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GENERALIZED SOFT FILTER AND SOFT NET

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Abstract

In this paper we introduce to generalized soft filter and generalized soft net with them properties and the relation between them.

Keywords: *Generalized soft topology, Generalized soft filter, gs-net.*

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1. Introduction

The real world is inherently uncertain, imprecise and vague. To solve complex problems in economics, engineering, environment, sociology, medical science, business management, etc. we cannot successfully use classical methods because of various uncertainties typical for those problems. In recent years, a number of theories have been proposed for dealing which such systems in an effective way. Some of these are theory of probability, theory of soft sets [3], etc. and these many be utilized as mathematical tools for dealing with diverse types of uncertainties and imprecision embedded in a system. But all these theories have their inherent difficulties. To overcome these kinds of difficulties, Molodtsov [3] proposed a completely new approach, which is called soft set theory, for modelling vagueness and uncertainty. Soft set theory is growing very rapidly nowadays. The basic properties of the theory may be found in [1].

In the present paper, firstly we give, as a preliminaries, some basis facts in soft set theory. Secondly, we introduce concepts of generalized soft filter by using soft sets on an universal set, and give several interesting properties. Finally, we study the relation between generalized soft net and generalized soft filter.

In this section, we give some preliminaries about soft set. We make some small modifications to some of them in order to make theoretical study in detail. Throughout this paper, X refers to an initial universal set, E is a set of all possible parameters, $P(X)$ is the power set of X. Moreover, $SS(X, E)$ denotes the family of all soft sets over X.

Definition (1.1) [1]: For $A \subseteq E$, the pair (F,A) is called a Soft Set over X, where F is a mapping given by $F:A \rightarrow P(X)$.

In other words, the soft set is a parametrized family of subsets of the set X. Every set $F(e)$, $e \in E$, from this family may be considered as the set of e-elements of the soft set (F, E) , or as the set of e-approximate elements of the soft set. Clearly, a soft set is not a set.

Definition(1.2) [2] : Assume that we have a binary operation, denoted by $*$, for subsets of the set X. Let (F, A) and (G, B) be soft sets over X. Then, the operation $*$ for soft sets is defined in the following way :

$(F, A) * (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) * G(\beta)$, $\alpha \in A$, $\beta \in B$ and $A \times B$ is the Cartesian product of the sets A and B .

This definition takes into account the individual nature of any soft set.

Definition(1.3) [3] : For two soft sets (F,A) and (G,B) in $SS(X,E)$, we say that (F, A) is a soft subset of (G, B) if $A \subseteq B$ and $F(e) \subseteq G(e)$, $\forall e \in E$.

Also, we say that the pairs (F,A) and (G,B) are soft equal if $(F,A) \tilde{\subseteq} (G,B)$ and $(G,B) \tilde{\subseteq} (F,A)$. Symbolically, we write $(F,A) = (G,B)$.

Definition (1.4) [3] : The union of two soft sets (F, A) and (G,B) over the common universe X is the soft set (H,C), where $C = A \cup B$ and for all $e \in C$,

$$H(e)=\begin{cases} F(e) & ,e\in A-B, \\ G(e) & ,e\in B-A, \\ F(e)\cup G(e) & ,e\in A\cap B . \end{cases}$$

Definition(1.5) [3]: The intersection of two soft sets (F,A) and (G,B) over the common universe X is the soft set (H,C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$. Note that, in order to efficiently discuss, we consider only soft sets (F,E) over a universe X in which all the parameter set E are same.

Definition(1.6) [1]: The complement of a soft set (F,E), denoted by $(F,E)^c$ is defined by $(F,E)^c = (F^c, E)$, $F^c : E \rightarrow P(X)$ is a mapping given by $F^c(e) = X - F(e)$, $\forall e \in E$ and F^c is called the soft complement function of F.

Definition(1.7) [1]: The difference of two soft sets (F,E) and (G,E) over the common universe X, denoted by $(F,E) - (G,E)$ is the soft set (H,E) where for all $e \in E$, $H(e) = F(e) - G(e)$.

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Definition(1.8) [1] : Let (F,E) be a soft set over X and $x \in X$. We say that $x \tilde{\in} (F,E)$ read as x belongs to the soft set (F,E), whenever $x \in F(\alpha)$ for all $\alpha \in E$.

