A Probabilistic Multiple Hypothesis Tracking System for Space Object Tracking

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Abstract

Role of satellite has become important in recent years for the purpose of communication, scientific application, industrial application, navigation, and space research. Due to solar radiation and electromagnetic radiation, tracking of multiple targets is a complex as well as challenging task. Tracking of multiple space targets is a difficult task in clutter and multiple target echoes. The performance of the conventional target tracking techniques such as nearest neighbor and probabilistic data association related algorithms degrades in the presence of high density clutter and multiple target echoes. In order to respond this problem a combination of probabilistic data association filter (PDAF) and multiple hypotheses tracking (MHT) filter called as probabilistic multiple hypothesis (PMHT) filter and is proposed in this paper. This algorithm combines the effectiveness of the PDAF to associate data in a cluttered environment and the pattern of the multiple hypotheses for multiple target tracking. The likelihood of the measurement-to-track association of the measurements is processed in batches, where each measurement is associated with a track based upon its probability of association. The efficiency of the algorithm is verified using simulations and also compared with conventional tracking algorithms.

Keywords: *Data Association, Estimation, Kalman Filter, MHT, Tracking, PDAF.*

1. Introduction

Tracking of space targets in presence of various space objects and cluttered is an extremely difficult task. Traditional approaches used to solve the tracking problem by solving the measurement to track association in low density clutter. It is accomplished by combining gating with nearest neighbor association which picks the closest measurement to the current target position estimate. The data association is a key problem for target tracking in a cluttered environment, and the main task is to match measurements and targets by estimating unknown parameters such as state vector, covariance matrix. In tracking targets with less-than-unity probability of detection in the presence of false alarms (FAs), data association is a challenging assignment.

Instead of using only one measurement among the received ones and discarding the others, an alternative approach is used to assign measurements with different weights (probabilities) to a particular track known as probabilistic data association (PDA) [1]. Data association becomes more difficult with multiple targets where the tracks are assigned with multiple measurements at a single scan. Here, in addition to a track validating multiple measurements as in the single-target case, a measurement itself can be validated by multiple tracks (i.e., contention occurs among tracks for measurements). Many algorithms exist to handle this contention. The joint PDA (JPDA) algorithm is used to track multiple targets by evaluating the measurement-to-track association probabilities and combining them to find the state estimate [2]. The multiple hypothesis tracking (MHT) is a more powerful (but much more complex) algorithm that handles the multi target tracking problem by evaluating the likelihood that there is a target given a sequence of measurements [3]. Kalman Filter (KF) is extensively used in target tracking applications due to its simplicity, optimality, tractability and robustness [4]. It performs optimally when the model describing the target motion is liner and specified properly [5]. One of the important problem in target tracking is state estimation. If the target dynamics and measurement models are linear and the probability distributions are Gaussian, then the Kalman filter represents an optimal estimator in the Bayesian framework.

A common shortcoming of most tracking approaches is that they suffer from a combinatorial growth in algorithmic complexity with the number of targets and the number of measurements. This growth is also exponential with time if batch processing is used. The reason for the complexity problem is that standard tracking algorithms assume a measurement model that allows, at most, one measurement per target. This infers a dependency between measurements, and the resulting assignment problem. The computation requirements makes it impractical to implement such algorithms without making approximations that may degrade performance [6]. In this paper the PMHT algorithmic is proposed for tracking space objects, where the complexity grows only linearly with these data size parameters. This makes the PMHT an attractive option for multi-target tracking applications.

2. Problem Formulation

Suppose that a sensor observes a particular surveillance region and reports measurements at irregular intervals, referred to as scans. At each scan, there are multiple measurements. Some of these measurements are caused by signals that are of interest to the sensor operator, and are referred to as target detections. The other measurements may be caused by various undesired interference and noise processes collectively referred to as clutter. Let Z denote the collection of all measurements from the sensor over a batch from scan 1 to scan T . Let τ_t denote the time at which scan t was collected and n_t denote the number of measurements formed by the sensor in scan t. Assume that $\tau_{t+1} > \tau_t$ for all t. This merely implies that the measurements are ordered in time sequence and that all measurements observed at a particular time are collated in a single scan. Define $Z_t$be the vector of all measurements in scan t.

