IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 3 Issue 8, August 2016
ISSN (Online) 2348 – 7968 | Impact Factor (2015) - 4.332
www.ijiset.com

Some new face and total face signed product cordial graphs

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Abstract

In this paper, face signed product cordial labeling of wheel graph W_n, f_n, switching of a vertex in cycle and path and alternative triangular snake $A(T_n)$ and total face signed product cordial labeling of wheel graph W_n, f_n, switching of a vertex in cycle and path are presented.

Keywords: *Face signed product cordial graph, Total Face signed product cordial graph, Alternative triangular snake, Switching of a vertex.*

1. Introduction

By a graph, we mean a simple, finite, planar and undirected unless otherwise specified. A (p,q) planar graph G means a graph G = (V,E), where V is the set of vertices with |V| = p, E is the set of edges with |E| = q and F is the set of interior faces of G with |F| = number of interior faces of G, for terms not defined here, we refer to Harary [4]. For standard terminology and notations related to graph labeling, we refer to Gallian [3]. The concept of cordial labeling of graph was introduced by Cahit [2]. In [1], Baskar Babujee et al introduced the concept of signed product cordial labeling of graph. Sedlacek [7] defined a graph to be magic if it had an edge-labeling, with range the real numbers, such that the sum of the labels around any vertex equals some constant, independent of the choice of vertex. In [6], Lih introduced magic labelings of planar graphs where labels extended to faces as well as edges and vertices. In [5], Lawrence et al introduced the concept of face and total face signed product cordial labeling of graph and they proved the graphs Pl_n, $n \geq 5$ and $Pl_{m,n}$, $m,n \geq 3$ are face signed product cordial graph and the graphs Pl_n, $n \geq 4$ and $Pl_{m,n}$, $m,n \geq 3$ are total face signed product cordial graph. The brief summaries of definition which are necessary for the present investigation are provided below.

Definition: 1.1

A mapping $f : V(G) \rightarrow \{0,1\}$ is called binary vertex labeling of G and f(v) is called the label of the vertex v of G under f. If for an edge e = uv, the induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Then $v_f(i)$ = number of vertices of having label i under f and $e_f(i)$ = number of edges of having label i under f*.

Definition: 1.2

A binary vertex labeling f of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial if it admits cordial labeling.

Definition: 1.3

Let G be a simple graph and $f : V(G) \rightarrow \{0,1\}$ be a vertex labeling. For each edge uv, assign the label f(u)f(v). The labeling f is called a product cordial labeling of G if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(i)$ and $e_f(i)$ denote the number of vertices and the number of edges respectively labeled with i (i = 0,1). A graph with a product cordial labeling is called a product cordial graph.

Definition: 1.4

Let G be a simple graph and $f : V(G) \rightarrow \{0,1\}$ be a vertex labeling. For each edge uv, assign the label f(u)f(v). The labeling f is called a total product cordial labeling of G if $|f(0) - f(1)| \leq 1$, where f(i) denotes sum of the number of vertices and the number of edges labeled with i (i = 0,1). A graph with a total product cordial labeling is called a total product cordial graph.

Definition: 1.5

A vertex labeling of graph G, $f : V(G) \rightarrow \{-1,1\}$ with induced edge labeling $f^* : E(G) \rightarrow \{-1,1\}$ defined by $f^*(uv) = f(u) f(v)$ is called a signed product cordial labeling if $|v_f(-1) - v_f(1)| \leq 1$ and $|e_f(-1) - e_f(1)| \leq 1$, where $v_f(-1)$ is the number of vertices labeled with –1, $v_f(1)$ is the number of vertices labeled with 1, $e_f(-1)$ is the number of edges labeled with –1 and $e_f(1)$ is the number of edges labeled with 1. A graph G is signed product cordial if it admits signed product cordial labeling.

Definition: 1.6

For a planar graph G, the vertex labeling function is defined as $g : V(G) \rightarrow \{-1,1\}$ and g(v) is called the label of the vertex v of G under g, induced edge labeling function $g^* : E(G) \rightarrow \{-1,1\}$ is given as if e = uv then $g^*(e) = g(u)\, g(v)$ and induced face labeling function $g^{**} : F(G) \rightarrow \{-1,1\}$ is given as if $v_1, v_2, \ldots, v_n$ and $e_1, e_2, \ldots, e_m$ are the vertices and edges of face f, then $g^{**}(f) = g(v_1)\, g(v_2) \dots g(v_n)\, g^*(e_1)\, g^*(e_2) \dots g^*(e_m)$. Let us denote $v_g(i)$ is the number of vertices of G having label i under g, $e_g(i)$ is the number of edges of G having label i under g^* and $f_g(i)$ is the number of interior faces of G having label i under g^{**} for $i = -1,1$. g is called face signed product cordial labeling of graph G if $|v_g(-1)-v_g(1)| \le 1$, $|e_g(-1)-e_g(1)| \le 1$ and $|f_g(-1)-f_g(1)| \le 1$.

A graph G is face signed product cordial if it admits face signed product cordial labeling.

