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M/G/1 Retrial Queue with Two Stage Heterogeneous Service, Non-Persistent Customers, Extended Vacation and Setup Time.

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Abstract

This paper pays attention on a single server non-Monrovian retrial queuing model with non persistent customers, two stage heterogeneous service, Bernoulli vacation. Customers arrive according to Poisson stream with arrival rate λ and are served one by one with first come first served basis. In this model the server provides two phases of service in which after the first phase service is complete, second phase service starts and both the service time follow general (arbitrary) distribution. The customer, who finds the server busy upon arrival, can either join the orbit with probability p or he/she can leave the system with probability 1-p. On completion of a service the server may go for a vacation with probability θ or stay back in the system to serve a next customer with probability 1-θ, if any. Also when the vacation period is over the server is assumed as it must spend some time to get ready for giving proper service, called set up time and which is arbitrarily distributed. We obtain the steady solutions of the model by using supplementary variable technique. Also we derive the system performance measures and reliability indices are obtained.

Subject Classification: AMS 60K25, 90B22, 60K30

Key words: Bernoulli vacation, Retrial time, Reliability, Set up time, Two stages service.

1. Introduction

The study of retrial queuing system has become an interesting area because of its wide applicability in telephone switching networks, telecommunication networks and computer networks. Retrial queues are characterized by that upon arrival customer finds the server busy or on vacation and as there is no place in front of the server he/she may join the group of blocked customer(called orbit) for trying their request later on or leave the system immediately. There is a vast literature on retrial queuing models . Kulkarni (1983) , Farahmand (1990) Choi(1990) studied queuing systems with retrials. For recent bibliographies and survey we can refer Yang and Templeton (1990), Artalejo (1999) and Fallin and Templeton(1997). Artalejo and Fallin (2002) discussed the comparison between standard and retrial queueing models. Krishna kumar et al. (2002) studied an M/G/1 retrial queue with two phase service and pre emptive resume. Retrial queues with vacations has been concentrated by many authors include Artalejo (1997, 1999), Krishna kumar (2002) and Atencia(2005) and Zhou(2005). Arumuganathan (2008) discussed single server batch arrival retrial queue with general vacation time under Bernoulli schedule and two phases of heterogeneous service Arumuganathan (2009) performed analysis on two phase heterogenous service and different vacation policies of an M/G/1 retrial queue with non-persistent calls . Wang (2008) studied repairable retrial queue with setup time and Bernoulli vacation. Yang et al.(1990) studied M/G/1 retrial queue with impatient customers. Choudhry(2009) studied both single and batch arrival of retrial queue with two phase of service and general retrial times.

In this paper, a single server retrial queue with impatient customers, two stages of heterogeneous service, extended vacation is considered. In addition to this after the vacation is over the server must take some time before giving service. By using supplementary variable technique this model is analysed. The rest of the is organised as follows section 2 describes the mathematical model, section 3 discusses the steady state of the model, section 4 gives the performance measures of the system.

