

Performance Analysis of Cascaded Multilevel Inverter Using Particle Swarm Optimization Algorithm

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Abstract

This paper presents particle swarm optimization (PSO) technique to compute the optimum switching angles for five and seven level cascaded multilevel inverter (MLI). The expression of total harmonic distortion (THD) of inverter output voltage has been considered as objective function for PSO. While minimizing the objective function the individual selective harmonics like 5th, 7th, 11th etc. can be controlled within the allowable limits by incorporating the constraints in PSO algorithm. In the presented paper the 5th and 7th harmonics of inverter output voltage have been eliminated. The PSO results are validated with numerical method.

Keywords: Multilevel inverter, Cascaded Multilevel inverter (CMLI), Particle Swarm Optimization (PSO), Selective harmonic elimination(SHE), Total harmonic distortion (THD).

1. Introduction

In recent years, due to the requirement of high-power apparatus in various industrial applications, multilevel inverter technology has emerged as an important alternative in high power conditions. Multilevel inverter utilizes solar cells, fuel cells, biomass energy and rectified output of wind turbines as DC sources for connecting distribution to an existing AC grid [1].

The staircase output voltage is the main feature of multilevel inverter as compared to traditional two level VSI (voltage source inverters). This advantage results in lower switching losses, higher efficiency, higher power quality, reduced harmonic contents and better electromagnetic compatibility thus eliminating need of a transformer at distribution, thereby decreasing the cost [2]. Multilevel converters are mainly divided into three major categories: flying capacitors (capacitor clamped), diode-clamped and cascaded H-bridge multilevel inverters [3]. Among different topologies for converters, the cascaded multilevel inverter has obtained special attention because of its simplicity of control and modularity.

A major challenge with multilevel inverter is to eliminate harmonics from output voltage. The output voltage of the multilevel inverter must meet maximum THD limitations

as specified in [4]. Various modulation technique and control parameters have been established for multilevel inverters such as space-vector pulse width modulation (PWM), sinusoidal pulse width modulation (SPWM), fundamental frequency switching method, selective harmonic elimination (SHE) and others. However, it is not possible to completely eliminate lower-order harmonics using PWM [5]. So selected lower-order harmonics can be suppressed by considering the output waveform's Fourier series which gives the $(N + 1)$ nonlinear equations. SHE or programmed PWM techniques refers to choice of switching angles so that certain higher order harmonics such as the 5th, 7th and 11th are eliminated in the output voltage. A fundamental issue associated with such method is to obtain the arithmetic solution of nonlinear transcendental equations which contain trigonometric terms and there is a possibility of multiple solutions. Practical method of solving this nonlinear equation is by trial and error method.

This set of equations can be solved by iterative methods such as Newton-Raphson [6]. However such techniques required the right choice of the initial guess for converging. Furthermore, this technique find only one set of solutions depending on the initial guess. Therefore, the Newton-Raphson method is not feasible to solve the SHE problem for a large number of switching angles if good initial guesses are not available.

The other choice is to use resultant theory where these equations are converted into polynomials. The complexity of the problem increases as the number of DC sources increases, making the degree of the polynomial as well as computational burden quiet high. A systematic approach to solve the SHE problem based on the mathematical theory of resultant is proposed in [7], where transcendental equations that describe the SHE problem are converted into an equivalent set of polynomial equations and then the mathematical theory of resultant is utilized to find all possible sets of solutions for this equivalent problem. In fact, the resultant theory is limited to find up to six switching angles for equal dc voltages and up to three switching angles for non-equal dc voltage cases. In [8], a

genetic algorithm is used for harmonic optimisation. However, with the increase in number of harmonics to be eliminated, the cost function becomes more difficult to optimise and requires increased number of iterations. Also, at modulation indices where no specific solution is available, the suggested method does not address the problem for online application.