Definition: 1.7

For a planar graph G, the vertex labeling function is defined as $g : V(G) \to \{-1,1\}$ and g(v) is called the label of the vertex v of G under g, induced edge labeling function $g^* : E(G) \to \{-1,1\}$ is given as if e = uv then $g^*(e) = g(u)\, g(v)$ and induced face labeling function $g^{**} : F(G) \to \{-1,1\}$ is given as if $v_1, v_2, \dots, v_n$ and $e_1, e_2, \dots, e_m$ are the vertices and edges of face f, then $g^{**}(f) = g(v_1)\, g(v_2) \dots g(v_n)\, g^*(e_1)\, g^*(e_2) \dots g^*(e_m)$. Let g(–1), g(1) be the sum of the number of vertices, edges and interior faces having labels –1 and 1 respectively. g is called total face signed product cordial labeling of graph G if $|g(-1)-g(1)| \le 1$.

A graph G is total face signed product cordial if it admits total face signed product cordial labeling.

Definition: 1.8

A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

2. Main Results

Theorem 2.1

The wheel graph $W_n = C_n + K_1$ is face signed product cordial graph for $n \equiv 0,1 \pmod 4$, where $n \ge 3$.

Proof.

Let G be a wheel graph $W_n = C_n + K_1$. Let v be the central vertex and $v_1, v_2, \dots, v_n$ be the vertices of C_n.

Then $|V(G)| = n+1$, $|E(G)| = 2n$ and $|F(G)| = n$.

Define : $V(G) \to \{-1, 1\}$ as follows.

$g(v) = 1$,

Case (i) : $n \equiv 0 \pmod 4$

$g(v_i) = -1$, for $i \equiv 1,2 \pmod 4$ and $1 \le i \le n$.

$g(v_i) = 1$, for $i \equiv 0,3 \pmod 4$ and $1 \le i \le n$.

In view of the above defined labeling pattern, we have

$v_g(1) = v_g(-1)+1 = \frac{n+2}{2}$, $e_g(1) = e_g(-1) = n$, and $f_g(1) = f_g(-1) = \frac{n}{2}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$.

Case (ii) : $n \equiv 1 \pmod 4$

$g(v_i) = -1$, for $i \equiv 1,2 \pmod 4$ and $1 \le i \le n$.

$g(v_i) = 1$, for $i \equiv 0,3 \pmod 4$ and $1 \le i \le n$.

In view of the above defined labeling pattern, we have

$v_g(1) = v_g(-1) = \frac{n+1}{2}$, $e_g(1) = e_g(-1) = n$, and $f_g(1) = f_g(-1)+1 = \frac{n+1}{2}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Case (iii) : $n \equiv 2 \pmod 4$

In order to satisfy the vertex condition for G, it is essential to assign label 1 and –1 to at least $\frac{n}{2}$ vertices. Any pattern assigning vertex labels satisfying vertex condition and edge condition will induce face labels for n number of faces in such a way that $|f_g(0) - f_g(1)| \ge 2$, that is face condition for G is violated. Thus the graph G under consideration is not a face signed product cordial graph.

Case (iv) : $n \equiv 3 \pmod 4$

In order to satisfy the vertex condition for G, it is essential to assign label 1 and –1 to at least $\frac{n}{2}$ vertices. Any pattern assigning vertex labels satisfying vertex condition will induce edge labels for 2n number of edges in such a way that $|e_g(0) - e_g(1)| \ge 2$, that is edge condition for G is violated. Thus the graph G under consideration is not a face signed product cordial graph.

Therefore, W_n is face signed product cordial graph for $n \equiv 0,1 \pmod 4$.

Example 2.1

The graph W_4 and its face signed product cordial labeling is shown in figure 2.1.

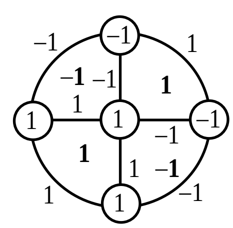

Figure 2.1

Theorem 2.2

The graph $f_n = P_n + K_1$ is face signed product cordial graph, $n \ge 3$.

Proof.

Let G be the planar graph f_n.

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Let $u, u_1, u_2, \ldots, u_n$ be the vertices, $e_1, e_2, \ldots, e_{2n-1}$ be the edges and $f_1, f_2, \ldots, f_{n-1}$ be the interior faces of f_n.

