Study on the Effects of Values of Magnification Factor, Main Reflector Diameter and Slant Factor of Feed Antenna, on the Optimum Size of Subreflector in Cassegrain Antenna System

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Abstract
In this paper design method of Cassegrain antenna system is reviewed. In this axisymmetrical antenna, subreflector and feed antenna are placed in front of the main reflector, so it is required in the design process to minimize the blockage effects. Three case studies are mentioned in this paper. Values of main reflector diameter, magnification factor and slant factor of feed antenna are separately changed in special ranges. In each case, optimum subreflector diameter is calculated and the ratio of subreflector to main reflector diameter is obtained and plotted versus these parameters to investigate their effects.
Keywords: *Cassegrain Antenna, Optimization, Conical Corrugated Horn Antenna, Magnification Factor, Blockage Efficiency.*

1. Introduction

Cassegrain antennas have vast applications in earth stations, TV relays, satellite and spacecraft communication antennas and etc. for different reasons. In these antennas, effective focal distance is larger than prime reflector antennas focal length, so the spurious cross polarization components in these antennas are less than prime reflector ones, results in higher cross polarization efficiency. Also positioning feed antenna near the main reflector apex in these antennas, permits easy adjustments and provides simpler support structure with respect to prime reflector antennas. Fig.1 illustrates a Cassegrain antenna used for communication with spacecrafts and is located in Canberra, Australia [1]. In 1984 a deep space dual band Cassegrain antenna is implemented in Japan with 64 (m) diameter, operating at S and X bands [2]. These antennas also have been utilized in imaging systems operating at 94 Ghz [3].
In Cassegran antennas, usually in order to reduce feed antenna cross polarization components, conical corrugated horn antenna is used as feed antenna. This kind of aperture antenna has equal E and H planes radiation patterns and is adequate for applications with circular polarization [4].

Fig. 1. Photo of a Cassegrain antenna used for spacecraft communications, located in Canberra, Australia [1].

The main disadvantage of Cassegrain antenna perhaps is the blockage effect caused by the feed antenna and subreflector which are located in front of the main reflector.
In analyzing this antenna theoretically, the method of physical optics is usually employed. To reduce the blockage effect, subreflector and feed antenna dimensions can be designed in such a way that subreflector diameter be equal to shadow diameter made by feed antenna on the main reflector. Gain of aperture antennas is obtained from

$$G = \frac{4\pi}{\lambda^2}\varepsilon A_p \qquad (1)$$

where A_P is physical area of aperture , λ is the free space wavelength , ε is the antenna efficiency and depends on field distribution on aperture plane, radiation field overflow from the edge of antenna , blockage effect and etc. The product of spillover and aperture efficiency is a function of field taper at the edge of antenna and becomes maximum when the tapering is about -11 dB [5]. Blockage efficiency is obtained from

$$\varepsilon_b = \left[1 - \frac{A_b}{\varepsilon_t A_p}\right]^2 \qquad (2)$$

Where A_b is the area of the blocked region against aperture plane and A_p is the physical area of aperture plane and ε_t is taper efficiency. So the blockage efficiency is related to the ratio of blocked area diameter to the main reflector antenna diameter.
In the second part of this paper, geometric structure of Cassegrain antenna system is explained and method in designing dimensions of feed antenna and subreflector is reviewed in order to minimize the blockage effect. Then in the next parts of the paper, the effects of choosing various system parameters such as main reflector diameter, magnification factor and slant factor of feed antenna, on the optimum blockage efficiency are investigated.