Note that for $x \in X$, $x \tilde{\notin} (F,E)$ if $x \notin F(\alpha)$ for some $\alpha \in E$.

Definition(1.9) [2]: A soft set (F,A) over X is said to be a null soft set, denoted by ϕ_A, if for all $e \in A$, $F(e)=\phi$ (null set), where $\phi_A(e)=\phi \quad \forall e \in A$

Definition(1.10) [1] : A soft set (F,A) over X is said to be an absolute soft set, denoted by X_A, if for all $e \in A$, $F(e)= X$. Clearly we have $X_A{}^c = \phi_A$ and $\phi_A^c = X_A$.

Definition(1.11) [2]: Let Λ be an arbitrary indexed set and $L = \{(F_i,E), i \in \Lambda\}$ be a subfamily of $SS(X,E)$.

(1)The union of L is the soft set (H,E), where $H(e) = \bigcup_{i\in\Lambda} F_i(e)$ for each $e \in E$. We write $(H,E) = \tilde{\bigcup}_{i\in\Lambda}(F_i, E)$.

(2)The intersection of L is the soft set (M,E), where $M(e) = \bigcap_{i\in\Lambda} F_i(e)$ for each $e \in E$. We write $(M,E) = \tilde{\bigcap}_{i\in\Lambda}(F_i, E)$.

Proposition(1.12) [1] : Let (F, A) and (G, B) be soft sets over X . Then

(i) $((F, A) \tilde{\cap} (G, B))^c = (F, A)^c \tilde{\cup} (G, B)^c$.

(ii) $((F, A) \tilde{\cup} (G, B))^c = (F, A)^c \tilde{\cap} (G, B)^c$.

Proposition(1.13) [1] : Let (F, A) be soft sets over X . Then

(i) $(F, A) \tilde{\cap} (F, A) = (F, A)$.

(ii) $(F, A) \tilde{\cup} (F, A) = (F, A)$.

(iii) $(F, A) \tilde{\cap} X_A = (F, A)$

(iv) $(F, A) \tilde{\cup} X_A = X_A$.

(v) $(F, A) \tilde{\cap} \phi_A = \phi_A$.

(vi) $(F, A) \tilde{\cup} \phi_A = (F, A)$.

Definition(1.14) [1],[2] : Let τ be a collection of soft sets over X with the fixed set E of parameters , then $\tau \subseteq SS(X,E)$. We say that the family τ defines a soft topology on X if the following axioms are true :

1- X_A , $\phi_A \in \tau$,

2- If (G,A) , $(H,A) \in \tau$, then $(G,A) \tilde{\cap} (H,A) \in \tau$,

3- If $(G_i, A) \in \tau$ for every $i \in \Lambda$, then $\tilde{\bigcup}_{i\in\Lambda} (G_i, A) \in \tau$.

Then τ is called a Soft Topology on X and the triple (X, τ, E) is called Soft Topological Spaces over X . τ is called a generalized soft topology if 1. $\phi_A \in \tau$ and 2. τ is closed under arbitrary union. By definition of τ, X need not be an element of τ.

Definition(1.15) [1] : Let(X,τ,E) be a soft topological space. The members of τ are said to be soft open sets in X. We denote the set of all soft open sets over X by $OS(X, \tau, E)$, or $OS(X)$ and the set of all soft closed sets, which are the complements of soft open sets by $CS(X,\tau,E)$, or $CS(X)$.

Definition(1.16) [1],[2] : Let (X, τ, E) be a soft topological space and $(F,E) \in SS(X, E)$. The soft closure of (F,E), denoted by $cl(F,E)$ is the intersection of all closed soft super sets of (F,E) that to say $cl(F,E) = \tilde{\cap}\{(H,E) ; (H,E)$ *is* closed soft set and $(F,E) \tilde{\subseteq} (H,E)\}$.

Definition (1.17) [1] : Let (X, τ, E) be a soft topological space and $(G,E) \in SS(X, E)$. The soft interior of (G,E), denoted by $int(G,E)$ is the union of all open soft subsets of (G,E) that to say $int(G,E) = \tilde{\cup}\{(H,E) ; (H,E)$ is an open soft set and $(H,E) \tilde{\subseteq} (G,E)\}$.

