IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 4 Issue 2, February 2017
ISSN (Online) 2348 – 7968 | Impact Factor (2016) – 5.264
www.ijiset.com

Design and Analysis of Optimal Control System Performance

Kamen Yanev[1]
[1] Department of Electrical Engineering, University of Botswana
Gaborone, Botswana

Abstract
The contribution of this research is in suggesting a procedure that establishes the optimal compensator corner frequencies and in this way bringing a control system to its best mode of operation. The design of a set-up for compensating a feedback control system is optimized to obtain the best performance that can be offered by the compensating device. The paper also proposes an analysis by the method of the Advanced D-partitioning, to examine extensively the effects of the parameters variation on the system's stability before and after the applied optimal compensation.
Keywords: **Compensation, Corner frequencies, Crossover frequencies, Optimization, Advanced D-partitioning.**

1. Introduction

Compensation networks are considered as the simplest types of controllers that can be applied to improve the system's degree of stability and performance. The design of phase-lag or phase-lead compensation networks can be optimized with intend to obtain their best operation. In the design process, it is essential to choose appropriately the compensator's corner frequencies ω_{c1} and ω_{c2} [1].

The existing design practice is to establish the limitations of the compensator's corner frequencies and to choose a value within these limits. These limitations are enforced mainly by the corner frequencies of the original plant open-loop transfer function, as well as some practical considerations for the physical realization of the compensator. By using this practice, the best compensation performance is difficult to predict.

The main contribution of this research is suggesting an optimization of the compensator's corner frequencies for obtaining the best system performance. The ITAE performance criterion is considered as a decisive factor, for the optimization strategy of compensator design.

Another contribution of this paper is analyzing the system with the aid of the Advanced D-partitioning method before and after the compensation. This will illustrate the effects of the parameters variation on system's performance. It defines graphically regions of stability in the space of the system's parameters displaying the effects of the compensation.

The amplitude and phase characteristics of linear systems are uniquely related according to the Bode's Theorems [2]. A specified slope of the amplitude-frequency curve $L(\omega)$ over a certain frequency interval, specifies and determines the corresponding phase-frequency characteristic $\varphi(\omega)$ over that same frequency interval.

Furthermore, the Bode's theorems state that the slope at the crossover frequency ω_c, where the $L(\omega)$ crosses the 0db line, is weighted more heavily towards determining system stability than the slopes more remote from this frequency. The crossover frequency ω_c is one of the two points that is checked to determine the degree of stability when using Bode diagrams. Specifically, the phase shift is measured at ω_c in order to determine the phase margin. A feedback control system whose slope at crossover ω_c is –20db/decade and whose other slope sections are relatively far away from ω_c implies ideally a phase shift of –90° in the vicinity of crossover ω_c and a corresponding phase margin of about 90°. That certainly implies a best case of a stable system. A slope at ω_c is –40db/decade, implies a phase shift around –180° and a corresponding phase margin of about 0°. This value of phase margin is related to a marginally stable system [2], [3].

In this research, an innovative procedure is suggested that can optimize the choice of the series phase-lead compensator corner frequencies. Identical procedure can be applied for the case of phase-lag compensation.

2. Performance of the Original System

A case study of a unity feedback control system is suggested. The open-loop transfer function of the system is of Type 1 and is presented as:

$$G_O(s) = \frac{180}{s(1+0{,}02s)(1+0{,}005s)} = \frac{180}{0.0001s^3 + 0.025s^2 + s} \qquad (1)$$

In accordance to the Bode stability criterion, the phase margin of the closed-loop system can be determined by plotting the Bode diagrams of the open-loop system $G_o(s)$, applying the following code:

```
>> Go=tf([0 180],[0.0001 0.025 1 0])
>> margin(Go)
```