$$Z_t = [z_1 z_2 \cdots z_{mk}] \tag{1}$$

Where z_{mk} is the r^{th} measurement at scan t. The set of all measurements Z can be written as the set of the scan vectors Z_t.

$$Z = [Z_1 Z_2 \cdots Z_T] \tag{2}$$

The probability density of the measurements, $P(Z)$, is a mixture of the measurement probability densities due to each of the various sources in the sensor, that is, $P(Z)$ is the weighted sum of the measurement probability densities of the sources. The weighting for each source is referred to as the mixing proportion for that source and the measurement probability for that source is a component of the probability density function of Z. Each of the source measurement probability densities has a known functional form with unknown time varying parameters, called the state. The goal is to find the dynamic mixture model that best describes the probability density function of the measurements Z, namely the states that maximize the probability of the observed measurements which is a Maximum Likelihood Criterion for estimating the states.

3. Kalman Filter Model for State Estimation

The suboptimal approach for filtering in nonlinear systems which is an extension of Kalman filter known as extended Kalman filter, a standard approach for nonlinear stochastic state estimation. State estimation is mainly to estimate position and velocity. Position includes range, azimuth, and elevation; Velocity includes speed and acceleration [7]. The Kalman filter is a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of a process, in a way that minimizes the mean of the squared error. In tracking systems, the state space of linear dynamic systems are represented by a state equation of the form [10]

$$x(k+1) = F(k)\, x(k) + v(k) \tag{3}$$

Measurement equation is given by

$$z(k) = H(k)x(k) + w(k) \tag{4}$$

$x(k)$ is the state of the system, $F(k)$ is the state transition matrix, $v(k)$ is the process noise, $w(k)$ is the measurement noise, $z(k)$ is the measurement vector and $H(k)$ is the measurement matrix. The process and measurement noise co-variances are considered to be uncorrelated, white, and Gaussian with zero mean and known co-variance matrices Q and R respectively as given in the following equations

$$\begin{aligned} &w_k \sim \mathcal{N}(0, Q_k) \\ &v_k \sim \mathcal{N}(0, R_k) \\ &E[w_k(i) {v_k}^T(j)] = 0 \\ &E[w_k(i) {w_k}^T(j)] = \delta_{ij}\, R(i) \\ &E[v_k(i) {v_k}^T(j)] = \delta_{ij}\, Q(i) \end{aligned} \tag{5}$$

The linear KF technique is applied to linearize the system using the following equations [4].

$$\begin{aligned} &x_{k+1|k} = f(x_k, v_k) \\ &P_{k+1|k} = F_k P_k {F_k}^T + Q_k \\ &K_k = P_{k+1|k} {H_k}^T {S_k}^{-1} \\ &S_k = H_k P_{k+1|k} {H_k}^T + R_k \\ &x_{k+1|k+1} = x_{k+1|k} + K_k\left(y_k - H x_{k|k}\right) \end{aligned} \tag{6}$$

4. Probabilistic Data Association Filter

Probability data association algorithm (PDA) was proposed by Bar-Shalom and Jaffer in 1972 [8]. Within the correlation gate, instead of the single target originated measurements there may have many effective measurements. Being different from nearest neighbor data association which selects the “nearest” measurement from prediction position, PDA considers each effective measurement comes from target has the different probability. This method gets posterior information of all the measurements within correlation gate, and then calculates the weighted sum to update target state.

On condition of knowing all the effective measurements from the 1^{st} to k^{th} scan, the correct correlation probability of the i^{th}measurement at the k^{th}scan can be defined as

$$P_i(k) = P\{\theta_i(k)/Z_k\} \qquad i = 1,2,\cdots,m_k \tag{7}$$

Where $\theta_i(k)$denotes correct measurement event from the 1 to $m_k$measurements at the k^{th} scan, Z_k that denotes all the effective measurements from the first to k^{th} scan, m_k denotes measurements number of the k^{th} scan. According to total probability formula, it can be proved that the state estimation at time k is optimal in the mean square sense.

