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Analysis of Channel Estimation Error and Feedback Delay in Closed-loop Transmit Antenna Diversity (TAD) Communication Systems

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Abstract

In this paper, we analyze the performance of closed-loop transmit diversity systems in the presence of feedback delay and channel estimation error in Rayleigh fading channels. We formulate both the feedback delay and the channel estimation error as a function of Doppler spread, and derive the average bit error probability under such imperfect channel information. It is shown that the channel estimation error is more sensitive to system performance than feedback delay in slow fading environments.

Keywords: *Channel Estimation, Feedback Delay, Rayleigh Fading, Transmit Diversity, Closed-loop.*

1. Introduction

Transmit antenna diversity (TAD) enhances the communications system capacity and the transmission quality as well as to reduce the system complexity of a mobile station [1][2]. There are representative two TAD systems; open-loop TAD systems and closed-loop TAD systems.

Under the assumption of the perfect channel state information (CSI) at transmit side, it is well known that the signal-to-noise ratio (SNR) gain of closed-loop over open-loop is $(10\times\log_{10} P)$ dB in frequency flat fading, where P denotes the number of the transmit antenna. However, in real frequency duplex system the perfect CSI assumption is impossible due to the inevitable feedback delays, CSI estimation error, quantization error of CSI, and etc. In [3], only the feedback delay is considered for performance analysis of closed-loop systems.

In this paper, we investigate the combined effect of channel estimation error and feedback delay on the performance of closed-loop TAD systems in frequency flat slow time-varying fading channels. The Maximal Likelihood (ML) channel estimator [4] is applied for the CSI estimation. We develop both the channel estimation and the feedback delay as a function of Doppler frequency, and derive an analytical bit error rate (BER) of the closed-loop TAD system. We also provide a simulation result with ML channel estimation in frequency flat slow time-varying fading channels.

2. System Model.

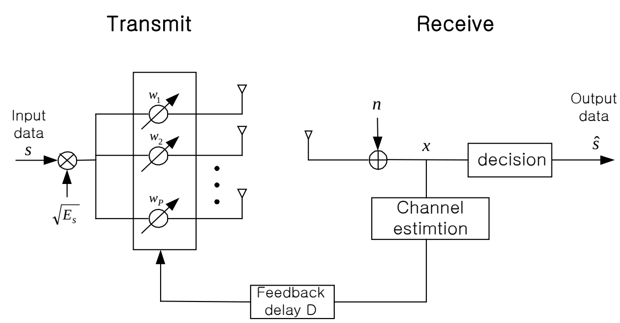

Fig. 1 Block diagram of a closed-loop TAD communication system.

We consider a closed-loop transmit diversity system with P transmit antennas, which is shown in Fig. 1. The complex baseband representation of the received signal, $x(t)$, at time t can be described by

$$x(t) = \sqrt{E_s}\,\boldsymbol{H}(t)\boldsymbol{W}(t)s(t) + n(t) \qquad (1)$$

where E_s is the symbol energy. Let $\boldsymbol{H}(t) = [h_1(t), h_2(t),..., h_P(t)]$ defines the $1\times P$ complex Gaussian channel gain vector. We assumed that each element of $\boldsymbol{H}(t)$ has independent identically distributed (i.i.d) zero mean wide-sense stationary (WSS) complex Gaussian and unit variance. $\boldsymbol{W}(t) = [w_1(t), w_2(t), ..., w_P(t)]^T$ is the $P\times 1$ transmit weighting vector where the superscript T denotes the transpose. We assume the power is constrained to $\|\boldsymbol{W}\|^2 = 1$, where $\|\bullet\|$ represents Euclidean norm. $s(t)$ is a transmit antipodal symbol, ± 1, with a symbol period T_s, and $n(t)$ is the zero mean complex

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WSS additive Gaussian noise with variance N_0 and independent of $\boldsymbol{H}(t)$.