2. Geometric structure of Cassegrain antenna

Geometric structure of antenna is depicted in Fig. 2. Assume that the origin of the coordinate system be located on the main reflector apex. This structure is made from a parabolic and a hyperbolic reflector. Focal point of the parabolic main reflector is indicated by F_1 and focal points of hyperbolic subreflector are named as F_1 and F_2. Distance between F_1 and F_2 is assumed to be equal to 2C. The phase center of feed antenna is placed on F_2 and waves emerging from this point, after reflection from subreflector travel spherically with F_1 origin. In this figure, the equivalent parabolic prime reflector is shown as the right dashed curve. Its focal distance F_{equal} is the distance between F_2 to the equivalent parabolic apex ,'A'. As is seen from the figure, effective focal length is increased with respect to F_1 and the ratio of F_{equal}/F_1 is defined as magnification factor, named as 'M'. Main reflector diameter is named as D_m and that's of subreflector is called D_S. Feed antenna diameter is indicated by D_f and shadow diameter of the feed on main reflector is shown by D_{fsh}. The amount of phase center displacement of the feed with respect to feed aperture plane is illustrated by d_{phc}.

Observing the figure, angles α , β , γ are respectively angles that passing rays from the edges of subreflector, main reflector and feed antenna, make with z axes. Considering Fig. 2, to optimize blockage efficiency, it is required that D_S becomes equal to D_{fsh}.

In order to analyze the structure with physical optics method, information on parabolic and hyperbolic curves are needed. Hyperbolic and parabolic cross sections are special cases of conic sections. A general conic section is shown in Fig. 3. These curves are generally specified by 2 geometric parameters called here as 'A' and 'C'. they are locus of points that the ratio of their distance to a fixed point ,called focal point, to their distance from a fixed line, called directrix , be equal to a constant 'e' , named as eccentricity. For parabola e=1 and for hyperbola e>1. Keeping these points in mind, assuming that the origin of the polar coordinate system and focal point of the curve, in Fig. 3 be located on point 'O', since in a parabola e=1, parabolic equation in polar coordinate may be written as

$$\frac{ON}{NM} = e = \frac{C-A}{A} = \frac{r}{C - r\cos(\theta)} = 1 \qquad (3)$$

so,

$$C = 2A \qquad (4)$$

and

$$C = r(1+\cos(\theta)) \Rightarrow r = \frac{2A}{2\cos^2\left(\frac{\theta}{2}\right)} \qquad (5)$$

if we call the focal distance , the distance from focal point to parabolic curve apex, as 'F' , considering Fig.3 in a parabola F=A, so from (5) we have,

$$r = \frac{F}{\cos^2\left(\frac{\theta}{2}\right)} \qquad (6)$$

The dashed line shown in Fig. 3, is the cross section of a plane perpendicular to the x axes, passing from focal point, called focal plane, with the curve plane. All rays that emits from focal point travel the same distance to the focal plane, so they are all in-phase, since in Fig. 3, we have:

$$ON + NP = r + r\cos(\theta) = 2r\cos^2\left(\frac{\theta}{2}\right) = 2F \qquad (7)$$

Similarly in a hyperbola with e>1, the polar equation may be obtained similar to (3) as follows,

$$r = \frac{Ce}{1+e\cos(\theta)} \qquad (8)$$

using (6) , (8) , in Fig. 2, respectively on parabolic and hyperbolic curves, D_S and D_{fsh} can be calculated versus β and γ. Then by equating , D_S and D_{fsh}, minimum blockage condition due to the simultaneous presence of feed antenna and subreflector in front of main reflector is obtained. In continue, the process of obtaining such an equation is explained.
To calculate D_S versus β, we have from (8) and Fig. 2,

$$D_S = 2r\sin(\beta) = \frac{2Ce\sin(\beta)}{1+e\cos(\beta)} \qquad (9)$$

to obtain D_{fsh} from γ, we have from Fig. 2 and (6),

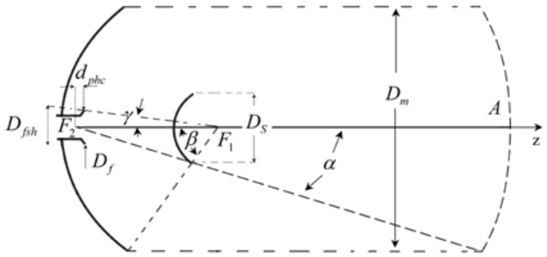

Fig.2 . Geometric structure of Cassegrain antenna.