Then $|V(G)| = n+1$, $|E(G)| = 2n-1$ and $|F(G)| = n-1$.
Define $g : V(G) \rightarrow \{-1,+1\}$ as follows
$g(v) = 1$

Case (i) : $n \equiv 0 \pmod 4$
$g(v_i) = -1$, for $i \equiv 1,2 \pmod 4$
$g(v_i) = 1$, for $i \equiv 0,3 \pmod 4$
In view of the above defined labeling pattern we have

$v_g(1) = v_g(-1)+1 = \frac{n+2}{2}$, $e_g(1) = e_g(-1)+1 = n$, and $f_g(1) = f_g(-1)+1 = \frac{n}{2}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Case (ii) : $n \equiv 1 \pmod 4$
$g(v_i) = -1$, for $i \equiv 1,2 \pmod 4$
$g(v_i) = 1$, for $i \equiv 0,3 \pmod 4$
In view of the above defined labeling pattern we have

$v_g(1) = v_g(-1) = \frac{n+1}{2}$, $e_g(-1) = e_g(1)+1 = n$, and $f_g(1) = f_g(-1) = \frac{n-1}{2}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Case (iii) : $n \equiv 2 \pmod 4$
$g(v_i) = -1$, for $i \equiv 1,2 \pmod 4$
$g(v_i) = 1$, for $i \equiv 0,3 \pmod 4$
In view of the above defined labeling pattern we have

$v_g(-1) = v_g(1)+1 = \frac{n+2}{2}$, $e_g(-1) = e_g(1)+1 = n$, and $f_g(1) = f_g(-1)+1 = \frac{n}{2}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Case (iv) : $n \equiv 3 \pmod 4$
$g(v_i) = -1$, for $i \equiv 1,2 \pmod 4$
$g(v_i) = 1$, for $i \equiv 0,3 \pmod 4$
In view of the above defined labeling pattern we have

$v_g(1) = v_g(-1) = \frac{n+1}{2}$, $e_g(-1) = e_g(1) + 1 = n$, and $f_g(1) = f_g(-1)+1 = \frac{n-1}{2}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Therefore, the graph f_n is face signed product cordial graph.

Example 2.2

The graph f_5 and its face signed product cordial labeling is shown in figure 2.2.

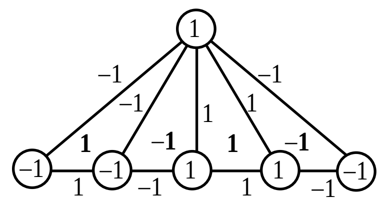

Figure 2.2

Theorem 2.3

Switching of a vertex in cycle C_n is face signed product cordial graph.

Proof.

Let $v_1, v_2, \ldots, v_n$ be the successive vertices of C_n.

G_v denotes graph is obtained by switching of vertex v of $G = C_n$.

Without loss of generality let the switched vertex be v_1.

Then $|V(G_{v_1})| = n$, $|E(G_{v_1})| = 2n-5$ and $|F(G_{v_1})| = n-4$.

Define vertex labeling $f : V(G_{v_1}) \rightarrow \{-1,+1\}$ as follows.

Case (i) : $n \equiv 0 \pmod 4$
$g(v_i) = 1$, for $i \equiv 0,1 \pmod 4$ and $1 \le i \le n$.
$g(v_i) = -1$, for $i \equiv 2,3 \pmod 4$ and $1 \le i \le n$.
In view of the above defined labeling pattern we have

$v_g(1) = v_g(-1) = \frac{n}{2}$, $e_g(-1) = e_g(1)+1 = n-2$, and $f_g(1) = f_g(-1) = \frac{n-4}{2}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Case (ii) : $n \equiv 1 \pmod 4$
$g(v_i) = 1$, for $i \equiv 0,1 \pmod 4$ and $1 \le i \le n$.
$g(v_i) = -1$, for $i \equiv 2,3 \pmod 4$ and $1 \le i \le n$.
In view of the above defined labeling pattern we have

$v_g(1) = v_g(-1) + 1 = \frac{n+1}{2}$, $e_g(1) = e_g(-1) + 1 = n-2$, and $f_g(-1) = f_g(1) + 1 = \frac{n-3}{2}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Case (iii) : $n \equiv 2 \pmod 4$

$g(v_i) = 1$, for $i \equiv 0,1 \pmod 4$ and $1 \le i \le n$.
$g(v_i) = -1$, for $i \equiv 2,3 \pmod 4$ and $1 \le i \le n$.

In view of the above defined labeling pattern we have

$v_g(-1) = v_g(1) = \frac{n}{2}$, $e_g(1) = e_g(-1) + 1 = n-2$, and $f_g(1) = f_g(-1) = \frac{n-4}{2}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Case (iv) : $n \equiv 3 \pmod 4$
$g(v_i) = 1$, for $i \equiv 0,1 \pmod 4$ and $1 \le i \le n$.
$g(v_i) = -1$, for $i \equiv 2,3 \pmod 4$ and $1 \le i \le n$.

In view of the above defined labeling pattern we have

$v_g(-1) = v_g(1)+1 = \frac{n+1}{2}$, $e_g(1) = e_g(-1)+1 = n-2$, and $f_g(-1) = f_g(1)+1 = \frac{n-3}{2}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Hence, the graph obtained by switching of a vertex in cycle C_n is a face signed product cordial graph.

Example 2.3

The face signed product cordial labeling of switching of a vertex in cycle C_8 is shown in figure 2.3.

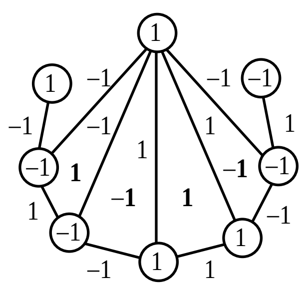

Figure 2.3

Theorem 2.4

Switching of a pendent vertex in path P_n is face signed product cordial graph.