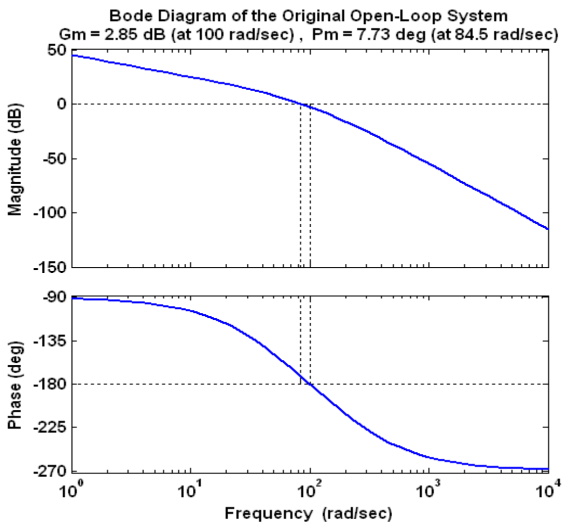

Fig. 1 Bode diagrams of the original open-loop system

As seen from Figure 1, the phase margin of the closed-loop system is Pm = 7.73°, implying a relatively low degree of stability. The crossover frequency is ω_c = 84.5 rad/sec, which is higher than the corner frequency ω_{o1} = 50 rad/sec and therefore at a slope of –40db/decade. Although the system is stable, its performance is quite poor. The evaluation of the closed loop system is achieved by the code:

```
>> Gofb=feedback(Go,1)
>> damp(Gofb)
      Eigenvalue              Damping      Freq. (rad/s)
 -5.19e+000 + 8.65e+001i     5.99e-002     8.67e+001
 -5.19e+000 - 8.65e+001i     5.99e-002     8.67e+001
 -2.40e+002                  1.00e+000     2.40e+002
```

From the evaluation, it is seen that the relative damping ratio of the closed loop system is ζ = 0.0599 and natural frequency of oscillation is ω_n = 86.7 rad/sec.

Control systems usually require performance criteria that consider simultaneously the response error $e(t)$ and the time t at which it occurs. A very useful criterion is the Integral of Time multiplied by the Absolute value of Error (ITAE). If a system is higher than the second order, a pair of dominant poles can represent the system dynamics. Then, ζ and ω_n can still be used to indicate the location of these poles and the damping ratio is referred as the relative damping ratio of the system. Meeting the ITAE criterion, the following objectives should be targeted in the optimization design [4], [5]:

$$\zeta = 0.707 \quad (2)$$

$$PMO \leq 4\% \quad (3)$$

$$t_s / t_m \leq 2.5 \quad (4)$$

The poor step response of the discussed closed-loop original system is shown in Figure 2 and is obtained by the following code:

```
>> Gofb = feedback (Go,1)
>> Step (Gofb)
```

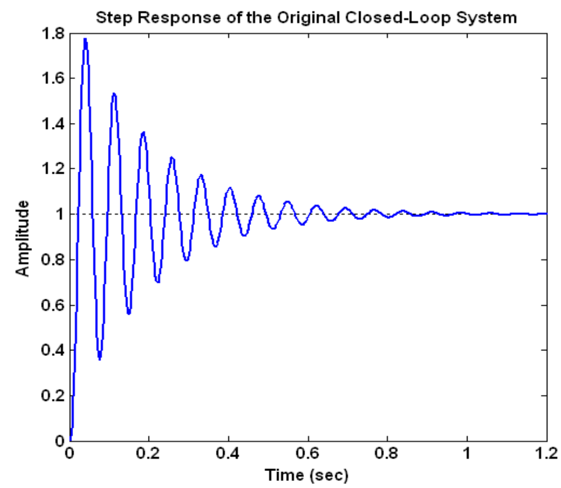

Fig. 2 Step response of the original closed-loop system

Considering the obtained results, the settling time $ts_{(1\%)}$, the time to maximum overshoot t_m and percent maximum overshoot PMO [4], [5] are obtained as follows:

$$t_{s(1\%)} = \frac{4.6}{\xi\omega_n} = \frac{4.6}{0.0599 \times 86.7} = 0.886\,\text{sec} \quad (5)$$