Proposition(1.18) [1] : Let (X, τ, E) be a soft topological space and (F,E) , $(H,E) \in SS(X, E)$. Then

(i) $cl(cl(F,E)=cl(F,E)$,

(ii) $(F,E) \tilde{\subseteq} cl(F,E)$,

(iii) $(F,E) \tilde{\subseteq} (H,E)$ *implies* $cl(F,E) \tilde{\subseteq} cl(H,E)$,

(iv) $cl\{(F,E) \tilde{\cup} (H,E)\}= cl(F,E) \tilde{\cup} cl(H,E)$,

(v) $cl\{(F,E) \tilde{\cap} (H,E)\} \tilde{\supseteq} cl(F,E) \tilde{\cap} cl(H,E)$.

Proposition(1.19) [1] : Let (X, τ, E) be a soft topological space and (F,E) , $(H,E) \in SS(X, E)$. Then

(i) $int(int(F,E)=int(F,E)$,

(ii) $int(F,E) \tilde{\subseteq} (F,E)$,

(iii) $(F,E) \tilde{\subseteq} (H,E)$ *implies* $int(F,E) \tilde{\subseteq} int(H,E)$,

(iv) $int(F,E) \tilde{\cup} int(H,E) \tilde{\subseteq} int\{(F,E) \tilde{\cup} (H,E)\}$,

(v) $int\{(F,E) \tilde{\cap} (H,E)\} = int(F,E) \tilde{\cap} int(H,E)$.

Definition(3.2.1)[1] : Let SS(X,E) and SS(Y,B) be families of soft sets over X and Y respectively , $u: X \to Y$ and $p: E \to B$ be mappings . Then the mapping f_{pu} :SS(X,E) $\to$ SS(Y,B) is defined as :

1- If $(F,E) \in SS(X,E)$, then the image of (F,E) under f_{pu} , written as $f_{pu}(F,E) = (f_{pu}(F), p(E))$ is a soft set in SS(Y,B) such that

$$f_{pu}(F)(b) = \begin{cases} \bigcup_{a \in p^{-1}(b) \cap E} u(F(a)) \ , & p^{-1}(b) \cap E \neq \phi. \\ \phi \ , & p^{-1}(b) \cap E = \phi. \end{cases} \quad \text{for all } b \in B .$$

2- If $(G,B) \in SS(Y,B)$, then the inverse image of (G,B) under f_{pu} , written as

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f_{pu}^{-1}(G,B) =(f_{pu}^{-1}(G),p^{-1}(B)) is a soft set in SS(X,E) , such that

$$f_{pu}^{-1}(G)(a)=\begin{cases} u^{-1}(G(p(a))) \ , & p(a)\in B\,. \\ \phi \ , & o.w\,. \end{cases} \quad \text{for all } a\in E\,.$$

is called a soft mapping .

Definition(3.2.1)[1] : A soft filter on (F,E) is a nonempty subfamily *F* of SS(X,E) having the following properties:

(1) Every soft subset of SS(X,E) which includes a soft set in *F* belongs to *F*.

(2) The intersection of each finite family of soft sets in *F* belongs to *F*.

(3) All soft sets in *F* are not null soft set.

1- Generalized soft filter and soft net

Definition (2.1) : Let $\tilde{F} \tilde{\subseteq} SS(X, E)$. Then $\tilde{F}$ is called a generalized soft filter on X if $\tilde{F}$ satisfies the following properties :

1- $\phi_E \notin \tilde{F}$

2- $\forall$ *(G,E)* $\in \tilde{F}$ and *(G,E)* $\tilde{\subseteq}$ *(H,E)* implies *(H,E)* $\in \tilde{F}$.

Example(2.2) : Let *X*={a,b,c} and E={ e_1 , e_2 }. Let $\tilde{F}$ ={(A,E),(B,E), X_E } where (A,E)={(e_1 ,X), (e_2 ,{b,c})} and (B,E)={(e_1 ,X),(e_2 ,{a,b})}. Then $\tilde{F}$ is generalized soft filter on X.

Example (2.3): Let *X*={a,b,c} and E={ e_1 , e_2 }. Let $\tilde{F}$ ={(A,E),(B,E), X_E }where (A,E)={(e_1,{a.b}),(e_2,{b,c})} and (B,E)={(e_1,X),(e_2,{a,b})}. Then $\tilde{F}$ is not generalized soft filter on X, since (A,E) $\tilde{\subseteq}$ {(e_1,X),(e_2,{b,c})} $\notin \tilde{F}$.