Reference [9] and [10] presented stochastic search technique, particle swarm optimisation, to minimise the overall THD of the output voltage of a multilevel inverter. In this paper, the PSO approach is developed to deal with the SHE problem with equal dc sources. PSO is a powerful tool for optimisation of non-linear functions, which was developed through simulation of swarm intelligence viz. bird flocking, fish schooling and so on. The objective function derived from the SHE problem is minimised using the PSO algorithm to compute the switching angles while lower-order harmonics are eliminated. The proposed method solves the asymmetry of the transcendental equation set, which has to be solved in cascade multilevel inverters. In addition, for a low number of switching angles, the proposed PSO approach reduces the computational burden to find the optimal solution compared with iterative methods and the resultant theory approach.

2. Cascaded Multilevel Inverter

A Cascaded MLI consists of several structures of *S* H-bridge (Single phase full bridge) inverters. Where value of *S* depends on level of inverter, for $2S+1$ level of inverter *S* number of H-bridge inverters is required. Fig. 1 shows the structure of single-phase cascaded 7-Level inverter with 3 H-bridges and Fig. 2 shows the 3-phase model of 7-level inverter [11]. Each individual inverter can generate three different voltage output $+V_{dc}$, 0 , $-V_{dc}$ by connecting dc input to ac source side by combinations of four active switches S_1 , S_2 , S_3 and S_4 . Switching sequence for various voltage levels is shown in Table 1.

Table 1: Switching sequence of 7-level cascaded multilevel inverter

Voltage level	1st H-bridge	2nd H-bridge	3rd H-bridge
$3V_{dc}$	S_1, S_4	S_1, S_4	S_1, S_4
$2V_{dc}$	S_1, S_4	S_1, S_4	S_4, D_3
V_{dc}	S_1, S_4	S_4, D_3	S_4, D_3
0	Off	Off	Off
$-V_{dc}$	S_3, D_4	S_3, D_4	S_2, S_3
$-2V_{dc}$	S_3, D_4	S_2, S_3	S_2, S_3

$-3V_{dc}$	S_2, S_3	S_2, S_3	S_2, S_3
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Here, $V_{dc1} = V_{dc2} = V_{dc3} = V_{dc}$. The output voltage of cascaded multilevel inverter is equal to sum of the output voltages of the individual bridges and can be controlled to produce a staircase waveform. The advantage of this topology of MLI is that the modulation, control and protection requirements of each bridge are modular.

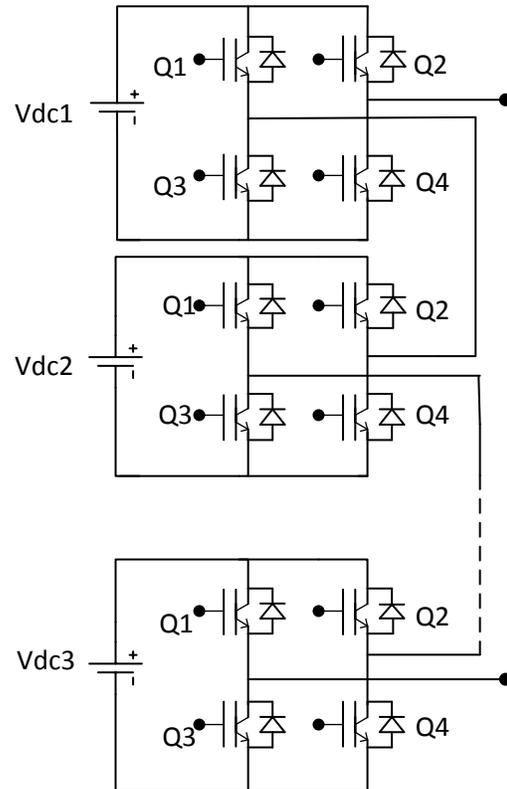


Fig. 1 Single phase cascaded MLI

3. Analysis of Multilevel Inverter

Fig. 3 shows the typical stepped waveform of the output phase voltage of a 7-level inverter with 3 equal dc sources. Assuming the symmetrical waveform only three angles are required to determine the full waveform α_1 , α_2 and α_3 . Where, α_1 , α_2 and α_3 are the switching angles of seven-level MLI.