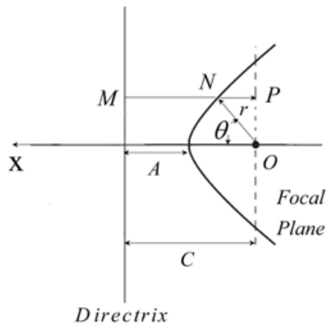

Fig. 3. Geometric structure of a conic section.

$$D_{fsh} = 2r\sin(\gamma) = 2\frac{F_1}{\cos^2\left(\frac{\gamma}{2}\right)}\sin(\gamma) = 4F_1\tan\left(\frac{\gamma}{2}\right) \qquad (10)$$

in Fig.2 , to get γ, knowing feed diameter D_f and the distance between F_1 and the feed aperture plane we have,

$$\gamma = \tan^{-1}\left(\frac{D_f/2}{2C - d_{phc}}\right) \qquad (11)$$

by equating $D_S = D_{fsh}$, using (9), (10) , (11) we have,

$$\frac{2Ce\sin(\beta)}{1+e\cos(\beta)} = 4F_1\tan\left(\frac{\tan^{-1}\left(\frac{D_f/2}{2C-d_{phc}}\right)}{2}\right) \qquad (12)$$

Equ. (12) contains several parameters like , C , e , D_f , d_{phc} , β, F_1 and is a complicated equation, requires numerical methods to be solved. In this paper we have chosen conical corrugated horn antenna as feed antenna, considering mechanical parameters like heaviness and being commercially fabricated and fitness of the structure dimensions. Also from the electrical point of view, this feed has less cross polarization components. In designing this kind of feed dimensions, considering the edge field tapering amount and beamwidth and slant radius, feed aperture diameter and phase center displacement with respect to the feed aperture plane can be computed using tables and curves provided in [6] versus different slant parameter of the feed called ‘S’ factor. This factor is defined as

$$S = \frac{a^2}{2\lambda R} \qquad (13)$$

where ‘a’ is the feed aperture radius, ‘R’ is slant radius of horn and λ is free space wavelength. $Y(a,\alpha)$ function may be defined as

$$Y(a,\alpha) = \frac{2\pi a}{\lambda}\sin(\alpha) \qquad (14)$$

where α , as is shown in Fig. 2, is the angle that passing rays from the subreflector edge make with the z axes. In this angle the power pattern intensity is assumed to be -11dB less than that of the aperture plane center. For tapering equal to -11 dB, and $0.2 \le S \le 0.6$, $Y(a,\alpha)$ is summarized in Table 1. A plot of $Y(a,\alpha)$ versus S is sketched in Fig.4.

Feed antenna phase center displacement with respect to the feed aperture, as indicated in Fig. 2 by d_{phc} , is also a function of horn slant radius and S factor. The ratio of d_{phc}/R versus different S factor values in the range $0.2 \le S \le 0.6$ can be obtained from Table 2 [6]. Diagram of d_{phc}/R versus S is depicted in Fig. 5.

Table 1- Variations of $Y(a,\theta)$, with respect to S factor, for tapering equal to -11 dB.

S	0.2	0.3	0.4	0.5	0.6
$Y(a,\alpha)$	3.9	4.1	4.5	5.2	5.9

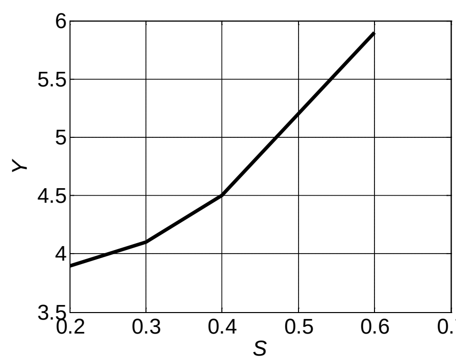

Fig. 4. Plot of $Y(a,\alpha)$ versus S factor as defined in (14).

Table 2. Variations of d_{phc}/R versus S factor for conical corrugated horn antenna.