The time-varying characteristics of the fading channel cause estimation error. Thus the channel gain vector can be written by

$$\boldsymbol{H}(t) = \hat{\boldsymbol{H}}(t) + \boldsymbol{\eta}(t) \quad (2)$$

where $\hat{\boldsymbol{H}}(t)$, the estimated channel gain, has the following relations [3, 5];

$$\hat{\boldsymbol{H}}(t) = \rho\hat{\boldsymbol{H}}(t-D) + \boldsymbol{\varepsilon}(t) \quad (3)$$

where $\hat{\boldsymbol{H}}(t-D)$ is the delayed channel gain vector and D is the feedback delay time. $\boldsymbol{\varepsilon}(t) = [\varepsilon_1(t), \varepsilon_2(t), \dots, \varepsilon_P(t)]$ is the error vector which has zero mean Gaussian distributed, and each element of $\boldsymbol{\varepsilon}(t)$ has variance σ_ε^2. The variance is defined by [6]

$$\sigma_\varepsilon^2 = \sigma_{\hat{H}}^2 (1 - |\rho|^2) \quad (4)$$

Without loss of generality, we can assume the variances of $H(t)$, $\hat{H}(t)$ and $\hat{H}(t-D)$ are same. The correlation coefficient, ρ, can be written by [5]

$$\rho = J_0(2\pi f_D D) \quad (5)$$

where $J_0()$ is the zeroth order Bessel function of the first kind and f_D is the maximum Doppler frequency. In (2), $\boldsymbol{\eta(t)} = [\eta_1(t), \eta_2(t), \dots, \eta_P(t)]$ is the $1 \times P$ channel estimation error vector which has zero mean Gaussian distribute. The variance, σ_η^2, of each element of $\boldsymbol{\eta(t)}$ can be obtained from that of the open-loop transmit system by replacing $(E_b/N_0)/P$ by E_b/N_0. Applying the result of open-loop transmit system in [4] to closed-loop transmit system,   can be represented by

$$\sigma_\eta^2 = \frac{1}{(E_s/N_0)d} + 2[\pi f_D T_s(\frac{2L-d-1}{2})]^2 \quad (6)$$

We assume a frame has L symbols which consist d training symbols and $L-d$ transmit data symbols. Note that the channel is estimated at the end of the training symbols and the estimated channel information is used during the data symbols. Consequently, (6) represents the variance of the channel estimation error for the Lth symbol in the given frame. The frame structure for ML channel estimation is shown in Fig. 2.

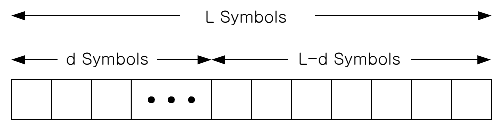

Fig. 1 Frame structure for ML channel estimation.

The optimum weighting vector is a normalized complex conjugate transpose of the estimated channel gain vector, and can be written by [7]

$$\boldsymbol{W_o}(t) = \frac{\hat{\boldsymbol{H}}^H(t-D)}{\left\|\hat{\boldsymbol{H}}(t-D)\right\|} \quad (7)$$

where the superscript $\underline{H}$ denotes the complex-conjugate transpose. Note that this optimum weighting vector includes the effect of the feedback delay and the channel estimation error. The variance of the channel estimation error variance is counted on the last data symbol in (6), $L-d$ symbols is regarded as feedback delay D.

By replacing (2), (3) and (7) in (1), we can obtain the received signal in the presence of feedback delay, and channel estimation error,

$$x(t) = \sqrt{E_s}\{\rho\hat{\boldsymbol{H}}(t-D) + \boldsymbol{\varepsilon}(t) + \boldsymbol{\eta}(t)\}\frac{\hat{\boldsymbol{H}}^H(t-D)}{\left\|\hat{\boldsymbol{H}}(t-D)\right\|}s(t) + n(t) \quad (8)$$

3. Performance Analysis

The instantaneous BER at sampling time $t = m$ can be written by

$$\begin{aligned} P_e &= Q(\sqrt{\frac{2E_s}{N_0}}\operatorname{Re}\{\boldsymbol{H}(m)\boldsymbol{W}_o(m)\}) \\ &= Q(\sqrt{\frac{2E_s}{N_0}}\operatorname{Re}[\{\rho\hat{\boldsymbol{H}}(m-D) + \boldsymbol{\varepsilon}(m) + \boldsymbol{\eta}(m)\} \\ &\quad \times \frac{\hat{\boldsymbol{H}}(m-D)}{\left\|\hat{\boldsymbol{H}}(m-D)\right\|}] \end{aligned} \quad (9)$$

where we assume the transmit signal $s(t)$ equals +1. Re() denotes Real part. And $Q()$ is defined by

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_u^\infty \exp(-\frac{u^2}{2})du \quad (10)$$

