(a_{32})=\frac{1}{4}[(-1+0+5+6)(0.4)+(1+2+3+4)(1-0.4)]$ $=\frac{1}{4}[(10)(0.4)+(10)(0.6)]=2.5$
a_{33}=(1,2,3,4,5,6,7,8)	$M_0^{oct}(a_{33})=\frac{1}{4}[(1+2+7+8)(0.4)+(3+4+5+6)(1-0.4)]$ $=\frac{1}{4}[(18)(0.4)+(18)(0.6)]=4.5$

Since the condition $a_1+a_2+a_7+a_8=a_3+a_4+a_5+a_6$ is satisfied by all the octagonal numbers for any value of k. We will get the same matrix as below.

Step 2:

The pay-off matrix is

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{pmatrix} 4.5 & 3.5 & 5.5 \\ 11.5 & 12.5 & 9.5 \\ 1.5 & 2.5 & 4.5 \end{pmatrix} \end{array}$$

Row I is dominated by Row II, So we omit Row I.

$$\begin{pmatrix} 11.5 & 12.5 & 9.5 \\ 1.5 & 2.5 & 4.5 \end{pmatrix}$$

Column II is dominated by column I. So we omit Column II.

$$\begin{pmatrix} 11.5 & 9.5 \\ 1.5 & 4.5 \end{pmatrix}$$

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{pmatrix} 11.5 & 9.5 \\ 1.5 & 4.5 \end{pmatrix} \end{array}$$

$$p_1 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$= \frac{4.5 - 9.5}{(11.5 + 4.5) - (9.5 + 1.5)}$$

$$= \frac{-5}{16 - 11}$$

$$= \frac{-5}{5} = -1$$

$p_2 = 1 - p_1 = 1 + 1 = 2$

$$q_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$= \frac{4.5 - 1.5}{(11.5 + 4.5) - (9.5 + 1.5)}$$

$$= \frac{3}{16 - 11}$$

$$= \frac{3}{5}$$

$q_2 = 1 - q_1$

$$= 1 - \frac{3}{5} = \frac{2}{5}$$

The optimum strategies are

$$S_A = (p_1, p_2) = (-1,2)$$

$$S_B = (q_1, q_2) = (\frac{3}{5}, \frac{2}{5})$$

$$V = \frac{a_{11}a_{22} - a_{12}a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$= \frac{(11.5 \times 4.5) - (9.5 \times 1.5)}{(11.5 + 4.5) - (9.5 + 1.5)}$$

$$= \frac{51.75 - 14.25}{16 - 11}$$

$$= \frac{37.5}{5}$$

$= 7.5$

Remark 1:

If the octagonal numbers are slightly modified so that the condition $a_1+a_2+a_7+a_8 \neq a_3+a_4+a_5+a_6$ is not satisfied, then for such a problem the solution for different values of $k(0<k<1)$ can be easily checked to lie in a finite interval.

Remark 2:

In the above two examples we have considered only 3×3 fuzzy games but the method applied here can be used to solve any m×n fuzzy game.

CONCLUSION

In this paper, a method of solving 3×3 fuzzy game problem using ranking of fuzzy numbers has been considered. The parameter k can be modified suitably by the decision maker to get the desired result. We may get different fuzzy game value for different values of k for the same fuzzy gamer

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