Fourier analysis of this waveform yield the expression:

$$V_n = \begin{cases} \frac{2\sqrt{2}V_{dc}}{n\pi} (\cos(n\alpha_1) + \cos(n\alpha_2) + \cos(n\alpha_3)), & (1) \\ 0, & \end{cases}$$

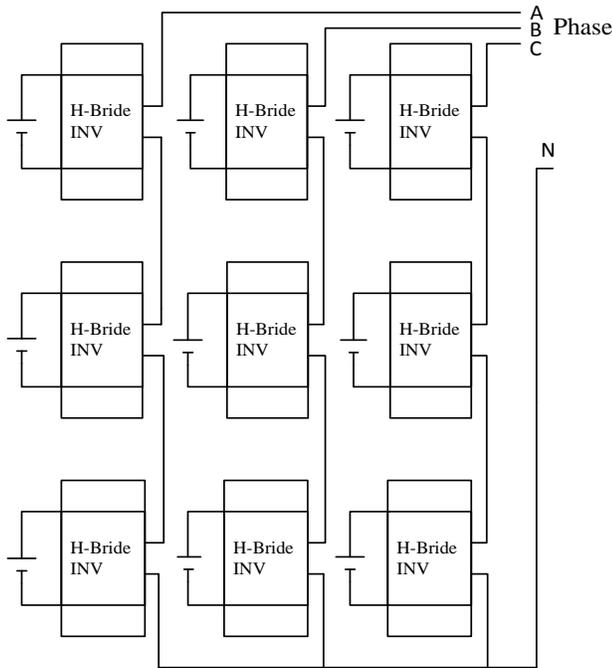


Fig. 2 Three phase structure of cascaded MLI

Owing to quarter wave symmetry in the waveform, the even harmonics are absent and thus only odd harmonics are present [12].

When α_1, α_2 and α_3 are all equal to zero the maximum possible value of fundamental components is obtained and given by:

$$V_1(\max) = \frac{6\sqrt{2}V_{dc}}{\pi} \quad (2)$$

Presenting the fundamental component in per unit (p.u.) based on its maximum value [13]

$$V_1^{pu} = \frac{V_1}{V_1(\max)} = \frac{1}{3}(\cos(\alpha_1) + \cos(\alpha_2) + \cos(\alpha_3)) \quad (3)$$

Normalised harmonic components is as follows:

$$\frac{V_n}{V_1(\max)} = \frac{1}{3n}(\cos(n\alpha_1) + \cos(n\alpha_2) + \cos(n\alpha_3)) \quad (4)$$

Considering the waveform in Fig. 2, the phase-voltage rms is calculated

$$V_{rms} = \sqrt{\frac{2}{\pi} \left(\int_0^{\frac{\pi}{2}} v^2 d\omega t \right)}$$

$$= V_{dc} \sqrt{\frac{2}{\pi} ((\alpha_2 - \alpha_1) + 4(\alpha_3 - \alpha_2) + 9(\frac{\pi}{2} - \alpha_3))} \quad (5)$$

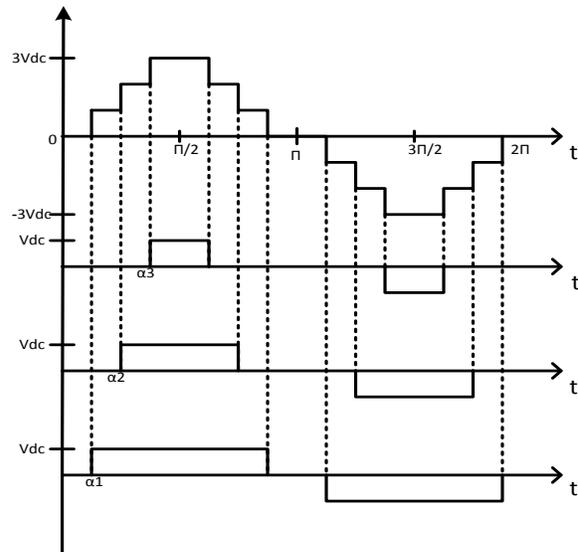


Fig. 3 Stepped waveform for cascaded MLI

Additionally, in per unit

$$V_{rms}^{pu} = \frac{V_{rms}}{V_1(\max)}$$

$$= \sqrt{\frac{\pi}{36} [(\alpha_2 - \alpha_1) + 4(\alpha_3 - \alpha_2) + 9(\frac{\pi}{2} - \alpha_3)]} \quad (6)$$