Proof.

Let $v_1, v_2, \ldots, v_n$ be the vertices of path P_n.

The graph G is obtained by switching of a pendent vertex in path P_n. v_1 and v_n are pendent vertex of path P_n.

Without loss of generality, let the switched vertex be v_1.

Then $|V(G)| = n$, $|E(G)| = 2n - 4$ and $|F(G)| = n - 3$.

Define $g : V(G) \to \{-1,+1\}$ as follows

Case (i) : $n \equiv 0,2 \pmod 4$
$g(v_i) = 1$, for $i \equiv 0,1 \pmod 4$ and $1 \le i \le n$
$g(v_i) = -1$, for $i \equiv 2,3 \pmod 4$ and $1 \le i \le n$.

In view of the above defined labeling pattern we have

$v_g(1) = v_g(-1) = \frac{n}{2}$, $e_g(1) = e_g(-1) = n-2$, and $f_g(-1) = f_g(1)+1 = \frac{n-2}{2}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Case (ii) : $n \equiv 1 \pmod 4$
$g(v_1) = 1$,
$g(v_i) = -1$, for $i \equiv 2,3 \pmod 4$ and $2 \le i \le n$
$g(v_i) = 1$, for $i \equiv 0,1 \pmod 4$ and $2 \le i \le n$

In view of the above defined labeling pattern we have

$v_g(1) = v_g(-1)+1 = \frac{n+1}{2}$, $e_g(-1) = e_g(1) = n-2$, and $f_g(1) = f_g(-1) = \frac{n-3}{2}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Case (iii) : $n \equiv 3 \pmod 4$
$g(v_i) = 1$, for $i \equiv 0,1 \pmod 4$
$g(v_i) = -1$, for $i \equiv 2,3 \pmod 4$

In view of the above defined labeling pattern we have

$v_g(-1) = v_g(1) = \frac{n+1}{2}$, $e_g(1) = e_g(-1) = n-2$, and $f_g(-1) = f_g(1) = \frac{n-3}{2}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Hence, G is face signed product cordial graph.

Example 2.4

The face signed product cordial labeling of switching of a pendent vertex in path P_6 is shown in figure 2.4.

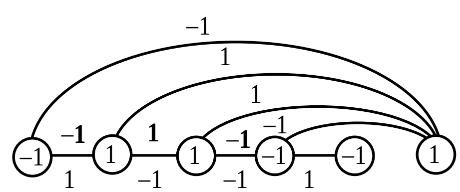

Figure 2.4

Theorem 2.5

The alternative triangular snake $A(T_n)$ is face signed product cordial graph, where $n \ge 3$.

Proof.

Let G be an alternative triangular snake $A(T_n)$.

Let $v_1, v_2, \ldots, v_n$ and $e_1, e_2, \ldots, e_{n-1}$ be the vertices and edges of the path P_n.

Case 1 : $n \equiv 0 \pmod 4$ and the first triangle start from v_1 and the last triangle ends with v_n.

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To construct alternative triangular snake $A(T_n)$ from path P_n by joining v_i and v_{i+1} alternatively with a new vertex u_i by edges $e'_{2i-1} = v_{2i-1}u_i$ and $e'_{2i} = u_i v_{2i}$ for $i = 1,2,\ldots,\frac{n}{2}$ and interior faces $f_i = v_{2i-1}\, u_i\, v_{2i}$ for $i = 1,2,\ldots,\frac{n}{2}$.

Then $|V(G)| = \frac{3n}{2}$, $|E(G)| = 2n - 1$ and $|F(G)| = \frac{n}{2}$.

Define $g : V(G) \rightarrow \{-1,+1\}$ as follows

$g(u_{2i-1}) = 1$, for $1 \le i \le \frac{n}{4}$

$g(u_{2i}) = -1$, for $1 \le i \le \frac{n}{4}$

$g(v_i) = 1$, for $i \equiv 0,1 \pmod 4$ and $1 \le i \le n$

$g(v_i) = -1$, for $i \equiv 2,3 \pmod 4$ and $1 \le i \le n$.

In view of the above defined labeling pattern, we have

$v_g(1) = v_g(-1) = \frac{3n}{4}$, $e_g(-1) = e_g(1)+1 = n$, and $f_g(-1) = f_g(1) = \frac{n}{4}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Thus, the alternative triangular snake $A(T_n)$ is face signed product cordial graph, when $n \equiv 0 \pmod 4$ and the first triangle start from v_1 and the last triangle ends with v_n.

Case 2 : $n \equiv 0 \pmod 4$ and the first triangle start from v_2 and the last triangle ends with v_{n-1}.