$$\hat{X}(k|k) = \sum_{i=0}^{m_k} P_i(k) \hat{X}_i(k|k) \tag{8}$$

Where $\hat{X}_i(k|k)$ is the state estimation on condition that effective measurement comes from target. $\hat{X}_0(k|k)$ is the state estimation on condition that effective measurement comes from false-alarm or clutter. Correlation probability is the metrics for effective measurement working on target state. PDA does not determinate which measurement is true, but calculates each measurement statistical probability weight on condition of thinking all effective measurements come from target or clutter. Under the Poisson distribution of clutter space density, the correlation probability of PDA can be given by [9]:

$$P_{ij} = \frac{a_{ij}}{a_{i0} + \sum_{i=1}^{mk} aij} \qquad i = 1,2,\cdots,m_k \tag{9}$$

$$a_{ij} = P_D \; exp\left[-\frac{1}{2}e_{ij}(k)S_i^{-1}(k)e_{ij}^{T}(k)\right] \quad j > 0 \tag{10}$$

$$a_{i0} = (2\pi)^{\frac{M}{2}}\,\lambda\,\sqrt{|S_i(k)|}(1-P_D) \qquad j = 0 \tag{11}$$

where M denotes measurement dimension, PDdenotes detection probability, $S_i(k)$ denotes covariance matrix of $e_{ij}(k)$, λ denotes Poisson distribution parameter.

5. Multi-Hypothesis Tracker

Multiple hypothesis tracker enumerates almost all of the possible combinations of measurement-to-track assignments. The most likely measurement is selected as the best estimate of the track are present. The problem complexity increases as the data combinations are growing exponentially as each new measurement batch is received. This requires a huge amount of memory and computing power. Furthermore, as the number of possibilities increase, some form of pruning must be done in order to keep the number of hypotheses within limits [10]. A key strategy in MHT is to delay data association decisions by keeping multiple hypotheses active until data association ambiguities are resolved. MHT maintains multiple track trees, and each tree represents all of the hypotheses that originate from a single target. At each frame, the track trees are updated from observations and each track in the tree is scored. In order to demonstrate the difference between these tracking techniques a practical is shown here. Assume that there are two tracks. Let at a single scan three measurements are received. By using the gating technique, measurement to track association is formed which is illustrated in Fig 1.

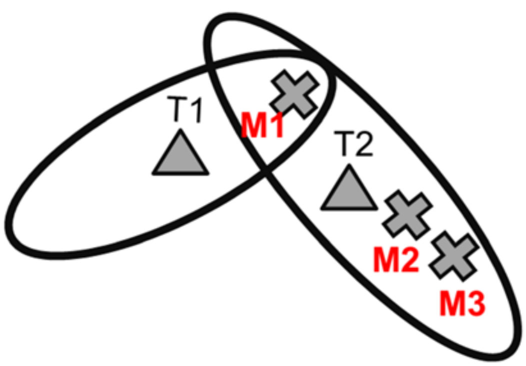

Fig 1. Gating procedure, tracks are drawn with triangles and measurements are denoted with crosses [4]

6. Probabilistic Multiple Hypothesis Tracking

Streit and Luginbuh [5] proposed a new algorithm called the probabilistic multi-hypothesis tracking (PMHT) algorithm for naval systems. Like the MHT, this algorithm processes data in batches, thereby giving it the advantage of future data before decisions are made. Each iteration of the PMHT algorithm begins with a set of track position estimates and a set of measurement probabilities. Then the weights are computed and centroids formed. These centroids are then used in the Kalman Smoother to update the track position estimates. With the new estimates, a new set of weights is computed. These weights will be similar to the previous weights, but they will be different because of the new estimates. After some iterations, the weights will converge. When convergence is reached, the current estimate is theoretically the optimal estimate for a given track.

6.1 Initialization

Measurement probabilities, $\Pi^{(0)} = \{\pi_{tm}\}$ must be assigned so that $\pi_{tm} > 0$. It is not critical what values are assigned to these measurement probabilities because in the first iteration they will be recalculated. Moreover, they do not have an adverse effect before they are recomputed. The $\pi_{tm}{}^{(i)}$ values specify the estimated probability that a measurement at scan t is assigned to target model m after i iterations of the PMHT algorithm. An initial target state $({x_{0m}}^{0}, {x_{1m}}^{0}, \cdots, {x_{Tm}}^{0})$ for each time increment and each of the M target models must be assigned. Here m specifies the target model $m = (1,2,\ldots\ldots,M)$, $t = (1,2,\ldots T)$ and i specifies the iteration index.