THD is defined as the ratio of sum of all harmonic components rms value to the rms value of fundamental component and is expressed as:

$$(THD)_p = \frac{\sqrt{\sum_{n=2}^{\infty} V_n^2}}{V_1} \quad (7)$$

Substituting V_1 and V in (7) by (3) and (6), the following analytical expression is obtained for the phase-voltage THD:

$$(THD)_p = \sqrt{\left[\frac{\pi}{4} \frac{[(\alpha_2 - \alpha_1) + 4(\alpha_3 - \alpha_2) + 9(\frac{\pi}{2} - \alpha_3)]}{(\cos(\alpha_1) + \cos(\alpha_2) + \cos(\alpha_3))^2} \right]^{-1}} \quad (8)$$

4. Harmonic Elimination

The number of harmonics which can be eliminated from the output phase voltage of inverter is $(S - 1)$. For example to eliminate the 7th order harmonic from seven-level inverter, Eq. (8) must be satisfied. The elimination of triple harmonics is not necessary as these are eliminated automatically from three-phase system [14].

$$\begin{aligned} \cos(\alpha_1) + \cos(\alpha_2) + \cos(\alpha_3) &= m \\ \cos(5\alpha_1) + \cos(5\alpha_2) + \cos(5\alpha_3) &= 0 \\ \cos(7\alpha_1) + \cos(7\alpha_2) + \cos(7\alpha_3) &= 0 \end{aligned} \tag{9}$$

Where switching angles α_1 to α_3 must satisfy the following conditions:

$$0 < \alpha_1 < \alpha_2 < \alpha_3 < \frac{\pi}{2} \tag{10}$$

And the modulation index M is defined from m . $M=m/s$, where s is the number of switching angles. Here, $s = 3$.

5. Particle Swarm Optimization

The PSO is a population based optimization technique which was originally developed by Kennedy and Eberhart in 1995 [15]. It is a stochastic search method for optimization problem, inspired by the behaviour of swarms such as schools of fish and flocks of birds. Every individual in the swarm is called as particles. PSO can be easily implemented, simple in concept and is an effective techniques when compared to other algorithms such as genetic algorithms (GA) and other iterative techniques.

PSO basically consists of two primary operators: Velocity vector and Position vector. During each iteration each particle searches for the optimal solution and accelerated to the particles global best position and the previously found best position, by comparing itself with the best position it has attained. At each generation a new velocity vector value is generated based on its present velocity, the distance from its previous best position and knowledge of global best position. All these particles move in multidimensional search and keep updating itself to find best solution (fitness). This fitness value decide the *gbest* (global best position) and *pbest* (local best position). The velocity and position vectors of PSO are shown below:

$$V_{i+1} = WV_i + \alpha \times rand_1(pbest_i - X_i) + \beta \times rand_2(gbest_i - X_i) \tag{11}$$

$$X_{i+1} = X_i + V_{i+1} \tag{12}$$

Where, α and β are the cognitive factors, $rand_1$ and $rand_2$ is the random numbers between 0 and 1, W is

the inertia weight to guarantee the speed of convergence and defined as:

$$W = (W_{max} - W_{min}) \times \frac{iter_{max} - iter}{iter_{max}} + W_{min} \tag{13}$$

6. PSO Algorithm for THD Minimization

Let $\alpha_i = [\alpha_{i1}, \alpha_{i2} \dots \alpha_{is}]$ be the initial trial vector of i^{th} particle of the swarm to be evolved. The elements of α_i are the solution of the harmonic minimization problem. The step-by-step by procedure to solve THD minimization problem using PSO is given as follows:

STEP 1: Get initial data for particles such as population size n , the maximum iteration count etc.

STEP 2: Generate initial conditions for each particle. Each particle position in the swarm is randomly initialized between 0 and $\pi/2$ according to Eq. (9); similarly the velocity vector of particle is initialized randomly from $-V_{max}$ to V_{max} based on the lower and upper bounds of the particle. The current searching point is set to *pbest* for each particle.