Proposition(2.4): Every soft filter is a generalized soft filter.

Proof: It is clear.

Remark(2.5): The converse of Proposition(2.5) need not be true by the following example.

Example(2.6): Let *X*={a,b,c} and E={ e_1 , e_2 }. Let $\tilde{F}$ ={(A,E),(B,E), X_E }where (A,E)={(e_1,X),(e_2,{b,c})} and (B,E)={(e_1,X),(e_2,{a,b})}. Then $\tilde{F}$ is generalized soft filter on X. But not soft filter, since *(A,E)* $\tilde{\cap}$ *(B,E)*={(e_1,X),(e_2,{b})} $\notin \tilde{F}$.

Proposition (2.7): Let $\left(\tilde{F}_i\right)_{i\in I}$ be a family of generalized soft filters over *X*. Then $\tilde{F} = \tilde{\bigcap}_{i\in I} \tilde{F}_i$ is a generalized soft filter over *X*.

Proof: Since $\phi_E \notin \tilde{F}_i$ for each $i \in I$. Then ϕ_E does not belong to $\tilde{F}$.

Let (G,E) $\in \tilde{F}$ and *(G,E)* $\tilde{\subseteq}$ *(H,E)*. Since (G,E) $\in \tilde{F}_i$ for each $i \in I$ and *(G,E)* $\tilde{\subseteq}$ *(H,E)*, we get (H,E) $\in \tilde{F}_i$ for each $i \in I$. Thus (H,E) $\in \tilde{F}$.

Proposition(2.8): Let $\left(\tilde{F}_i\right)_{i\in I}$ be a family of generalized soft filters over *X*. Then $\tilde{F} = \tilde{\bigcup}_{i\in I} \tilde{F}_i$ is a generalized soft filter over *X*.

Proof: Since $\phi_E \notin \tilde{F}_i$ for each $i \in I$. Then ϕ_E does not belong to $\tilde{F}$.

Let (G,E) $\in \tilde{F}$ and *(G,E)* $\tilde{\subseteq}$ *(H,E)*. Since (G,E) $\in \tilde{F}_i$ for some $i \in I$ and *(G,E)* $\tilde{\subseteq}$ *(H,E)*, we get (H,E) $\in \tilde{F}_i$ for some $i \in I$. Thus (H,E) $\in \tilde{F}$.

Proposition(2.9): Let f_{pu} :*SS(X, E)* $\rightarrow$ SS(Y,K) be a soft mapping where *SS(X, E)* and SS(Y,K) are two families of soft sets on X and Y respectively. Suppose that $\tilde{F}$ is a generalized soft filter on X. Then $f_{pu}(\tilde{F})$ ={(A,K); $f_{pu}^{-1}(A,K)\in\tilde{F}$ } is a generalized soft filter on Y.

Proof: Since $f_{pu}^{-1}(\phi_K)=\phi_E \notin \tilde{F}$ then ϕ_K does not belong to $f_{pu}(\tilde{F})$.

Let (A,K) $\in f_{pu}(\tilde{F})$ and *(A,K)* $\tilde{\subseteq}$ *(H,K)*. then $f_{pu}^{-1}(A,K)\in\tilde{F}$, *then* $f_{pu}^{-1}\{(A,K)\tilde{\subseteq}(H,K)\}$ = $f_{pu}^{-1}(A,K)\tilde{\subseteq}f_{pu}^{-1}(H,K)$. Therefore $f_{pu}^{-1}(H,K) \in \tilde{F}$. Hence (H,K) $\in f_{pu}(\tilde{F})$. Thus $f_{pu}(\tilde{F})$ is a generalized soft filter on Y.

Definition (2.10) : Let $\tilde{F}$ be a generalized soft filter in a generalized soft topological space (X, τ,E). $\tilde{F}$ is said to be convergent to an element $x\in$ X if every soft open set containing *x* belong to $\tilde{F}$. We write $\tilde{F} \rightarrow x$. *x* is called a limit of $\tilde{F}$.

Theorem(2.11): Let SS(X,E) and SS(Y,K) be the families of all soft sets on X and Y, respectively. Suppose that f_{pu} :*(X,τ,E)* $\rightarrow$ SS(Y, τ^*,K) is a soft mapping, where *u*: X $\rightarrow$ Y and p: E $\rightarrow$ K are two mappings. If f_{pu} is a soft continuous function and $\tilde{F}$ is a generalized soft filter on X such that $\tilde{F} \to x$, then we have $f_{pu}(\tilde{F}) \to u(x)$.