S	0.2	0.3	0.4	0.5	0.6
d_{phc}/R	0.124	0.275	0.464	0.643	0.753

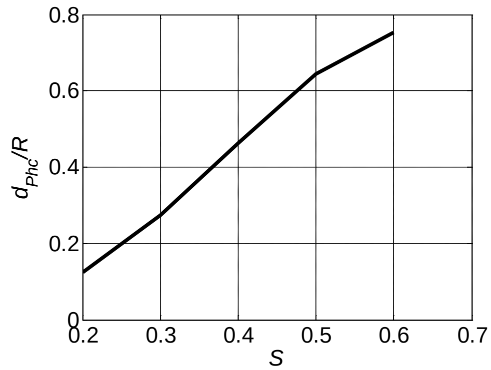

Fig. 5. Plot of d_{phc}/R against S factor for conical corrugated horn antenna according to Table 2.

In continue, we use the above information to solve (12), and investigate the effects of choosing different values for parameters of the structure, on the $\frac{D_S}{D_m}$ ratio and blockage efficiency of antenna. Three case studies are illustrated next.

3- Case study 1: investigating the effects of different values of main reflector diameter on the optimum ratio of $\frac{D_S}{D_m}$

In designing the antenna, the main reflector diameter is usually calculated considering the budget link, antenna gain, the required amount of received power and receiver sensitivity. In this section, (12) is solved for three different values of $D_m = 5$, 7.5, 10 (m). The optimum ratio of $\frac{D_S}{D_m}$ is obtained for each value. In solving (12), it is assumed that the antenna operation frequency is f=10 Ghz, $F_1/D_m = 0.5$, $S = 0.2$, antenna magnification factor is arbitrary chosen as M=5 and the amount of tapering is considered equal to -11 dB. Neglecting the thickness of feed antenna cover, feed diameter illustrated by D_f in Fig. 2 would be equal to 2a, where 'a' is the feed aperture radius defined in (13) , (14). To solve (12), at first we have similar to (10),

$$D_m = 4F_1 \tan\left(\frac{\beta}{2}\right) \Rightarrow \beta = 2\tan^{-1}\left(\frac{D_m}{4F_1}\right) = 2\tan^{-1}(0.5) = 53.1^\circ \quad (15)$$

from Table 1, for S=0.2, we have Y=3.9. To get 'a' from (14), α must be obtained first. Taking into account magnification factor, M, from Fig. 2 and similar to (10) we get,

$$D_m = 4F_{equal} \tan\left(\frac{\alpha}{2}\right) \Rightarrow \alpha = 2\tan^{-1}\left(\frac{D_m}{4F_{equal}}\right) = 2\tan^{-1}\left(\frac{D_m}{4MF_1}\right) = 11.4^\circ \quad (16)$$

from (14) , (16) ,

$$3.9 = \frac{2\pi a}{0.03}\sin(11.4^\circ) \Rightarrow a = 9.4\ (cm) \Rightarrow D_f = 2a = 0.19\ (m) \quad (17)$$

and from (17) , (13) ,

$$S = \frac{a^2}{2\lambda R} \Rightarrow R = \frac{a^2}{2\lambda S} = \frac{(0.094)^2}{2\times 0.03\times 0.2} = 0.74\ (m) \quad (18)$$

from Table 2, for S=0.2 ,

$$d_{phc}/R = 0.124 \Rightarrow d_{phc} = 0.124\times 0.74 = 9.2\ (cm) \quad (19)$$

we have the relation among eccentricity 'e' and magnification factor 'M' as ,

$$M = \frac{e+1}{e-1} \Rightarrow e = \frac{M+1}{M-1} \Rightarrow e = \frac{6}{4} = 1.5 \quad (20)$$

now by solving (12) for various main reflector diameter D_m, optimum size of the subreflector diameter D_s would be obtained. One may solve this equation by MATLAB software [7]. Changing D_m, only alters F_1 in this equation and after solving (12) versus variable 'C', D_s would be gotten from (9). The calculated optimum values of D_s versus different D_m amounts are provided in Table 3 in summary, and are plotted in Fig. 3, for D_m in the range $5 \le D_m \le 10$.