$$t_m = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{86.7\sqrt{1-0.0599^2}} = 0.0363\,\text{sec} \quad (6)$$

$$PMO = 100e^{-(\pi\xi\sqrt{1-\xi^2})} = 59.52\% \quad (7)$$

$$\frac{t_s}{t_m} = 24.41 \quad (8)$$

In comparison with the ITAE objectives, demonstrated with equations (3) and (4), the PMO = 59.52%, as well as the ratio t_s / t_m = 24.41 are considerably higher, which is another proof of the poor system performance.

3. Analysis of the Original System with the Aid of the Advanced D-Partitioning

In addition to the unsatisfactory performance, the system may experience instability, due to uncertain parameters. It is considered that its gain and one of its time-constants may be variable as a result of external disturbances.

In a number of previously published research [6], [7], [8], [9], [10], the author of this paper, further expanded the initial ideas of Neimark to a method of **Advanced D-Partitioning,** developing a generalized stability analysis tool. By implementing an interactive MATLAB procedure methodology, the Advanced D-Partitioning created by the author, is based on innovative transparent graphical display of regions of stability and instability in the space of system's variable parameters. The basic principle, suggested by the author, is introducing the system's characteristic equation in a format that exposes the variable parameter.

If the variable parameter is presented as a complex number, the D-Partitioning regions can be obtained graphically in the complex plane of this parameter, by varying the frequency within the range $-\infty \le \omega \le +\infty$. The D-Partitioning curve in terms of one variable parameter can be plotted in the complex plane within the frequency range $-\infty \le \omega \le +\infty$, facilitated by MATLAB the "nyquist" m-code.

To avoid any misinterpretation of the D-Partitioning procedure, the "nyquist" m-code is modified into a "dpartition" m-code with the aid of the MATLAB Editor and a proper formatting. The "dpartition" m-code can plot the curve of a specific system parameter in terms of the frequency variation from $-\infty$ to $+\infty$. In order to benefit from the Advanced D-Partitioning analysis suggested by the author, the wider engineering community can still use the "nyquist" m-code for the purpose of plotting the D-Partitioning curve.

3.1 Case of Variable System Gain *K*

Initially, the variation of the system's gain K is explored with the aid of the method of the method of the Advanced D-partitioning. Taking into account the system's forward transfer function as:

$$G_{ok}(s) = \frac{K}{0.0001s^3 + 0.025s^2 + s} \tag{9}$$

The characteristic equation of the closed-loop system is determined by:

$$\begin{aligned} A_k(s) &= 0.0001s^3 + 0.025s^2 + s + K = \\ &= P(s) + KQ(s) = 0 \end{aligned} \tag{10}$$

The D-partitioning curve of the variable parameter K is obtained by substituting $s = j\omega$ in its Laplace format $K(s) = -P(s)/Q(s)$ and varying the frequency within the range $-\infty \le \omega \le +\infty$ following the code [8], [9], [10]:

```
>> Gok = tf([1],[0.0001 0.025 1 0])
>> [den,num]=tfdata(-Gok,'v')
>> K=tf(num,den)
>> nyquist(K)
```

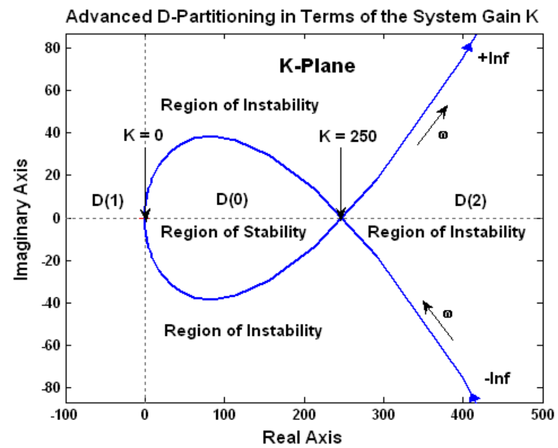

Fig. 3 Advanced D-partitioning in terms of the gain *K* (original system)

As seen from Figure 3, the D-partitioning determines three regions on the K-plane: $D(0)$, $D(1)$ and $D(2)$. Only $D(0)$ is the region of stability, being the one, always on the left-hand side of the curve for a frequency variation from $-\infty$ to $+\infty$ [7], [8], [9], [10]. The system is stable within the gain range $0 \le K \le 250$.