If we define

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$$g(\hat{\boldsymbol{H}}(m-D),\boldsymbol{\varepsilon}(m),\boldsymbol{\eta}(m)) = \rho\left\|\hat{\boldsymbol{H}}(m-D)\right\| + \frac{\mathrm{Re}[\hat{\boldsymbol{H}}^{H}(m-D)\{\boldsymbol{\varepsilon}(m)+\boldsymbol{\eta}(m)\}]}{\left\|\hat{\boldsymbol{H}}(m-D)\right\|}, \tag{11}$$

then, $g(\hat{\boldsymbol{H}}(m-D),\boldsymbol{\varepsilon}(m),\boldsymbol{\eta}(m))$ is Gaussian distributed with mean $\rho\left\|\hat{\boldsymbol{H}}(m-D)\right\|$ and variance $(\sigma_\varepsilon^2+\sigma_\eta^2)/2$ for a given $\hat{\boldsymbol{H}}(m-D)$. And $\hat{\boldsymbol{H}}(m-D)$ is Complex Gaussian distributed with zero mean and unit variance. The average BER is denoted by

$$\begin{aligned} P_{avg} &= E[Q(\sqrt{\frac{2E_s}{N_0}}\mathrm{Re}\{\boldsymbol{H}(m)\boldsymbol{W_o}(m)\})] \\ &= E_{\hat{H}(m-D)}[E_{\varepsilon(m)+\eta(m)} \\ &\{Q(\sqrt{\frac{2E_s}{N_0}}g(\hat{\boldsymbol{H}}(m-D),\boldsymbol{\varepsilon}(m),\boldsymbol{\eta}(m))\big|\hat{\boldsymbol{H}}(m-D)\}] \end{aligned} \tag{12}$$

where $E_X[Y]$ denotes the expectation of random variable Y with respect to random variable X. By applying the estimation error variance to the average BER of E. Onggosanusi et all's [3], the average BER of closed-loop transmit diversity system with feedback delay and channel estimation error can be obtained by

$$P_{avg} = \frac{1}{2}[1-\frac{1}{2}\sum_{p=0}^{P-1}\sum_{k=0}^{p}\left(-\frac{1}{2}\right)^k\binom{p}{k}\frac{1}{\pi} \times \int_0^{\pi}\frac{d\theta}{(a+b\cos\theta)^{k+1}}] \tag{13}$$

and

$$\begin{aligned} a &= b+\frac{1}{2} \\ b &= \frac{\mu(1-\rho^2+\sigma_\eta^2)+2}{4\mu\rho^2} \end{aligned} \tag{14}$$

where μ is E_s/N_0 and equals to E_b/N_0 when BPSK modulation. If there is no channel estimation error (i.e. $\boldsymbol{H}(t)=\hat{\boldsymbol{H}}(t)$), the above results are coincident with (26) in [3].

4. Numerical Results and Discussions

For numerical examples we assumed 12.2 Kbps symbol rate (T_s = 81.97 μsec) and carrier frequency of 2.15 GHz in WCDMA [2]. To generate a time correlated Rayleigh fading channel in our simulation, the Jakes fading model [5] is used.

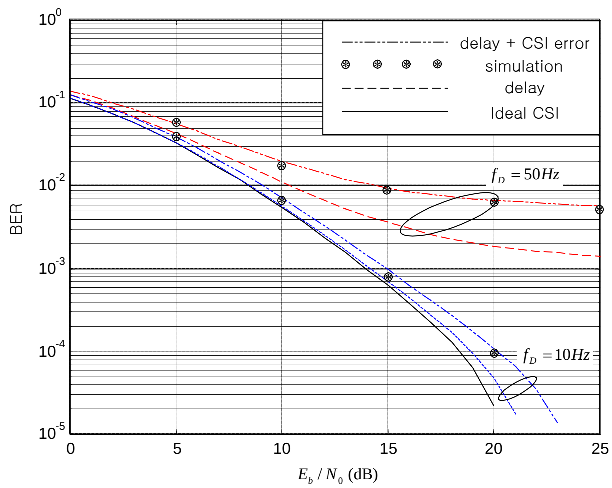

Fig. 3 Average bit error rate for f_D = 10 Hz and 50 Hz ($P=2, d=4, d/L=20$).