Proof: Let $u(x) \tilde{\in}$ (G,K) such that (G,K)∈ τ^*. Thus, we have $x \tilde{\in} f_{pu}^{-1}(F,K)$. Since f_{pu} is a soft continuous function, then $f_{pu}^{-1}(F,K)$ is soft open in X such that $x \tilde{\in} f_{pu}^{-1}(F,K)$ and hence $f_{pu}^{-1}(F,K) \in \tilde{F}$. This implies that (G,K)∈ $f_{pu}(\tilde{F})$ and so $u(x) \tilde{\in}$ (G,K)∈ $f_{pu}(\tilde{F})$. It follows that have $f_{pu}(\tilde{F}) \to u(x)$.

Definition(2.12): Let $\tilde{F}$ be a generalized soft filter on X. Let $\tilde{S} \subseteq SS(X, E)$ is called a base for the generalized soft filter $\tilde{F}$ if $\tilde{S} \subseteq \tilde{F}$ and every element of $\tilde{F}$ is superset of some elements of $\tilde{S}$.

$\tilde{F}$ is called the generalized soft filter generated by $\tilde{S}$ and denoted it by $\tilde{F}(\tilde{S})$.

Proposition(2.13) : Let $\tilde{S} \subseteq SS(X, E)$. The generalized soft filter $\tilde{F}(\tilde{S})$ is the coarsest generalized soft filter which contains $\tilde{S}$.

Proof: Suppose that $\tilde{F}_1$ be a generalized soft filter such that $\tilde{S} \subseteq \tilde{F}_1$. For every (A,E) $\in \tilde{F}(\tilde{S})$, then (A,E) is super soft set for some elements of $\tilde{S}$.
Say (B,E). but $\tilde{S} \subseteq \tilde{F}_1$. Then (B,E) $\in \tilde{F}_1$ and (B,E) $\tilde{\subseteq}$ (A,E). Hence (A,E) $\in \tilde{F}_1$. Therefore $\tilde{F}(\tilde{S}) \subseteq \tilde{F}_1$. we get $\tilde{F}(\tilde{S})$ contains $\tilde{S}$.

Proposition (2.14): Any collection of nonempty soft sets in SS(X,E) is a generalized soft filter base.

Proof: Suppose that Ω be any collection of non empty soft sets in SS(X,E). Let $\tilde{F}$ =SS(X,E) is a generalized soft filter, Then $\Omega \tilde{\subseteq} \tilde{F}$ and every element of $\tilde{F}$ is superset of some elements of $\tilde{S}$. Thus Ω) is a generalized soft filter base.

Definition (2.15): Let (X,τ,E) be a generalized soft topological space. Let S be the collection of all soft open sets containing a fixed point x. The generalized soft filter generated by S is called the Neighborhood generalized soft filter of x and it is denoted by $\tilde{N}(x)$.

Corollary(2.16): Let (X, τ,E) be a generalized soft topological space. Then $\tilde{N}(x) \to x$.

Proof: Let (G,E) be a soft open sets which contains x. Then (G,E) ∈ S $\subseteq \tilde{N}(x)$. Therefore $x \tilde{\in}$ (G,E) ∈ $\tilde{N}(x)$. Thus $\tilde{N}(x) \to x$.

Definition (2.17): A generalized soft filter base S is said to converge to a point x if the generalized soft filter generated by S converges to x. We write S → x.

Proposition(2.18): Let (X,τ,E) be a generalized soft topological space. Let S be a generalized soft filter base. S→x if and only if for every soft open set (F,E) containing x, there exist an element (G,E) in S such that (G,E) $\tilde{\subseteq}$ (F,E).

Proof: Let S→x. Then the generalized soft filter $\tilde{F}$ which is generated by S converges to x. If (F,E) is any soft open set containing x, since $\tilde{F}$ converges to x, (F,E) $\in \tilde{F}$. Since $\tilde{F}$ is generated by the generalized soft filter base S, there exists (A,E)∈S such that (A,E) $\tilde{\subseteq}$ (F,E). Conversely, let $\tilde{F}$ be the generalized soft filter generated by S. Let (F,E) be any soft open set containing x, then there exists (A,E)∈S such that (A,E) $\tilde{\subseteq}$ (F,E), which implies (F,E) $\in \tilde{F}$. Hence $\tilde{F} \to x$ which gives S→x.