6.2 Weight Computation

For every target and measurement combination at each scan, a weight is computed. The value of the likelihood function (assuming normal distribution) evaluated at the error between the current estimated position and each measurement is used for the weight:

$${w_{ntr}}^{i} = \frac{exp\left[-\frac{1}{2}{Z_{ntr}}^{i\,T}{S_i}^{-1}(k){Z_{ntr}}^{i}\right]}{(2\pi)^{\frac{M}{2}}\,\lambda\,\sqrt{|\Sigma|}} \tag{12}$$

$${w_{ntr}}^{i+1} = \frac{{w_{ntr}}^{i}}{\sum {\pi_{tm}}^{i}{w_{ntr}}^{i}} \tag{13}$$

Where $Z_{ntr} = Z_i - {\hat{Z}_{ntr}}^{i}$ is the error between the current estimate and measurement. Σ is the weighting matrix defined as $\Sigma = CPC^T + R$. C is the measurement matrix and R is the covariance matrix.

6.3 Calculation of Measurement Centroid

The measurement centroid $\bar{Z}_{tr}$ can be computed as

$$\bar{Z}_{tr} = \frac{1}{nt\,{w_{ntr}}^{i+1}}\sum_{r=1}^{nt}{w_{ntr}}^{i+1}Z_i \tag{14}$$

This measurement centroid will be used in the Kalman filtering for the state update, where

$${w_{ntr}}^{i+1} = \frac{1}{nt}\sum_{r=1}^{nt}{w_{ntr}}^{i+1} \tag{15}$$

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6.4 Target Measurement Probability Update

The target measurement probability can be updated by the following equation with the help of measurement centroid which is used in the Kalman filter state estimation and can be represented by

$$\pi_{ntr}{}^{i+1} = w_{ntr}{}^{i+1}\pi_{ntr}{}^{i} \quad (16)$$

The forward recursion equations of the filtering equations are obtained as

$$P_{k+1|k} = F_k P_{k|k} F_k{}^T + Q$$
$$G_{k+1} = n_{k+1}\pi_{ntr}{}^{i} P_{k+1|k} C^T \left[n_{k+1}\pi_{ntr}{}^{i}\, C\, P_{k+1|k} C^T + R\right]^{-1}$$
$$P_{k+1|k+1} = P_{k+1|k} - G_{k+1}\, C\, P_{k+1|k} \quad (17)$$
$$y_{k+1|k+1} = F_k y_{k+1|k} + G_{k+1}\left(\bar{Z}_{tr} - C F_k y_{k+1|k}\right)$$
$$x_{k+1|k+1}{}^{i+1} = y_{k+1|k+1} + P_{k+1|k+1} F_k P_{k+1|k}{}^{-1}\left[x_{k+1|k}{}^{i+1} - F_k y_{k+1|k}\right]$$

The equations in this subsection make up a bank of M Kalman Smoothers which can be run in parallel, although these filters are not independent because they are linked by the weights.

7. Simulation Results and Discussion

In this section space target motion trajectory is simulated with constant velocity model for a period of $20s$ with a sampling interval of 0.5. The sensor measurements for a maneuvering target is generated in polar coordinates. The starting position of the targets in x, and y coordinates are (900, 950) with a velocity of 50 m/sec in x-axis, 40 m/sec in y-axis respectively.