2. Mathematical Model

In this paper, we consider a single server retrial queuing system in which the primary customers arrive according to Poisson process with rate λ, and upon arrival if the customer finds the server busy or on vacation joins the orbit with probability p or leaves the service area with probability 1-p, being impatient. The server provide the two stages service in succession to all customers. After the second stage service is over the server may apt for a vacation with probability θ or stay continue in the system to serve a new customer if any with probability 1-θ. The server vacation The retrial time that is the time between successive repeated attempts of each customer in orbit is assumed to be generally distributed with distribution function A(x), density function a(x), the mean value a, and Laplace transform $\bar{a}(s)$. Assuming that retrial times begin either at the completion instants of service or setup times so that the distribution of the remaining retrial time is $A_e(x) = \frac{1}{a}\int_0^\infty (1 - A(x))x$ and $a_e(x) = \frac{1 - A(x)}{a}$. The conditional completion rate time for retrials is given by $\eta_e(x) = \frac{a_e(x)}{(1 - A(x))}$ .Let $B_1(x)$,b_1 (x)and $B_2(x)$, b_2 (x)be the distribution function and the density function of the both stage service time respectively. Both the service times are independent to each other. The Laplace transform of total service is given by $\bar{s}(s)$. Let $\mu_i(x)$ be the conditional probability density function of service completion of i[th] service during the interval (x,x+dx] given that the elapsed time is x, so that $\mu_i(x) = \frac{b_i(x)}{1 - B_i(x)}$; i=1,2. After the completion of second stage service , the server may apt for a vacation with probability θ or staying back in the system to provide service to new customer if any , with probability 1-θ. Let V_i (x)and $v_i(x)$ be the distribution function and density function respectively and its Laplace transform is $\bar{v}(s)$. Let $\gamma_i(x)$ be the conditional probability density function of service completion of i[th] phase vacation during the interval (x,x+dx] given that the elapsed time is x, so that $\gamma_i(x) = \frac{v_i(x)}{1 - V_i(x)}$; i=1,2. After the first phase of vacation is over the server may go for second phase of vacation with probability r or return to the system with probability 1-r. After the vacation period is over, the server spend some time for setup and the setup time is arbitrarily distributed. Let D(x), d(x) be its distribution function, density function respectively and its Laplace transform is $\bar{d}(s)$. Let δ(x) be the conditional probability density function of setup time during the interval (x,x+dx] given that the elapsed time is x, so that $\delta(x) = \frac{d(x)}{1 - D(x)}$. All the stochastic processes are independent to each other.

Let N(t) be denote the number of customers in the orbit at time t and C(t) be the state of the server and which is given by:

C(t)=0 if the server is idle
1 if the server is busy with first stage service
2 if the server is busy with second stage service
3 if the server is on first phase vacation
4 if the server is on second phase vacation
5 if the server is in set up

Define the following probability functions

$Q_n(x,t)dt = \Pr\{N(t)= n, C(t)=0\}$, n>0

$P_n^{(i)}(x,t)dt = \Pr\{N(t)= n, C(t)=i\}$, $n\geq 0$; i=1,2

$V_n^{(i)}(x,t)dt = \Pr\{N(t)= n, C(t)=i\}$, $n\geq 0$;i=3,4

$S_n(x,t)dt = \Pr\{N(t)= n, C(t)=5\}$, $n\geq 0$

3 Steady State System Size Distribution

The steady state equations are obtained for our model are

$$\lambda Q_0 = (1-\theta)\int_0^\infty p_0^{(2)}(x)\mu(x)dx + \int_0^\infty S_0(x)\delta(x)dx; n\geq 1 \tag{1}$$

$$\left(\frac{d}{dx}+\lambda+\eta_e(x)\right)Q_n(x)=0; n\geq 1 \tag{2}$$

$$\left(\frac{d}{dx}+p\lambda++\mu_1(x)\right)P_n^{(1)}(x) = p\lambda P_{n-1}^{(1)}(x); n\geq 0 \tag{3}$$

$$\left(\frac{d}{dx}+p\lambda++\mu_2(x)\right)P_n^{(2)}(x) = p\lambda P_{n-1}^{(2)}(x); n\geq 0 \tag{4}$$

$$\left(\frac{d}{dx}+p\lambda+\gamma_1(x)\right)V_n^{(1)}(x) = p\lambda V_n^{(1)}(x); n\geq 0 \tag{5}$$

$$\left(\frac{d}{dx}+p\lambda+\gamma_2(x)\right)V_n^{(2)}(x) = p\lambda V_n^{(2)}(x); n\geq 0 \tag{6}$$

$$\left(\frac{d}{dx}+p\lambda+\delta\ (x)\right)S_n(x) = p\lambda S_n(x); n\geq 0 \tag{7}$$

Subject to the boundary conditions

$$I_n(0) = (1-\theta)\int_0^\infty P_n^{(2)}\ (x)\mu(x)dx + \int_0^\infty S_n(x)\gamma(x)dx; n\geq 1 \tag{8}$$

$$P_O^{(1)}\ (0) = \int_0^\infty Q_1(x)\eta_e(x)dx + \lambda Q_0 \tag{9}$$

$$P_n^{(1)}\ (0) = \int_0^\infty Q_{n+1}(x)\eta_e(x)dx + \lambda\int_0^\infty Q_n(x)dx; n\geq 1 \tag{10}$$