Proposition(2.19): Let (X,τ,E) be a generalized soft topological space. Let (F,E) ∈SS(X,E) and (F,E) $\neq \phi_E$. Then x $\tilde{\in}$ cl((F,E)) if and only if ∃ generalized soft filter $\tilde{F} \to$x where (F,E) $\tilde{\cap}$ B $\in \tilde{F}$ for every B $\in \tilde{F}$.

Proof: Suppose $x \tilde{\in}$ cl((F,E)). Consider P= {(F,E) ∩(G,E)/ (G,E) is a soft open set containing x} Since $x \tilde{\in}$ cl((F,E)), every element of P is non empty and hence P generates a generalized soft filter say $\tilde{F}$. Since (F,E) $\tilde{\cap}$ (G,E) belongs to $\tilde{F}$ for every soft open set (G,E) containing x and $\tilde{F}$ is a generalized soft filter, we have $\tilde{F}$ contains every soft open set (G,E) containing x. Hence $\tilde{F}$ converges to x. Now $\tilde{F}$ is the required generalized soft filter.
Conversely, suppose there exists a generalized soft filter $\tilde{F} \to x$ where (F,E) $\tilde{\cap}$ (B,E) $\in \tilde{F}$ for every (B,E) $\in \tilde{F}$. Since $\tilde{F} \to x$, every soft open set (H,E) containing x belongs to $\tilde{F}$. Now (F,E) $\tilde{\cap}$ (H,E) $\in \tilde{F}$ and hence (F,E) $\tilde{\cap}$ (H,E) is non empty. Hence for every soft open set (H,E) containing x, we have (H,E) $\tilde{\cap}$ (F,E) $\neq \phi_E$ which implies $x \tilde{\in}$ cl((F,E)).

Theorem(2.20): Let (X, τ,E) be a generalized soft topological space. Let (A,E) $\neq \phi_E$. (A,E) is soft closed if and only if every x∈X satisfies the condition, if ∃ a

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generalized soft filter $\tilde{F} \to x$ with (A,E) $\tilde{\cap}$ (B,E) $\in \tilde{F}$ for every (B,E) $\in \tilde{F}$, then $x \tilde{\in}$ (A,E).
Proof: Follows from previous theorem.
Theorem(2.21): Let (X, τ,E) be a generalized soft topological space. Let (A,E) $\neq \phi_E$. (A,E) is soft open if and only if (A,E) belongs to every generalized soft filter which converges to a point of (A,E).

Proof: Let (A,E) be a soft open over X. If $\tilde{F}$ is a generalized soft filter converging to $x_0 \tilde{\in}$ (A,E), then every soft open set containing $x_0 \tilde{\in}$ (A,E) which implies (A,E) $\in \tilde{F}$. Conversely, suppose (A,E) belongs to every generalized soft filter converging to a point of (A,E). Take any $a \tilde{\in}$ (A,E). Let $\tilde{N}(a)$ be the neighborhood generalized soft filter of a. Since $\tilde{N}(a) \to a$, (A,E) $\in \tilde{N}(a)$. Hence there exist a soft open set $(O,E)_a \tilde{\subseteq}$ (A,E). Now to each element $a \tilde{\in}$ (A,E), ∃ a soft open set $(O,E)_a \tilde{\subseteq}$ (A,E). Since (A,E) is the union of all $(O,E)_a$ where $a \tilde{\in}$ (A,E), (A,E) is soft open set.
Definition: (i) A pre-order set is a pair (D,≥), where D is a nonempty set and ≥ is a binary relation in D which is reflexive and transitive.
(ii) Let X be a universal set, E be the set of parameters and (D,≥) be a pre-order set. A mapping S:D→SS(x,E), where SS(x,E) is the collection of all soft elements over X, is called generalized soft net (gs-net) over X. A gs-net usually denoted by $\{(x^n_{\alpha_n}, E)\}_{n\in D}$, where (x^n, A) is the support and α_n is the value of its nth member. If (A,E) is a soft set over X and $\{(x^n_{\alpha_n}, E)\} \leq (A, E)$, for each $n \in D$, then we say that the gs-net $\{(x^n_{\alpha_n}, E)\}_{n\in D}$ in in (A,E).