3.2 Case of Variable System Time-constant *T*

Assuming a system gain of $K = 240$, close to the marginal case, one of the system's time-constants, T_1, is considered as variable. The forward transfer function of the system is presented as follows:

$$G_{OT}(s) = \frac{240}{s(1 + T_1 s)(1 + 0{,}005s)} \tag{11}$$

Then the characteristic equation of the closed-loop system is determined as:

$$A_T(s) = s(1 + T_1 s)(1 + 0{,}005s) + 240 = 0 \tag{12}$$

By implementing the Advanced D-partitioning method, the regions of stability of the system can be determined in terms of the variable time-constant T_1. From equation (12), the variable parameter is presented as follows:

$$T_1 = -\frac{0{,}005s^2 + s + 240}{0{,}005s^3 + s^2} \tag{13}$$

The D-partitioning curve in terms of the variable time-constant T_1 is obtained as follows [7], [8]:

```
>> T1 = tf([-0.005 -1 -240],[0.005 1 0 0])
>> nyquist(T1)
```

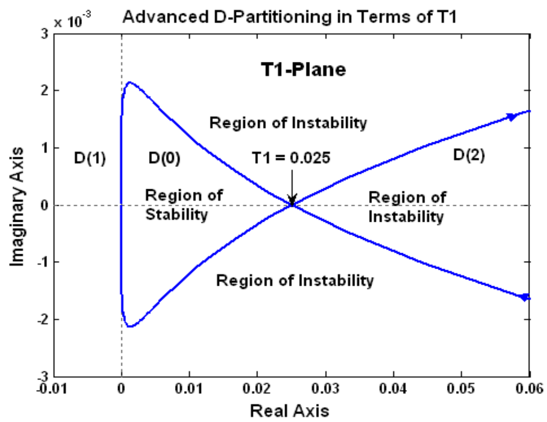

Fig. 4 Advanced D-partitioning in terms of the time-constant (original system)

As seen from Figure 4, the D-partitioning determines three regions on the T1-Plane: *D(0), D(1)* and *D(2)*. Only *D(0)* is the region of stability, being always on the left-hand side of the D-partitioning curve for the frequency variation within the range $-\infty \le \omega \le +\infty$ [8]. This rule implies that for the considered case, where $K = 240$, the system will be stable if the parameter T_1 is within the limited range 0 sec $\le T_1 \le$ 0.025 sec. A simultaneous variation of T_1 and K results in a strong interaction between these two parameters [7], [8], [11].

4. Design of Optimal Series Compensation

The system insufficient performance is due mainly to the large system gain. If for proper operation larger gain is still required, the system performance can be improved by applying series compensation. The system becomes stable and its performance is improved if a compensator network is connected in series [12]with the open-loop system. The transfer function of the phase-lead compensator is:

$$G(s) = \frac{1+\alpha Ts}{\alpha + \alpha Ts}, \text{ where } \alpha > 1, \ \frac{1}{\alpha T} < \frac{1}{T} \tag{14}$$

Pole and a zero values are determined to satisfy the design criteria of the closed loop system. The range of frequencies where the compensator's corner frequencies $\omega_{C1} = 1/\alpha T$ and $\omega_{C2} = 1/T$ can be placed is very limited. The following conditions should be satisfied [5], [9]:

$$\omega_{01} < \omega_{C1} < \omega_{oc} \text{ and } \omega_{02} < \omega_{C2} \tag{15}$$