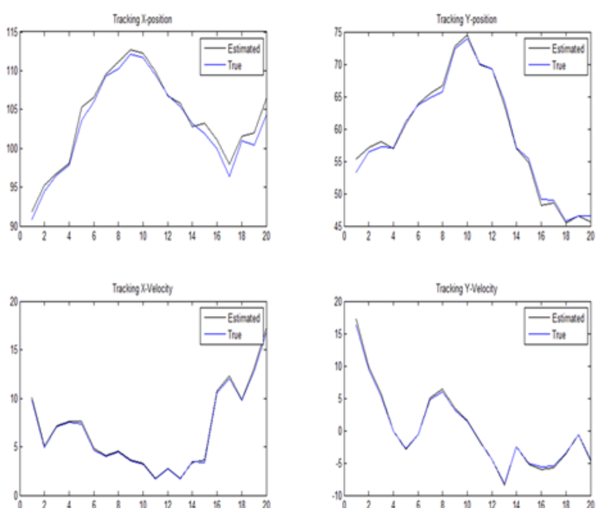

Figure 2. Tracking with PMHT of a constant velocity model

Random noise with variances of 150 m^2, 52 m/sec^2, 2 deg^2, 5 deg/sec^2 is added to range, range rate, elevation and elevation rate respectively. Tracking performance using PMHT and Kalman Filtering without presence of clutter is shown in Fig. 2. Tracking performance of PDAF and PMHT in presence of low density clutter is shown in Fig. 3. Here in this simulation tracking of a single target is considered. The performance of tracking and state estimation in high density clutter has shown in Fig. 4 with both PDAF and PMHT. The clutter density variations in tracking performance has shown in Fig. 5, where it can be seen that as the clutter density increases the PMHT algorithm gives better result as compare to the PDAF.

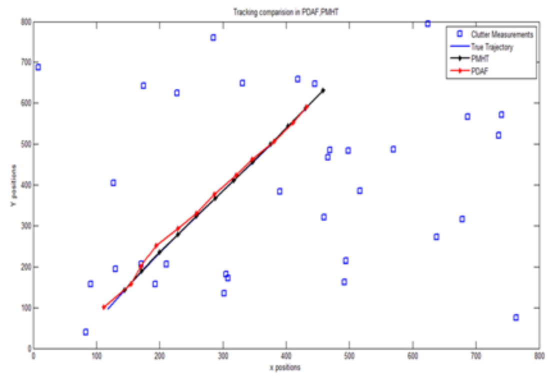

Figure 3. Tracking in PDAF and PMHT with low clutter density

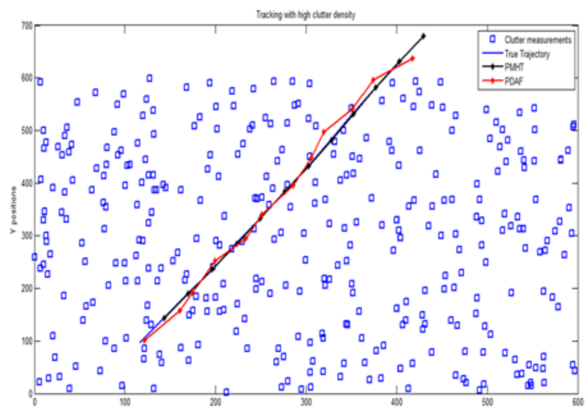

Figure 4. Tracking in PDAF and PMHT with low clutter density

The performance of the algorithms are compared for probabilistic data association filter and probabilistic multi hypothesis tracking filter in Fig. 5.

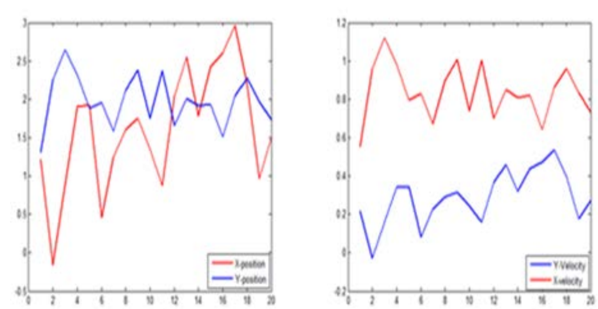

Figure 5. Performance comparison of PDAF and PMHT

It has been analyzed that the performance of the probability association and the track formation done by the probabilistic multiple hypothesis density filter performance is better. This technique will be one of the efficient technique for the tracking of multiple space objects like satellites.

8. Conclusion

In this paper the tracking performance of the probabilistic data association filter (PDAF) and the probabilistic multiple hypothesis tracking (PMHT) algorithm for a single space target is considered. The above simulations can conclude that as the density of clutter increases the PMHT data association algorithm performs better as compare to that of PDAF. In this paper only one target is considered and the maneuvering of the target is not considered. This algorithm can be extended for the tracking of multiple space target scenario with multiple transmitters and receiver arrangements.

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