$$P_n^{(2)}\ (0) = \int_0^\infty P_n^{(1)}{}_n(x)\mu_1(x)dx; n\geq 0 \tag{11}$$

$$V_n^{(1)}(0) = \theta\int_0^\infty P_n^{(1)}(x)\mu_1(x)dx;; n \ge 0 \tag{12}$$

$$V_n^{(2)}(0) = r\int_0^\infty V_n^{(1)}(x)\gamma_1(x)dx;; n \ge 0 \tag{13}$$

$$S_n(0) = (1-r)\int_0^\infty V_n^{(1)}(x)\gamma_1(x)dx + r\int_0^\infty V_n^{(2)}(x)\gamma_2(x)dx; n \ge 0 \tag{14}$$

Normalising condition

$$1 = Q_0 + \sum_{n=1}^\infty\int_0^\infty Q_n(x)dx + \sum_{n=0}^\infty\int_0^\infty P_n^{(1)}(x)dx + \sum_{n=0}^\infty\int_0^\infty P_n^{(2)}(x)dx + \sum_{n=0}^\infty\int_0^\infty V_n^{(1)}(x)dx + \sum_{n=0}^\infty\int_0^\infty V_n^{(2)}(x)dx + \sum_{n=0}^\infty\int_0^\infty S_n(x)dx \tag{15}$$

To solve the above equations, we define the following generating functions:

$$Q_q(x,z) = \sum_{n=1}^\infty Q_n(x)z^n; \tag{16}$$

$$P_q^{(i)}(x,z) = \sum_{n=0}^\infty P_n^{(i)}(x)z^n; i = 1,2 \tag{17}$$

$$V_q^{(i)}(x,z) = \sum_{n=0}^\infty V_n^{(i)}(x)z^n; i = 1,2 \tag{18}$$

$$S_q(x,z) = \sum_{n=0}^\infty S_n(x)z^n; \tag{19}$$

Multiply equations (2) –(7) by appropriate powers of z and apply the generating function defined above

$$\left(\frac{d}{dx} + \lambda + \eta_e(x)\right)Q_q(x,z) = 0 \tag{20}$$

$$\left(\frac{d}{dx} + p\lambda(1-z) + \mu_1(x)\right)P_q^{(1)}(x,z) = 0 \tag{21}$$

$$\left(\frac{d}{dx} + p\lambda(1-z) + \mu_2(x)\right)P_q^{(2)}(x,z) = 0 \tag{22}$$

$$\left(\frac{d}{dx} + p\lambda(1-z) + \gamma_1(x)\right)V_q^{(1)}(x,z) = 0 \tag{23}$$

$$\left(\frac{d}{dx}+p\lambda(1-z)+\gamma_2(x)\right)V_q^{(2)}\ (x,z)=0 \tag{24}$$

$$\left(\frac{d}{dx}+p\lambda(1-z)+\delta(x)\right)S_q(x,z)=0 \tag{25}$$

$$Q_q(0,z)=(1-\theta)\int_0^\infty p_q^{(2)}\ (x,z)\mu_2(x)dx+\int_0^\infty S_q(x,z)\delta(x)dx-\lambda I_0 \tag{26}$$

$$zP_q^{(1)}\ (0,z)=\int_0^\infty Q_q(x,z)\eta_e(x)dx+\lambda z\int_0^\infty Q_q(x,z)dx+\lambda Q_0 z \tag{27}$$