Therefore, for the considered control system:

$$50 rad/\sec < \omega_1 < 85 rad/\sec \tag{17}$$

For the physical realization of the compensation network it is accepted that $\alpha \le 10$. The lowest time-constant T in the range is obtained by:

$$T = 1/\alpha\omega_1 = \frac{1}{10 \times 85} = 0.0012[s] \tag{18}$$

The highest value of the time-constant T in the range is obtained as:

$$T = 1/\alpha\omega_1 = \frac{1}{10 \times 50} = 0.002[s] \tag{19}$$

Assuming $\alpha = 7$, an array code to determine the optimal compensator time-constant [8], [9], [10] is applied. The results are shown in Table I.

```
>> a =7
>> T=[0.0012:0.0001:0.002]
>> for n=1:length(T)
Gc_array(:,:,n)=tf([a*T(n) 1],[a*T(n) a])
End
>> Gsc = (Go*Gc_array)
>> [Gm,Pm]=margin(Gsc)
```

Table 1: Array results specifying a time-constant for optimal compensation performance

n	T[s]	Gm[dB]	Pm[º]
1	0.0012	55.16	67.64
2	0.0013	51.68	68.36
3	0.0014	48.33	69.07
4	**0.0015**	**45.17**	**69.78**
5	0.0016	42.25	70.48
6	0.0017	39.57	71.17
7	0.0018	37.13	71.85
8	0.0019	34.91	72.53
9	0.0020	32.89	73.20

From the results seen in Table 1, it is obvious that after the compensation, the phase margin has great improvement compared with the original phase margin of Pm = 7.73°.

The fourth case, at a time-constant T = 0.0015[s] has a phase margin Pm = 69.78° that is the closest match to a

IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 4 Issue 2, February 2017
ISSN (Online) 2348 – 7968 | Impact Factor (2016) – 5.264
www.ijiset.com

damping ratio $\zeta \approx 0.707$ of all the values in the range. It will be used for the design of the optimal phase-lead compensation. The transfer function of an optimal compensator and of the open-loop system can be determined by:

```
>> Gc_4=Gc_array(:,:,4)
 Transfer function:
0.0105 s + 1
------------
0.0105 s + 7
>> Gsc_4= (Go*Gc_4)
 Transfer function:
1.89 s + 180
-----------------------------------------------
1.05e-006 s^4 + 0.0009625 s^3 + 0.1855 s^2 + 7 s
```

Applying the optimal time constant of T = 0.0015[s], an array to verify the optimal compensator constant α is created [8], [9], [10]. The results are presented in Table 2.

```
>> a=[4:1:12]
>> for n=1:length(a)
Gc_array(:,:,n)=tf([a*T(n) 1],[a*T(n) a])
end
>> Gsc = (Go*Gc_array)
>> [Gm,Pm]=margin(Gsc)
```

Table 2: Array results specifying the constant α for optimal compensation performance

n	α	Gm[dB]	Pm[°]
1	4	55.1614	67.6423
2	5	51.6778	68.3600
3	6	48.3291	69.0725
4	**7**	**45.1712**	**69.7783**
5	8	42.2488	70.4773
6	9	39.5709	71.1693
7	10	37.1295	71.8541
8	11	34.9086	72.5316
9	12	32.8892	73.2015

The results from Table 2 confirm that the optimal value for the compensator constant is α = 7.

The transfer function of the close-loop compensated system and its transient response are determined as:

```
>> Wfb_4=feedback(Gsc_4,1)
Transfer function:
 1.89 s + 180
---------------------------------------------------------
1.05e-006 s^4 + 0.0009625 s^3 + 0.1855 s^2 + 8.89 s + 180
>> step(Wfb_4, Gofb)
```