To construct alternative triangular snake $A(T_n)$ from path P_n by joining v_i and v_{i+1} alternatively with a new vertex u_i by edges $e'_{2i-1} = v_{2i}u_i$ and $e'_{2i} = u_i v_{2i+1}$ for $i = 1,2,\ldots,\frac{n-2}{2}$ and interior faces $f_i = v_{2i}\, u_i\, v_{2i+1}$ for $i = 1,2,\ldots,\frac{n-2}{2}$. Then $|V(G)| = \frac{3n-2}{2}$, $|E(G)| = 2n - 3$ and $|F(G)| = \frac{n-2}{2}$.

Define $g : V(G) \rightarrow \{-1,+1\}$ as follows

$g(u_{2i-1}) = 1$, for $1 \le i \le \frac{n}{4}$

$g(u_{2i}) = -1$, for $1 \le i \le \frac{n-4}{4}$

$g(v_i) = -1$, for $i \equiv 1,3 \pmod 4$ and $1 \le i \le n$

$g(v_i) = 1$, for $i \equiv 0,3 \pmod 4$ and $1 \le i \le n$.

In view of the above defined labeling pattern, we have

$v_g(1) = v_g(-1)+1 = \frac{3n}{4}$, $e_g(1) = e_g(-1)+1 = n - 1$, and $f_g(-1) = f_g(1)+1 = \frac{n}{4}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Thus, the alternative triangular snake $A(T_n)$ is face signed product cordial graph, when $n \equiv 0 \pmod 4$ and the first triangle start from v_2 and the last triangle ends with v_{n-1}.

Case 3 : $n \equiv 2 \pmod 4$ and the first triangle start from v_1 and the last triangle ends with v_n.

Let path P_n having vertices $v_1, v_2,\ldots,v_n$ and edges $e_1,e_2,\ldots,e_{n-1}$.

To construct alternative triangular snake $A(T_n)$ from path P_n by joining v_i and v_{i+1} alternatively with a new vertex u_i by edges $e'_{2i-1} = v_{2i-1}u_i$ and $e'_{2i} = u_i v_{2i}$ for $i = 1,2,\ldots,\frac{n}{2}$ and interior faces $f_i = v_{2i-1}\, u_i\, v_{2i}$ for $i = 1,2,\ldots,\frac{n}{2}$.

Then $|V(G)| = \frac{3n}{2}$, $|E(G)| = 2n - 1$ and $|F(G)| = \frac{n}{2}$.

Define $g : V(G) \rightarrow \{-1,+1\}$ as follows

$g(u_{2i-1}) = 1$, for $1 \le i \le \frac{n+2}{4}$

$g(u_{2i}) = -1$, for $1 \le i \le \frac{n-2}{4}$

$g(v_i) = 1$, for $i \equiv 0,1 \pmod 4$ and $1 \le i \le n$

$g(v_i) = -1$, for $i \equiv 2,3 \pmod 4$ and $1 \le i \le n$.

In view of the above defined labeling pattern, we have

$v_g(1) = v_g(-1) + 1 = \frac{3n+2}{4}$, $e_g(-1) = e_g(1) + 1 = n$, and $f_g(-1) = f_g(1) + 1 = \frac{n+2}{4}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Thus, the alternative triangular snake $A(T_n)$ is face signed product cordial graph, when $n \equiv 2 \pmod 4$ and the first triangle start from v_1 and the last triangle ends with v_n.

Case 4 : $n \equiv 2 \pmod 4$ and the first triangle start from v_2 and the last triangle ends with v_{n-1}.

To construct alternative triangular snake $A(T_n)$ from path P_n by joining v_i and v_{i+1} alternatively with a new vertex u_i by edges $e'_{2i-1} = v_{2i}u_i$ and $e'_{2i} = u_i v_{2i+1}$ for

$i = 1,2,\ldots,\frac{n-2}{2}$ and interior faces $f_i = v_{2i}\,u_i\,v_{2i+1}$ for $i = 1,2,\ldots,\frac{n-2}{2}$.

Then $|V(G)| = \frac{3n-2}{2}$, $|E(G)| = 2n-3$ and $|F(G)| = \frac{n-2}{2}$.

Define $g : V(G) \rightarrow \{-1,+1\}$ as follows

$g(u_{2i-1}) = 1$, for $1 \le i \le \frac{n-2}{4}$

$g(u_{2i}) = -1$, for $1 \le i \le \frac{n-2}{4}$

$g(v_1) = -1$,

$g(v_i) = 1$, for $i \equiv 1,2 \pmod 4$ and $2 \le i \le n$

$g(v_i) = -1$, for $i \equiv 0,3 \pmod 4$ and $2 \le i \le n$.

In view of the above defined labeling pattern, we have $v_g(1) = v_g(-1) = \frac{3n-2}{4}$, $e_g(-1) = e_g(1) + 1 = n-1$, and $f_g(-1) = f_g(1) + 1 = \frac{n-2}{4}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Thus, the alternative triangular snake $A(T_n)$ is face signed product cordial graph, when $n \equiv 2 \pmod 4$ and the first triangle start from v_2 and the last triangle ends with v_{n-1}.

Case 5 : $n \equiv 1 \pmod 4$ and the first triangle start from v_1 and the last triangle ends with v_{n-1}.