$$P_q^{(2)}\ (0,z)=\int_0^\infty P_q^{(1)}\ (x,z)\mu_1(x)dx \tag{28}$$

$$V_q^{(1)}\ (0,z)=\theta\int_0^\infty P_q^{(2)}\ (x,z)\mu_2(x)dx \tag{29}$$

$$V_q^{(2)}\ (0,z)=r\int_0^\infty V_q^{(1)}\ (x,z)\gamma_1(x)dx \tag{30}$$

$$S_q\ (0,z)=\int_0^\infty V_q^{(2)}\ (x,z)\gamma_2(x)dx \tag{31}$$

0n solving

$$Q_q(x,z)=Q_q(0,z)e^{-\lambda x}(1-R_e(x)) \tag{32}$$

$$P_q^{(1)}\ (x,z)=P_q^{(1)}(0,z)e^{-\lambda p(1-z)x}(1-B_1(x)) \tag{33}$$

$$P_q^{(2)}\ (x,z)=P_q^{(2)}(0,z)e^{-\lambda p(1-z)x}(1-B_2(x)) \tag{34}$$

$$V_q^{(1)}\ (x,z)=V_q^{(1)}(0,z)e^{-\lambda p(1-z)x}(1-V_1(x)) \tag{35}$$

$$V_q^{(2)}\ (x,z)=V_q^{(2)}(0,z)e^{-\lambda p(1-z)x}(1-V_2(x)) \tag{36}$$

$$S_q(x,z)=S_q(0,z)e^{-\lambda p(1-z))x}(1-S(x)) \tag{37}$$

$$Q_q(0,z) = (1-\theta)P_q^{(1))}(0.,z)\bar{B}_1(\lambda p(1-z))\bar{B}_2(\lambda p(1-z)) + \theta P^{(1)}{}_q(0,z)\bar{B}_1(\lambda p(1-z))\bar{B}_2(\lambda p(1-z))[(1-r)\bar{V}_1(\lambda p(1-z)) + r\bar{V}_2(\lambda p(1-z))]\bar{S}(\lambda p(1-z)) - \lambda I_0 \tag{38}$$

Let $H(z) = \bar{B}_1(\lambda p(1-z))\bar{B}_2(\lambda p(1-z))$ and

$$M(z) = [(1-r)\bar{V}_1(\lambda p(1-z)) + r\bar{V}_2(\lambda p(1-z))]\bar{S}(\lambda p(1-z))$$

$$Q_q(0,z) = [(1-\theta)H(z) + \theta M(z)H(z)]P_q^{(1)}(0,z) - \lambda Q_0 \tag{39}$$

$$P_q^{(1)}(0,z) = \frac{\lambda Q_0(1-z)\bar{R}_e(\lambda)}{Dr} \tag{40}$$

$$P_q^{(2)}(0,z) = \frac{\lambda Q_0(1-z)\bar{R}_e(\lambda)\bar{B}_1(\lambda p(1-z))}{Dr} \tag{41}$$

$$V_q^{(1)}(0,z) = \frac{\theta\lambda Q_0(1-z)\bar{R}_e(\lambda)\bar{B}_1(\lambda p(1-z))\bar{B}_2(\lambda p(1-z))}{Dr} \tag{42}$$

$$V_q^{(2)}(0,z) = \frac{r\theta\lambda Q_0(1-z)\bar{R}_e(\lambda)\bar{B}_1(\lambda p(1-z))B_2(\lambda p(1-z))\bar{V}_1(\lambda p(1-z))}{Dr} \tag{43}$$

$$S_q(0,z) = \frac{\theta\lambda Q_0(1-z)\bar{R}_e(\lambda)\bar{B}_1(\lambda p(1-z))\bar{B}_2(\lambda p(1-z))[(1-r)\bar{V}_1(\lambda p(1-z)) + r\bar{V}_2(\lambda p(1-z))]}{Dr} \tag{44}$$

$$Q_q(0,z) = \frac{\lambda Q_0 z(1-\bar{R}_e(\lambda))\{1-[(1-\theta)+\theta M(z)]H(z)\}}{Dr} \tag{45}$$

Where Dr is given by

$$\text{Dr} = \bar{R}_e(\lambda)(1-z)\{[(1-\theta)+\theta M(z)]H(z)\} - z\{1-[(1-\theta)+\theta M(z)]H(z)\} \tag{46}$$

The steady probabilities are given by

$$P_q^{(1)}(z) = \frac{Q_0\bar{R}_e(\lambda)(1-B_1(\lambda\bar{p}(1-z)))}{pDr} \tag{47}$$

$$P_q^{(2)}(z) = \frac{Q_0\bar{R}_e(\lambda)B_1(\lambda p(1-z))(1-B_2(\lambda\bar{p}(1-z)))}{pDr} \tag{48}$$

$$V_q^{(1)}(z) = \frac{\theta Q_0 \bar{R_e}(\lambda) B_1(\lambda p(1-z)) B_2(\lambda p(1-z))(1 - V_1(\lambda \bar{p}(1-z)))}{pDr} \quad (49)$$