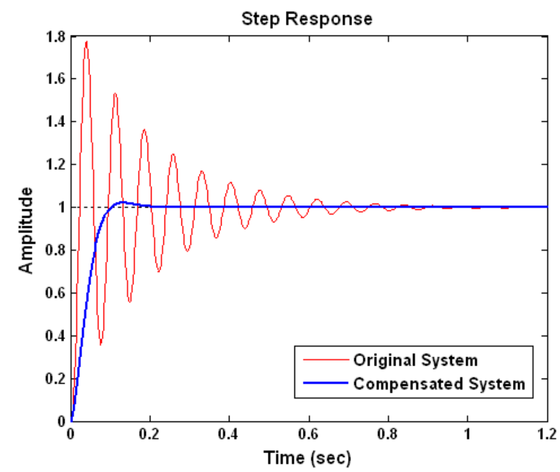

Fig. 5 Comparison between the step responses of the compensated and the original system

The result is also proved by applying the Bode stability criterion as follows:

```
>> margin(Gsc_4)
```

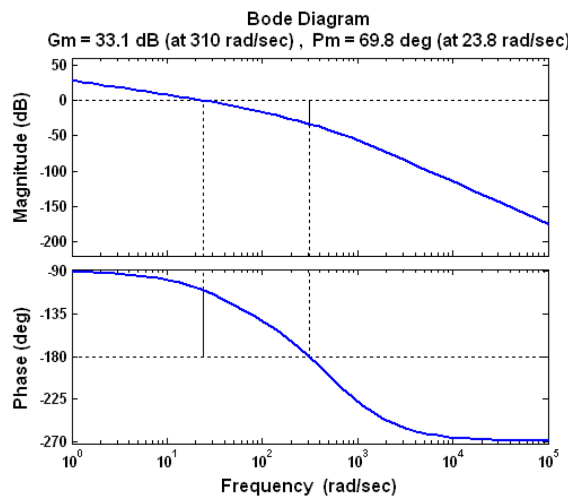

Fig. 6 Bode diagrams of the compensated open-loop system

The evaluation of the closed loop compensated system is performed as follows:

```
damp(Wfb_4)
Eigenvalue                    Damping     Freq. (rad/s)
 -2.91e+001 + 2.28e+001i      7.17e-001   3.70e+001
 -2.91e+001 - 2.28e+001i      7.17e-001   3.70e+001
 -1.87e+002                   1.00e+000   1.87e+002
 -6.72e+002                   1.00e+000   6.72e+002
```

The pair of dominant poles that represents the dynamics of the compensated system enforces a relative damping ratio of $\zeta = 0.717$.

The ratio of the settling time $t_{S(1\%)}$, time to maximum time to overshoot t_m and percent maximum overshoot PMO of the compensated system are obtained as follows [11], [12]:

$$t_{S(1\%)} = \frac{4.6}{\xi\omega_n} = \frac{4.6}{0.717 \times 37} = 0.173\text{sec} \qquad (21)$$

$$t_m = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{37\sqrt{1-0.717^2}} = 0.121\text{sec} \qquad (22)$$

$$PMO = 100e^{-(\pi\xi\sqrt{1-\xi^2})} = 2.06\% \qquad (23)$$

$$\frac{t_s}{t_m} = 1.42 \qquad (24)$$

The achieved results from the optimal compensation and the objectives are compared in Table 3.

Table 3: Performance results before and after the compensation

Properties	Objectives	Before Compensation	After Compensation
ζ	0.707	0.0599	0.717
t_s / t_m	≤2.5	24.41	1.42
PMO	≤4%	59.52%	2.06%

It is obvious that after applying compensation, the performance of the system is very close or better than the required performance objectives.