To construct alternative triangular snake $A(T_n)$ from path P_n by joining v_i and v_{i+1} alternatively with a new vertex u_i by edges $e'_{2i-1} = v_{2i-1}u_i$ and $e'_{2i} = u_i v_{2i}$ for $i = 1,2,\ldots,\frac{n-1}{2}$ and interior faces $f_i = v_{2i-1}u_i v_{2i}$ for $i = 1,2,\ldots,\frac{n-1}{2}$.

Then $|V(G)| = \frac{3n-1}{2}$, $|E(G)| = 2n-2$ and $|F(G)| = \frac{n-1}{2}$.

Define $g : V(G) \rightarrow \{-1,+1\}$ as follows

$g(u_{2i-1}) = 1$, for $1 \le i \le \frac{n-1}{4}$

$g(u_{2i}) = -1$, for $1 \le i \le \frac{n-1}{4}$

$g(v_i) = 1$, for $i \equiv 0,1 \pmod 4$ and $1 \le i \le n$

$g(v_i) = -1$, for $i \equiv 2,3 \pmod 4$ and $1 \le i \le n$.

In view of the above defined labeling pattern, we have $v_g(1) = v_g(-1) + 1 = \frac{3n+1}{4}$, $e_g(1) = e_g(-1) = n-1$, and $f_g(1) = f_g(-1) = \frac{n-1}{4}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Thus, the alternative triangular snake $A(T_n)$ is face signed product cordial graph, when $n \equiv 1 \pmod 4$ and the first triangle start from v_1 and the last triangle ends with v_{n-1}.

Case 6 : $n \equiv 1 \pmod 4$ and the first triangle start from v_2 and the last triangle ends with v_n.

To construct alternative triangular snake $A(T_n)$ from path P_n by joining v_i and v_{i+1} alternatively with a new vertex u_i by edges $e'_{2i-1} = v_{2i}u_i$ and $e'_{2i} = u_i v_{2i+1}$ for $i = 1,2,\ldots,\frac{n-1}{2}$ and interior faces $f_i = v_{2i}\,u_i\,v_{2i+1}$ for $i = 1,2,\ldots,\frac{n-1}{2}$.

Then $|V(G)| = \frac{3n-1}{2}$, $|E(G)| = 2n-2$ and $|F(G)| = \frac{n-1}{2}$.

Define $g : V(G) \rightarrow \{-1,+1\}$ as follows

$g(u_{2i-1}) = 1$, for $1 \le i \le \frac{n-1}{4}$

$g(u_{2i}) = -1$, for $1 \le i \le \frac{n-1}{4}$

$g(v_i) = -1$, for $i \equiv 1,2 \pmod 4$ and $1 \le i \le n$

$g(v_i) = 1$, for $i \equiv 0,3 \pmod 4$ and $1 \le i \le n$.

In view of the above defined labeling pattern, we have $v_g(-1) = v_g(1) + 1 = \frac{3n+1}{4}$, $e_g(1) = e_g(-1) = n-1$, and $f_g(1) = f_g(-1) = \frac{n-1}{4}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Thus, the alternative triangular snake $A(T_n)$ is face signed product cordial graph, when $n \equiv 1 \pmod 4$ and the first triangle start from v_2 and the last triangle ends with v_n.

Case 7 : $n \equiv 3 \pmod 4$ and the first triangle start from v_1 and the last triangle ends with v_{n-1}.

To construct alternative triangular snake $A(T_n)$ from path P_n by joining v_i and v_{i+1} alternatively with a new vertex u_i by edges $e'_{2i-1} = v_{2i-1}u_i$ and $e'_{2i} = u_i v_{2i}$ for $i = 1,2,\ldots,\frac{n-1}{2}$ and interior faces $f_i = v_{2i-1}u_i v_{2i}$ for $i = 1,2,\ldots,\frac{n-1}{2}$.

Then $|V(G)| = \frac{3n-1}{2}$, $|E(G)| = 2n-2$ and $|F(G)| = \frac{n-1}{2}$.

Define $g : V(G) \rightarrow \{-1,+1\}$ as follows

$g(u_{2i-1}) = 1$, for $1 \le i \le \frac{n+1}{4}$

$g(u_{2i}) = -1$, for $1 \le i \le \frac{n-3}{4}$

$g(v_i) = 1$, for $i \equiv 0,1 \pmod 4$ and $1 \le i \le n$

IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 3 Issue 8, August 2016
ISSN (Online) 2348 – 7968 | Impact Factor (2015) - 4.332
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$g(v_i) = -1$, for $i \equiv 2,3 \pmod 4$ and $1 \le i \le n$.

In view of the above defined labeling pattern, we have

$v_g(-1) = v_g(1) + 1 = \frac{3n+1}{4}$, $e_g(1) = e_g(-1) = n-1$, and

$f_g(-1) = f_g(1)+1 = \frac{n+1}{4}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Hence, the alternative triangular snake $A(T_n)$ is face signed product cordial graph, when $n \equiv 3 (\mathrm{mod}\ 4)$ and the first triangle start from v_1 and the last triangle ends with v_{n-1}.