Table 3- Variations of D_S/D_m versus main reflector diameters D_m.

D_m	5 (m)	7.5 (m)	10 (m)
D_S	0.57 (m)	0.7 (m)	0.8 (m)
D_S/D_m	0.11	0.093	0.08

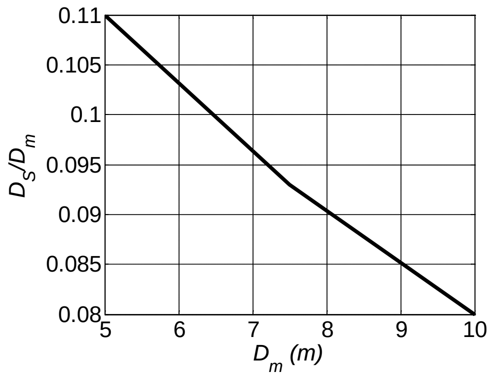

Fig.6. Depicts the optimum calculated values of D_S / D_m against main reflector diameter variations D_m . Design frequency is 10 GHz, magnification factor is M=5, S is considered as 0.2 and field tapering is equal to -11 dB.

As it is observed from Fig. 6 and Table 3, increasing the main reflector antenna diameter, decreases the optimum ratio of D_S / D_m and thus increases the blockage efficiency.

4- Case study 2: to study the effects of magnification factor , 'M', values on the optimum ratio of D_S / D_m

As stated before, choosing M has direct impact on cross polarization field component values and thus cross polarization efficiency. Increasing M increases the cross polarization efficiency. In this section we investigate the effect of M on blockage efficiency and the ratio of D_S / D_m for M=2,3,4 ,5 values. In this case, similar to the earlier case study, design frequency is taken at 10 GHz, tapering is equal to -11 dB, S=0.2, but the main reflector diameter is considered fix equal to 10 (m). β angle can be gotten from (15) , but α angle varies with M as following,

$$\alpha = 2\tan^{-1}\left(\frac{D_m}{4MF_1}\right) = 2\tan^{-1}\left(\frac{1}{2M}\right) \quad (21)$$

from (14), 'a' is calculated and would be a function of α angle as,

$$3.9 = \frac{2\pi a}{0.03}\sin(\alpha) \Rightarrow a = \frac{3.9\times 0.03}{2\pi\sin(\alpha)}, \quad D_f = 2a \quad (22)$$

slant radius of horn obtains from,

$$S = \frac{a^2}{2\lambda R} \Rightarrow R = \frac{a^2}{2\lambda S} = \frac{a^2}{2\times 0.03\times 0.2} \quad (23)$$

and d_{phc} calculates as

$$d_{phc} / R = 0.124 \Rightarrow d_{phc} = 0.124\times R \quad (24)$$

Eccentricity, 'e', and magnification factor 'M' are related by (20). After running computer program for different M values, the optimum amounts of D_S / D_m are calculated and summarized in Table 4. These values are also plotted in Fig. 7 versus M to further illustrate the variations.

Table 4- Variations of feed antenna aperture diameter,'a', and the ratio of D_S / D_m for various magnification factors ,'M'.

M	a (m)	$D_S\ (m)$	D_S / D_m
2	0.04	0.59	0.059
3	0.057	0.66	0.066
4	0.076	0.73	0.073
5	0.094	0.8	0.08

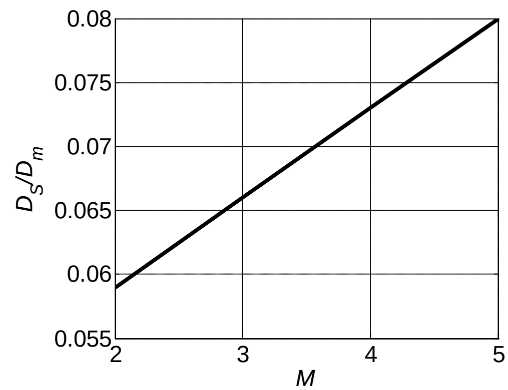

Fig. 7. The optimum calculated values of D_S / D_m against magnification factor ' M' . Design frequency is 10 GHz, main reflector diameter is $D_m = 10\ (m)$, S is considered 0.2 and field tapering is equal to -11 dB.