5. Design Advanced D-Partitioning Analysis of the Compensated system

Considering uncertain gain K, D-partitioning in terms of K of the compensated system is explored. The characteristic equation of the compensated closed-loop system is determined as:

$$\begin{aligned} A_{ck}(s) &= 0.000000105s^4 + 0.0009625s^3 + \\ &+ 0.1855s^2 + 8.89\,s + K = \\ &= P(s) + KQ(s) = 0 \end{aligned} \qquad (25)$$

>> K=tf([-0.000000105 -0.0009625 -0.1855 -8.89],[0 1])

>> nyquist(K)

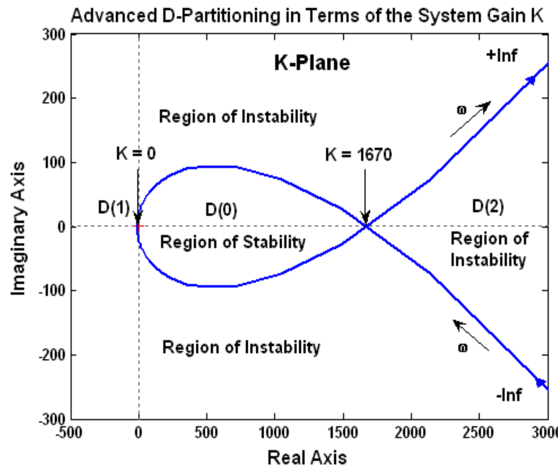

Fig. 7 Advanced D-partitioning in terms of the gain K (compensated system)

As seen from Figure 7, the region of stability D(0). It is on the left-hand side of the D-partitioning curve for a frequency variation from $-\infty$ to $+\infty$ and determines much larger gain range $0 \le K \le 1670$ at which the system remains stable.

Further, the uncertainty of the system's time-constants $T1$ is considered. Assuming a system gain $K = 240$, T_1 is determined from the characteristic equation of the compensated system. A procedure similar to the analysis of the original system is followed. As a result, the D-partitioning curve in terms of the variable time-constant T_1 is obtained with the procedure:

>> T1 = tf([-0.00053 -0.0455 -9.52 -240],[0.000053 0.046 7 0 0])

>> nyquist(T1)

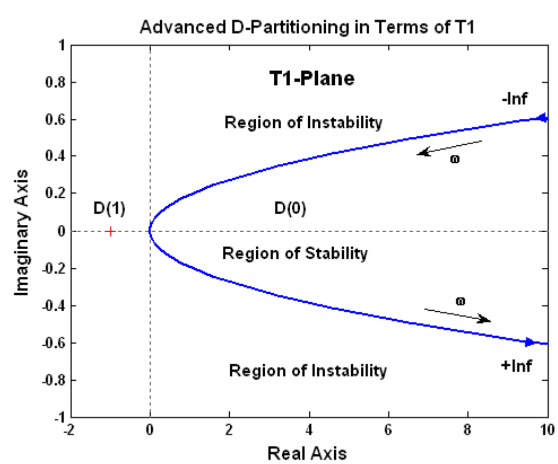

Fig. 8 Advanced D-partitioning in terms of the variable time-constant related to the compensated system

As seen from Figure 8, D(0) is the region of stability, being on the left-hand side of the D-partitioning curve for the frequency variation within $-\infty \leq \omega \leq +\infty$. This implies that if the parameter T_1 is within the range of $0 \leq T_1 \leq +\infty$ the system will be always stable. However this result is correct for the system gain of K = 240. Due to the interaction between T_1 and K [8],]12], larger system gain will again limit the range of T_1 at which the system will be stable.

6. Conclusions

Many control systems experience oscillation of their output signal when subjected to a step input. This may be due to different reasons, but usually a high system gain causes low phase margin and consequently a poor relative damping ratio. In some extreme cases this may lead even to system instability. Although, there are different methods of improving the system performance, this research is focusing mainly on the phase-lead series compensation and its optimization with the aid of proper MATLAB procedures. The suggested technique could be applied to phase-lag compensation as well.

One of the contributions of this paper is suggesting a procedure for optimization of the compensator's corner frequencies resulting in the best system performance. The achieved results show that by applying the array search within a specified area of interest, it is possible to find the optimal compensator parameters that will deliver the finest system performance under the current circumstances. Phase-lead compensation is chosen in this research, since it is preferred for the higher system gain within the higher frequency range [4, 12]. The suggested array procedure proved to be a powerful tool for optimization design of system controllers.