Case 8 : $n \equiv 3 (\mathrm{mod}\ 4)$ and the first triangle start from v_2 and the last triangle ends with v_{n-1}.

To construct alternative triangular snake $A(T_n)$ from path P_n by joining v_i and v_{i+1} alternatively with a new vertex u_i by edges $e'_{2i-1} = v_{2i-1}u_i$ and $e'_{2i} = u_i v_{2i}$ for $i = 1,2,\ldots, \frac{n-1}{2}$ and interior faces $f_i = v_{2i-1}u_i v_{2i}$ for $i = 1,2,\ldots, \frac{n-1}{2}$.

Then $|V(G)| = \frac{3n-1}{2}$, $|E(G)| = 2n - 2$ and $|F(G)| = \frac{n-1}{2}$.

Define $g : V(G) \to \{-1,+1\}$ as follows

$g(u_{2i-1}) = 1$, for $1 \le i \le \frac{n+1}{4}$

$g(u_{2i}) = -1$, for $1 \le i \le \frac{n-3}{4}$

$g(v_i) = -1$, for $i \equiv 1,2 \pmod 4$ and $1 \le i \le n$

$g(v_i) = 1$, for $i \equiv 0,3 \pmod 4$ and $1 \le i \le n$.

In view of the above defined labeling pattern, we have

$v_g(-1) = v_g(1) + 1 = \frac{3n+1}{4}$, $e_g(1) = e_g(-1) = n-1$, and

$f_g(-1) = f_g(1)+1 = \frac{n+1}{4}$.

Thus $|v_g(1) - v_g(-1)| \le 1$, $|e_g(1) - e_g(-1)| \le 1$ and $|f_g(1) - f_g(-1)| \le 1$

Hence, the alternative triangular snake $A(T_n)$ is face signed product cordial graph, when $n \equiv 3 (\mathrm{mod}\ 4)$ and the first triangle start from v_2 and the last triangle ends with v_{n-1}.

Therefore, the alternative triangular snake $A(T_n)$ is face signed product cordial graph, where $n \ge 3$.

Example 2.5

The alternative triangular snake $A(T_6)$ with first triangle start from first vertex and its face signed product cordial labeling is shown in figure 2.5.

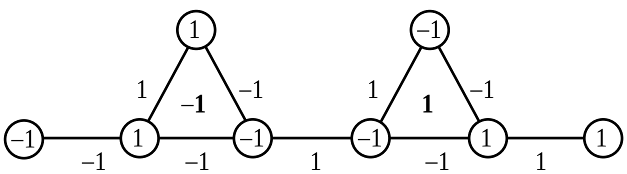

Figure 2.5

Theorem 2.6

The wheel graph $W_n = C_n + K_1$ is total face signed product cordial graph, where $n \ge 3$.

Proof.

Let G be a wheel graph $W_n = C_n + K_1$. Let v be the central vertex and $v_1, v_2, \ldots, v_n$ be the vertices of C_n.

Then $|V(G)| = n+1$, $|E(G)| = 2n$ and $F(G) = n$.

Define $g : V(G) \to \{-1,+1\}$ as follows.

$g(v) = -1$,

$g(v_i) = 1$, for $1 \le i \le n$.

In view of the above defined labeling pattern we have

$v_g(1) = n$, $v_g(-1) = 1$, $e_g(1) = n$, $e_g(-1) = n$, $f_g(1) = 0$ and $f_g(-1) = n$.

Then, $g(-1) = g(1)+1 = 2n$.

Thus, $|g(-1) - g(1)| \le 1$.

Therefore, W_n is total face signed product cordial graph.

Example 2.6

The total face signed product cordial labeling of W_5 is shown in figure 2.6.

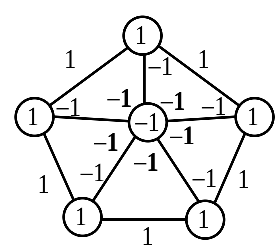

Figure 2.6

Theorem 2.7

The planar graph f_n is total face signed product cordial graph, $n \ge 3$.

Proof.

Let G be the planar graph f_n.

Let $u, u_1, u_2, \ldots, u_n$ be the vertices, $e_1, e_2, \ldots, e_{2n-1}$ be the edges and $f_1, f_2, \ldots, f_{n-1}$ be the interior faces of f_n.

Then $|V(G)| = n+1$, $|E(G)| = 2n-1$ and $|F(G)| = n-1$.

Define $g : V(G) \to \{-1,+1\}$ as follows

$g(v) = -1$

$g(v_i) = 1$, for $1 \le i \le n$

In view of the above defined labeling pattern we have

ISSN (Online) 2348 – 7968 | Impact Factor (2015) - 4.332

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Then, $v_g(1) = n$, $v_g(-1) = 1$, $e_g(1) = n-1$, $e_g(-1) = n$, $f_g(1) = 0$ and $f_g(-1) = n-1$.

Therefore $g(-1) = g(1)+1 = 2n$.