Fig.3 shows analytical and simulated BER of the closed-loop transmit system for maximum Doppler frequency of 10 Hz and 50 Hz with $P=2, d=4, d/L=20$ %. The ideal CSI means no feedback delay and no estimation error. Simulation is performed under the condition of feedback delay with channel estimation error. At maximum Doppler frequency of 10 Hz, the performance degradation is 1.0 dB at BER of 10^{-3} when the delay and CSI estimation error exist. While Doppler frequency of 50 Hz, the performance degrades more severely and cannot be reached to BER of 10^{-3} . These Doppler frequency 10 Hz and 50 Hz at 2.15 GHz carrier frequency correspond to mobile speeds of 5 Km/h and 25.1 Km/h, respectively. Therefore, closed-loop transmit diversity scheme is adequate to slowly moving vehicles.

The performance degradation of delay only case and that of delay with estimation error case are 0.03 dB and 0.6 dB at BER of 10^{-2} with Doppler frequency 10 Hz, respectively. However, this degradation increases to 2.0 dB and 6.6 dB with Doppler frequency 50 Hz. This implies the channel estimation error is more sensitive to the system performance than feedback delay.

In Fig. 4, we assume the training symbols are equal to the transmit antennas; the training symbols of the ML estimator for a multiple transmit system must be greater than or equal to transmit antennas [4]. The BER performance is improved by the increase of the number of the transmit antennas. This is because the time variation of the channel fading diminishes as the diversity order increases. As we expected, the performance gain is not linearly proportional to the number of the transmit antennas.

The performance degradation at high BER is small compared to that at low BER. It is interpreted that the system performance is more sensitive to the feedback delay with channel estimation error at high BER.

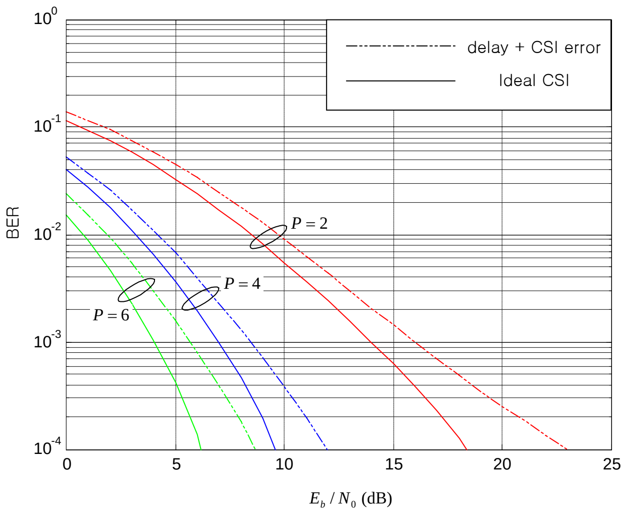

Fig. 4 BER versus the number of transmit antennas for f_D = 30 Hz ($d = P$, $d / L = 20$ %).

5. Conclusions

We investigated the effect of the feedback delay and the channel estimation errors on the performance of closed-loop transmit diversity systems. The BER performance was analytically derived and verified by the simulation. From our numerical examples, we note that the dominant factor of the performance degradation is the estimation error rather than feedback delay.

Acknowledgments

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References

[1] R. T. Derryberry, S. D. Gray, D. M. Lonescu, G. Mandayam, and B. Raghothaman, “Transmit diversity in 3G CDMA systems,” IEEE Commun. Mag., Apr. 2002, pp.68-75.
[2] 3rd Generation Partnership Project, Technical Specification Group Radio Access Network; UE Radio Transmission and Reception (FDD), Release 5, Sep. 2002.
[3] E. N. Onggosanusi, A. Gatherer, A. G. Dabak, and S. Hosur, “Performance analysis of closed-loop transmit diversity in the presence of feedback delay,” IEEE Trans. Commun., Vol.49, No.9, Sep. 2001, pp.1618-1630.
[4] Q. Sun, D. C. Cox, “Estimation of continuous flat fading MIMO channels,” in WCNC’02, 2002, Vol. 1, pp.189-193.
[5] W. C. Jakes, Microwave mobile communications, New York: Wiley, 1974.
[6] J. K. Cavers, “An analysis of pilot symbol assisted modulation for Rayleigh fading channels,” IEEE Trans. Veh. Technol., Vol.40, Nov. 1991, pp.686-693.
[7] J. Choi, “Performance analysis for transmit antenna diversity with/without channel information,” IEEE Trans. Veh. Technol., Vol.51, No.1, Jan. 2002, pp.101-113.

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