From comparing the achieved performance results with the require performance criteria objectives, it is obvious that after applying compensation, the performance of the system is even better than the required performance objectives. The graphical comparison of the system's transient responses before and after the compensation also proves the considerable improvement of the system performance.

D-partitioning analysis, applied for the original control system, demonstrates that the system's parameters variation and uncertainty may easily cause deteriorating performance or even instability. It is also seen from the results that the range of variation of the system's gain or a time-constant is quite limited. After implementing the suggested optimized compensation, the D-partitioning analysis reveals graphically considerable better range of parameters margins at of the system's range of stability.

The outcome and the analysis shown in this research can be easily extended for system optimization in case of feedback compensation and also in case of optimizing any other type of controllers.

References

[1] Shinners S., "Modern Control System Theory", New York, USA, McGraw-Hill, 2004.
[2] Driels M., "Linear Control System Engineering", New York, USA, McGraw-Hill, 2006.
[3] Kuo B., "Automatic Control Systems", John Wiley & Sons, INC., New York, USA, 2012.
[4] Yanev K.M., O. Opok, "Improved Technique of Multi-Stage Compensation", First African Control Conference AFRICON 2003, Cape Town, South Africa, p.51-66.
[5] Ogata K. "Modern Control Engineering", Prentice- Hall International, Inc. London, UK, 2014.
[6] Yanev K.M., "Analysis of Systems with Variable Parameters and Robust Control Design", Proceedings of the 6th IASTED International Conference on Modeling, Simulation and Optimization, 2006 pp. 75-83.
[7] Yanev K. M., "Application of the Method of D-Partitioning for Stability of Control Systems with Variable Parameters", BJT, Vol.16, Number 1, 2007, pp.51-58.
[8] Yanev, K.M., Anderson G.O, Masupe S., "D-Partitioning Analysis and Design of a Robust Photovoltaic Light Tracker System", BIE 12th Annual Conference, 2011, pp. 60-61.
[9] Yanev K.M., Advanced Interactive Tools for Analysis and Design of Nonlinear Robust Control Systems, International Review of Automatic Control, Naples, Italy, 2013 Vol. 6, N. 6, pp. 720-727.
[10] Yanev K. M., Anderson G. O., Masupe S., Strategy for Analysis and Design of Digital Robust Control Systems, ICGST-ACSE Journal, Vol. 12, Issue 1, 2012, pp. 37-44.
[11] Yanev K.M, Anderson G.O., Masupe S., "Multivariable System's Parameters Interaction and Robust Control Design", Journal of International Review of Automatic Control, Vol. 4, N.2, 2011, pp.180-190.
[12] Yanev K.M, "Strategy for Design of Optimal Control System Compensation", International Journal of Energy Systems, Computers and Control, Volume 1, No. 2, 2010, pp. 113–126,

[1]**Prof. Kamen Yanev** has a M.Sc. in Control Systems Engineering. His Ph.D. is in the area of Systems with Variable Parameters and Robust Control Design. He started his academic career in 1974, following a number of academic promotions and being involved in considerable research, service and teaching at different Commonwealth Universities around the world. Prof. Yanev worked in a number of outstanding universities in Europe and in Africa, in countries like Nigeria, Zimbabwe and Botswana. Currently he works as Associate Professor in Control and Instrumentation Engineering at the Department of Electrical Engineering at University of Botswana. His major research is in the field of Automatic Control and Systems Engineering. He has 112 publications in the area of Control Systems, Electronics and Instrumentation. Most of his latest publications and current research interests are in the field of Electronics, Analysis of Control Systems with Variable Parameters and Robust Control Design. He is a member of the Institution of Electrical and Electronic Engineering (IEEE), a member of the Academic Community of International Congress for Global Science and Technology (ICGST), USA, where he was awarded a Golden Membership in 2016 based on one of his best publications.