Thus, $|g(-1) - g(1)| \leq 1$.

Therefore, the planar graph f_n is total face signed product cordial graph.

Example 2.7

The planar graph f_5 and its total face signed product cordial labeling is given in figure 2.7.

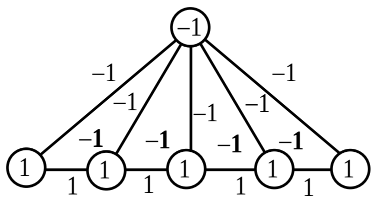

Figure 2.7

Theorem 2.8

Switching of a pendent vertex in path P_n is total face signed product cordial graph.

Proof.

Let $v_1, v_2, \ldots, v_n$ be the vertices of path P_n.

The graph G is obtained by switching of a pendent vertex in path P_n. v_1 and v_n are pendent vertex of path P_n.

Without loss of generality, let the switched vertex be v_1.

Then $|V(G)| = n$, $|E(G)| = 2n - 4$ and $|F(G)| = n - 3$.

Define $g : V(G) \rightarrow \{-1,+1\}$ as follows

$g(v_1) = 1$,

$g(v_i) = -1$, for $2 \leq i \leq n$

In view of the above defined labeling pattern we have

$v_g(1) = 1$, $v_g(-1) = n-1$, $e_g(1) = n-2$, $e_g(-1) = n-2$, $f_g(1) = n-3$ and $f_g(-1) = 0$.

Therefore $g(-1) = g(1)+1 = 2n-3$ and $|g(-1)- g(1)| \leq 1$.

Hence, G is total face signed product cordial graph.

Example 2.8

The total face signed product cordial labeling of switching of a pendent vertex in path P_6 is shown in figure 2.8.

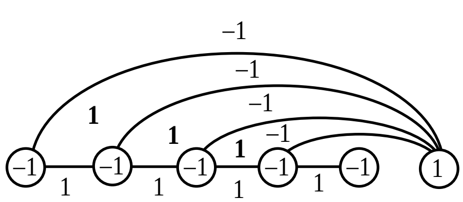

Figure 2.8

Theorem 2.9

Switching of a vertex in cycle C_n is total face signed product cordial graph.

Proof.

Let $v_1, v_2, \ldots, v_n$ be the successive vertices of C_n.

G_v denotes graph is obtained by switching of vertex v of $G = C_n$.

Without loss of generality let the switched vertex be v_1.

Then $|V(G_{v_1})| = n$, $|E(G_{v_1})| = 2n-5$ and $|F(G_{v_1})| = n-4$.

Define vertex labeling $f:V(G_{v_1}) \rightarrow \{-1,+1\}$ as follows.

$g(v_1) = 1$,

$g(v_i) = -1$, $\quad 2 \leq i \leq n$.

In view of the above defined labeling pattern we have

$v_g(1) = 1$, $v_g(-1) = n-1$, $e_g(1) = n-2$, $e_g(-1) = n-3$, $f_g(1) = n-4$ and $f_g(-1) = 0$.

Therefore $g(-1) = g(1)+1 = 2n-4$ and $|g(-1)-g(1)| \leq 1$.

Hence, the graph obtained by switching of a vertex in cycle C_n is total face signed product cordial graph.

Example 2.9

The total face signed product cordial labeling of switching of a vertex in cycle C_8 is shown in figure 2.9.

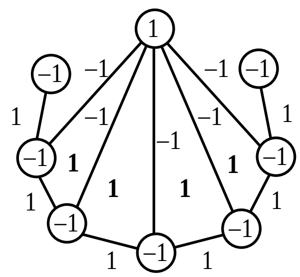

Figure 2.9

4. Conclusions

In this paper, we present the face signed product cordial labeling of wheel graph W_n, f_n, switching of a vertex in cycle and path and alternative triangular snake $A(T_n)$ and total face signed product cordial labeling of wheel graph W_n, f_n, switching of a vertex in cycle and path.

References

[1]. J. Baskar Babujee and Shobana Loganathan, On signed product cordial labeling, Applied Mathematics, Vol 2, pp 1525-1530, 2011.

[2]. I. Cahit, Cordial graphs: A weaker version of graceful and harmonious graphs, Ars Combinatoria, Vol 23, pp. 201-207, 1987.

[3]. J. A. Gallian, A dynamic survey of graph labeling, The Electronic Journal of Combinatorics, 16, # DS6, 2015.

[4]. F. Harary, Graph theory, Addison Wesley, Reading, Massachusetts, 1972.

[5]. P. Lawrence Rozario Raj and R. Lawrence Joseph Manoharan, Face and total face signed product cordial

labeling of planar graphs, Asia Pacific Journal of Research, Vol. I. Issue XLI, pp.16-23, 2016.

[6]. K. W. Lih , On magic and consecutive labelings of plane graphs, *Utilitas Math*. Vol 24, pp. 165-197, 1983.

[7]. J. Sedlacek, Problem 27, In Proc. Symposium on Theory of Graphs and its Applic., pp. 163-167, 1963.