STEP 3: Now, for each particle the fitness value is calculated by using the objective function. The THD Eq. (7) is considered to be the objective function. To reduce the overall THD in output voltage waveform, the objective function has to be minimized with the constraints.

$$\begin{aligned} \text{Minimise } F(\alpha) &= F(\alpha_1, \alpha_2, \dots, \alpha_m) = \frac{\sqrt{\sum_{n=3,5,7}^m V_n^2}}{V_1} \\ \text{Subject to: } &0 < \alpha_1 < \alpha_2 < \dots < \alpha_m < (\pi/2) \\ &V_1 = m; \\ &V_5 \leq \epsilon_1; \\ &V_7 \leq \epsilon_2; \end{aligned} \tag{14}$$

According to Eq. (9) value of V_1, V_5, V_7 are obtained. Where the modulation index M is defined as $M=m/s$, where s is the number of switching angles here, $s = 3$. The min evaluated fitness value becomes the *gbest* for the swarm.

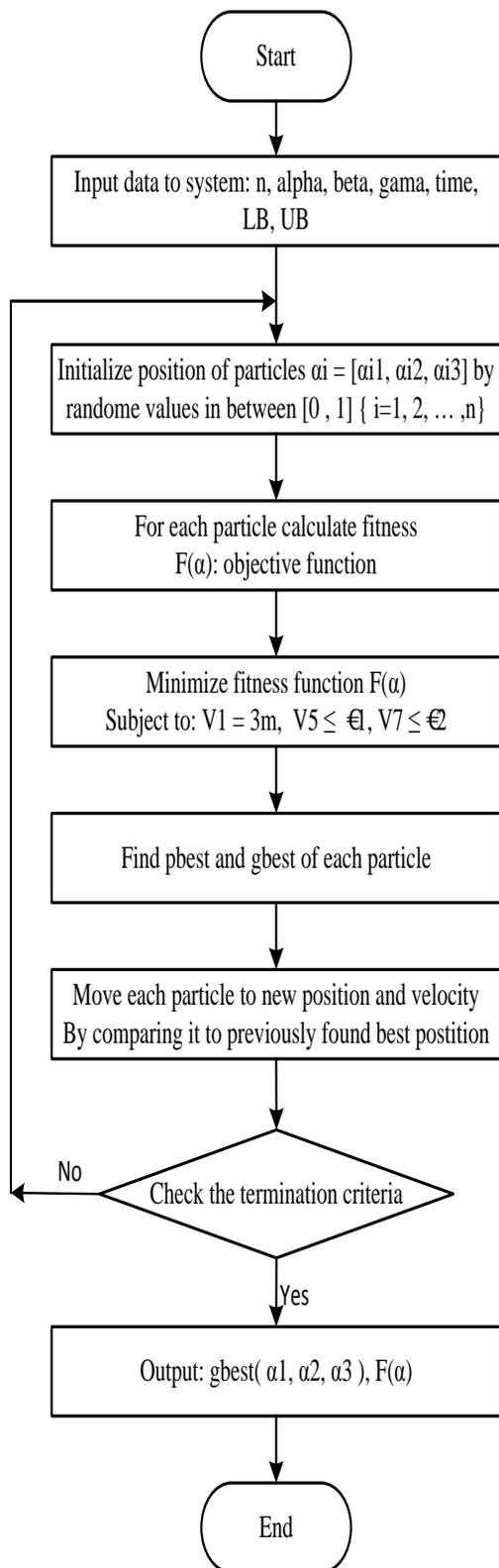


Fig. 4 Flow chart for PSO algorithm

STEP 4: Each particle of the swarm moves to new position and velocity, according to Eq. (11), (12) based on the *pbest* and *gbest* of the system, by updating itself. It also updated the personal best (*pbest*) if the current position is better than the previous position.

STEP 5: Check for the termination criteria. If the iteration count reaches the maximum iteration, stop; else, go back to step (2).