$$V_q^{(2)}(z) = \frac{r\theta Q_0 \bar{R_e}(\lambda) B_1(\lambda p(1-z)) B_2(\lambda p(1-z)) V_1(\lambda p(1-z))(1 - V_2(\lambda \bar{p}(1-z)))}{pDr} \quad (50)$$

$$S_q(z) = \frac{\theta Q_0 \bar{R_e}(\lambda) B_1(\lambda p(1-z)) B_2(\lambda p(1-z))[(1-r)V_1(\lambda p(1-z)) + rV_2(\lambda p(1-z))](1 - S\ (\lambda \bar{p}(1-z)))}{pDr} \quad (51)$$

$$Q_q(z) = \frac{Q_0 z(1 - \bar{R_e}(\lambda))\{1 - [(1-\theta) + \theta M(z)]H(z)\}}{Dr} \quad (52)$$

To find Q_0 using the normalizing condition at z=1

$$Q_0 + P_q^{(1)}(1) + P_q^{(2)}(1) + V_q^{(1)}(1) + V_q^{(2)}(1) + S_q(1) = 1 \quad (53)$$

$$Q_0 = \frac{\bar{R_e}(\lambda) - p\rho}{\bar{R_e}(\lambda)[1 + \rho(1-p)]} \quad (54)$$

Where $\rho=\lambda[(E(S_1+S_2)+\theta(E(V_1)+rE(V_2)+E(D)]$

The necessary and sufficient condition for the stability condition is

$$\bar{R_e}(\lambda) > p\rho \quad (55)$$

4. **Performance Measures of the System**

In this section we give some system queuing measures of the system.

(1)Probability that the server is idle= $Q_0 = \dfrac{\bar{R_e}(\lambda) - p\rho}{\bar{R_e}(\lambda)[1 + \rho(1-p)]}$ (56)

(2)Probability that the server is idle but the orbit is not empty= $Q_q(1) = \dfrac{(1 - \bar{\bar{R}}_e(\lambda))p\rho}{\bar{R_e}(\lambda)[1 + \rho(1-p)]}$ (57)

(3)Probability that the server is busy with either first stage or second stage= $P_q^{(1)}(1) + P_q^{(2)}(1) = \dfrac{\lambda(E(S_1) + E(S_2))}{[1 + \rho(1-p)]}$ (58)

(4)Probability that the server is on first phase vacation $= V_q^{(1)}(1) = \dfrac{\lambda\theta E(V_1)}{[1 + \rho(1-p)]}$ (59)

(5)Probability that the server is on second phase vacation= $V_q^{(2)}(1) = \dfrac{r\lambda\theta E(V_2)}{[1+\rho(1-p)]}$ (60)

(6)Probability that the server is on setup period $S_q(1) = \dfrac{\lambda\theta E(D)}{[1+\rho(1-p)]}$ (61)

(7)The average number of customers in the orbit =L $_q$=

$$\frac{\lambda^2 p\{E(S^2)+2E(S)\theta[E(V)+E(D)]+\theta[E(V^2)+E(D^2)]\}\{(1-p)\bar{R}_e(\lambda)+p\}}{2[1+\rho(1-p)][\bar{R}_e(\lambda)-p\rho]} + \left[\frac{p\rho(1-\bar{\bar{R}}_e(\lambda))}{\bar{R}_e(\lambda)-p\rho}\right]$$

(62)

Where E(S)=E(S_1)+E(S_2); E(S^2)=E(S_1^2)+2E(S_1)E(S_2)+E(S_2^2);

E(V)=E(V_1)+rE(V_2); E(V^2)=E(V_1^2)+2rE(V_1)E(V_2)+rE(V_2^2);

(8)The expected waiting time in the orbit $W_q = \dfrac{L_q}{\lambda p}$ (63)

(9)The blocking probability =1-[Q_0+Q_1]= $= \dfrac{\rho}{[1+\rho(1-p)]}$ (64)

(10) The steady state availability of the server = A = $1-\left\{\dfrac{\lambda\theta[E(V_1)+rE(V_2)+E(D)]}{1+\rho(1-P)}\right\}$ (65)

Conclusion

In this paper, a single server retrial queue with two stage heterogeneous service, extended server vacation and setup time is considered . We studied the steady state distribution with performance measures of system .

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