As illustrated by Table 4 and Fig.7, reducing M will result in simultaneous reduction of feed antenna aperture radius and optimum D_S / D_m ratio.

5- Case study 3: inspecting the impacts of different S factor values on the optimum ratio of D_S / D_m .

S factor is defined in (13) and depends on physical parameters of feed like its aperture diameter and the slant radius of horn. In selecting proper feed, its mechanical specification, dimensions, weight and also its cost must be considered to design proper support structure for holding the feed in place. In this section we solve (12) for various S factors in the range S=0.2, 0.4, 0.6. Similar to earlier cases, optimum ratio of D_S / D_m is calculated and plotted against S factor. We may assume almost the same conditions with cases 1 , 2 , for better comparison of the results. So design frequency is 10 GHz, magnification factor is considered fixed as M=5, main reflector diameter is assumed constant equal to $D_m = 10\ (m)$ and tapering is -11 dB. On these conditions, angles α , β would be gotten from (15) and (16) but feed aperture diameter is a function of Y, where Y itself is a function of S as follows. Their functionalities investigated already in the second section of the paper.

$$Y = \frac{2\pi a}{0.03}\sin\left(11.4^\circ\right) \Rightarrow a = \frac{Y \times 0.03}{2\pi \sin\left(11.4^\circ\right)} \qquad (25)$$

The horn slant radius, also is related to 'a' and S by (13). Knowing 'R', d_{phc} may be determined from d_{phc} / R by getting information of Table 2 and Fig.5. In this case, the eccentricity, from (20) is e=1.5.

Now all parameters in (12) are specified except for 'C', and by numerical solving of (12), 'C' and D_S are obtained. The optimum results of D_S / D_m versus different S parameters are depicted in Table 5 and Fig. 8.

Table 5- Variations of feed antenna aperture diameter,'a', and the ratio of D_S / D_m for various S factors.

S	a (m)	$D_S\ (m)$	D_S / D_m
0.2	0.094	0.8	0.08
0.4	0.11	0.9	0.09
0.6	0.142	1.1	0.11

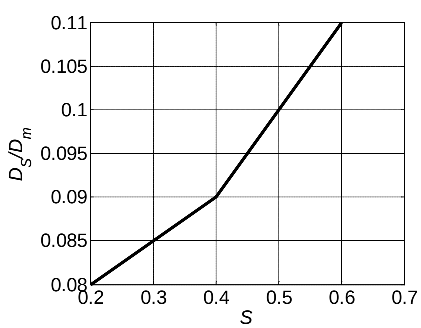

Fig. 8. The optimum calculated values of D_S / D_m versus S factor . Design frequency is 10 GHz, main reflector diameter is $D_m = 10\ (m)$, Magnification factor is M=5 and field tapering is equal to -11 dB.

As are illustrated by Fig. 8 and Table 5, increasing S, results in boosting feed aperture dimension and the ratio of D_S / D_m , so the blockage efficiency decreases.

6- Conclusion

In this paper the design process of feed antenna and subreflector dimensions in Cassegrain antenna system is reviewed in order to optimize the blockage efficiency. Three different case studies are considered to study the effects of changing different system parameters on the blockage efficiency. In the first case study, main reflector antenna diameter is taken as variable and variations of the ratio of subreflector diameter to main reflector diameter are calculated and plotted versus that. In the second case study, parabolic reflector diameter is assumed fixed, but the magnification factor of antenna is taken as variable and its effect on the antenna blockage efficiency is explored. In the third case study, the main reflector diameter and magnification factor are considered constant and the effects of changing S factor of the feed is investigated.

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