7. Results

By using PSO method, all possible solution sets for a five and seven level cascaded multilevel inverters are computed and a complete analysis is presented in following paragraphs. Switching angles ($\alpha_1, \alpha_2, \alpha_3$) are taken in range of 0° to 90° . The modulation index value is varies for the linear range from 0.1 to 0.95.

7.1 7-Level CMLI

For seven level cascaded inverter it is required to eliminate the 5th and 7th order harmonics and get the minimization in total harmonic distortion. The Fig.5 shows the switching angles sets and Fig.6 shows the corresponding least THD for various modulation indices from both numerical method and PSO for 7-level. It can be seen from Fig. 6 that THD is minimum at range 0.7 to 0.8 of modulation index.

7.2 5-Level CMLI

For five level it is required to eliminate the 5th harmonics. The Fig.7 shows the switching angles sets and Fig.8 shows the corresponding least THD for various modulation indices from both numerical method and PSO for 5-level. Fig. 9 compares the THD for five and seven level CMLI for same modulation index.

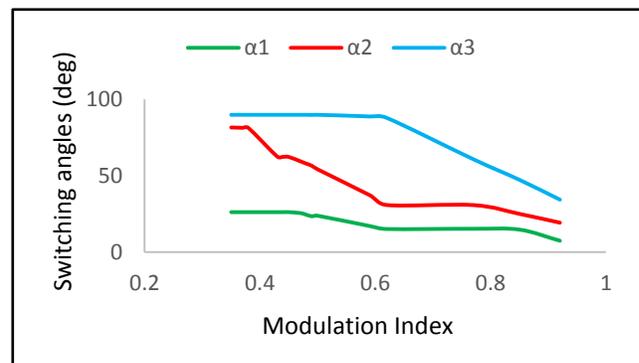


Fig. 5 Switching angles vs Modulation index for 7-level CMLI

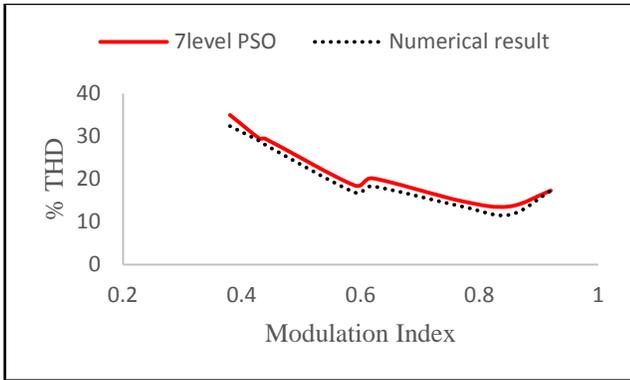


Fig. 6 THD vs Modulation index for 7-level CMLI

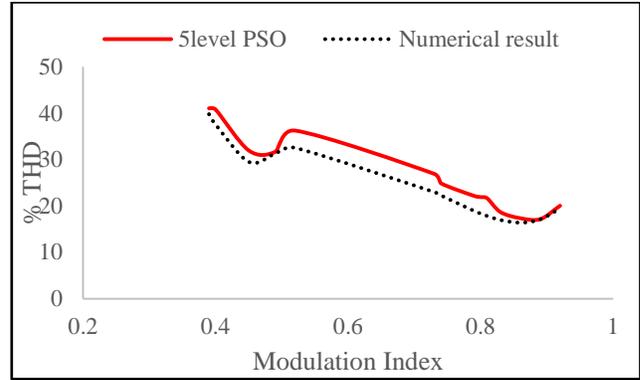


Fig. 8 THD vs Modulation index for 5-level CMLI

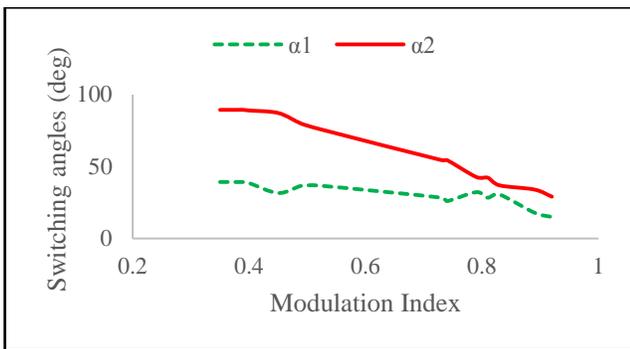


Fig. 7 Switching angles vs Modulation index for 5-level CMLI

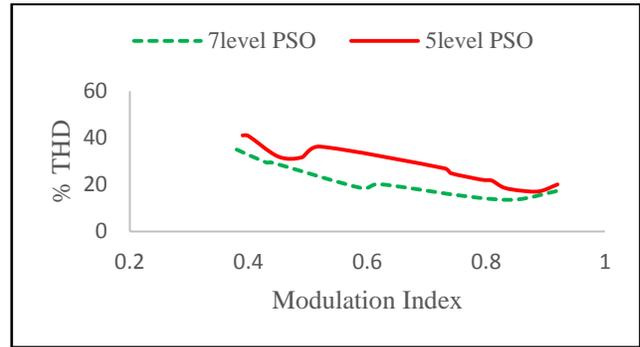


Fig. 9 Comparison of THD for five and seven level CMLI

Table 2: Comparison of THD obtained from PSO and numerical method in case of 7-level CMLI

MI	Switching angles in degrees						%THD	
	$\alpha 1$ (PSO)	$\alpha 1$ (Numerical method)	$\alpha 2$ (PSO)	$\alpha 2$ (Numerical method)	$\alpha 3$ (PSO)	$\alpha 3$ (Numerical method)	(PSO)	(Numerical method)
0.38	26.2335	19.0911	81.3055	78.8137	89.9427	89.9427	35.00	32.39
0.43	26.2335	18.1949	62.5671	70.184	89.9427	89.9427	29.45	28.90
0.44	26.2335	18.0105	62.4589	68.4077	89.9427	89.9427	29.40	27.97
0.59	17.1627	11.7628	37.3804	37.8145	88.8593	89.9427	18.53	16.90
0.62	15.1922	11.4783	30.89	36.7743	87.8914	85.4689	20.20	18.32
0.77	15.3642	10.2631	30.8573	31.7883	60.8373	61.5755	14.87	13.62
0.85	14.779	8.5061	25.0833	26.6197	47.3599	48.1641	13.62	11.66
0.92	7.5197	6.2796	19.3351	19.7798	34.4113	34.4115	17.34	17.29

Table 3: Comparison of THD obtained from PSO and numerical method in case of 5-level CMLI

MI	Switching angles in degrees				%THD	
	$\alpha 1$ (PSO)	$\alpha 1$ (Numerical method)	$\alpha 2$ (PSO)	$\alpha 2$ (Numerical method)	(PSO)	(Numerical method)
0.39	39.2299	38.9222	89.427	89.8854	41.08	39.82
0.4	38.54	37.0605	88.9785	89.8854	40.8	37.58
0.52	36.7886	19.4384	76.1637	84.4336	36.27	32.54
0.74	25.9769	16.4636	54.477	58.6006	24.91	22.29
0.8	33.6128	14.9836	39.8967	50.6541	21.92	18.37
0.92	14.9615	9.9364	29.0849	31.2403	20.07	19.47

8. Validation of Results

The results obtained from the PSO algorithm are found very close with those from the numerical method as explained in section 3. Numerical methods are lengthy and tedious to solve. On the other hand PSO converges to optimum value in much less steps. The dotted points in all the graphs shows the numerical solutions which coincides with the PSO solutions validating correctness of PSO. Comparison of PSO and numerical method can be seen in table 2 and table 3.

9. Conclusion

In this paper, application of PSO algorithm is investigated for reduction in THD of CMLI. The simulation is carried out for five and seven level inverter to eliminate harmonics using PSO. PSO converges faster and gives results very close to numerical method. From the result, we can observe that THD is much reduced for 7-level inverter as compared to 5-level. Also, between the modulation indexes in the range of 0.7 to 0.8, the magnitudes of 5th and 7th harmonics are less. This work can be further extended by increasing the level of CMLI, thereby increasing the number of switching angles. As switching angles are increased, more number of harmonics can be reduced.

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