Remark: Let D be the collection of all soft sets over X. Then D is a pre-ordered set under the ordering of inverse set inclusion, that is, under ≥ where (A,E) ≥(B,E) if and only if (A,E) $\tilde{\subseteq}$ (B,E).

3- Relation between generalized soft Net and generalized soft Filter

Now we see the interlink between generalized soft net and generalized soft filter.
Theorem (3.1): Let SS(X,E) be a nonempty family of soft sets. Let $\{(x^n_{\alpha_n}, E)\}_{n\in D}$ be a generalized soft net. Then it induces a generalized soft filter.

Proof: Let $(t_{\lambda 0},E)$ = $\{(x_\lambda,E); \lambda \geq \lambda_0\}$, a tail of the generalized soft net. Let P = $\{(t_{\lambda 0},E); \lambda_0 \in D\}$. It is clear that every element of P is non empty and hence it generates a generalized soft filter $\tilde{F}$ = {(A,E) ; (A,E) $\tilde{\supseteq}$ $(t_{\lambda 0},E)$ for some λ_0}.Hence the generalized soft net $\{(x^\lambda_{\alpha_\lambda}, E)\}_{\lambda\in D}$ induces a generalized soft filter.
Theorem (3.2): Let (X, τ,E) be a generalized soft topological space. Let $\{(x^\lambda_{\alpha_\lambda}, E)\}_{\lambda\in D}$ be a generalized soft net over X. Let $\tilde{F}$ be the generalized soft filter induced by $\{(x^\lambda_{\alpha_\lambda}, E)\}_{\lambda\in D}$. Then $\{(x^\lambda_{\alpha_\lambda}, E)\}_{\lambda\in D} \to x$ if and only if $\tilde{F} \to x$.
Proof: Let $(t_{\lambda 0},E)$ = $\{(x_\lambda,E); \lambda \geq \lambda_0\}$. $\tilde{F}$ = {(A,E);(A,E) $\tilde{\supseteq}$ $(t_{\lambda 0},E)$ for some λ_0}.It is clear that $\tilde{F}$ is a generalized soft filter. Suppose $\{(x^\lambda_{\alpha_\lambda}, E)\}_{\lambda\in D}$ →x, we have to prove that $\tilde{F} \to x$. If (U,E) is a soft neighborhood of x, since $\{(x^\lambda_{\alpha_\lambda}, E)\}_{\lambda\in D} \to x$, $\exists \lambda_0 \in$ D such that $(t_{\lambda 0},E) \tilde{\subseteq}$ (U,E), $\forall \lambda \geq \lambda_0$. In particular $(t_{\lambda 0},E) \tilde{\subseteq}$ (U,E) and this implies (U,E) $\in \tilde{F}$. Hence every neighborhood of x is an element of $\tilde{F}$ which implies $\tilde{F}$ →x.
Conversely, suppose $\tilde{F} \to x$, if (U,E) is a soft neighborhood of x, then (U,E) $\in \tilde{F}$. Hence ∃ $(t_{\lambda 0},E) \tilde{\subseteq}$ (U,E) which implies $(x_\lambda,E) \tilde{\in}$ (U,E), $\forall \lambda \geq \lambda_0$. This implies that $\{(x^\lambda_{\alpha_\lambda}, E)\}_{\lambda\in D} \to x$.
Theorem (3.3): Let $\tilde{F}$ be a generalized soft filter in X. Then $\tilde{F}$ induces a generalized soft net in X.
Proof: Let $\tilde{F}$ be a generalized soft filter in X. Let D = {(x, (A,E)) ; $x \tilde{\in}$ (A,E), (A,E) $\in \tilde{F}$ }. In D we define a relation ≤ as $(x_1, (A,E)_1) \leq (x_2, (A,E)_2)$ if $(A,E)_2 \tilde{\subseteq} (A,E)_1$. It is clear that ≤ is reflexive and transitive and hence D is a poset. Now we define f: D→X as $f((x, (A,E))) = x$. It is clear that f is a generalized soft net over X. Hence every generalized soft filter induces a generalized soft net over X.
Theorem (3.4): Let $\tilde{F}$ be a generalized soft filter in (X, τ,E), a generalized soft topological space. Let f be the induced generalized soft net. Then $\tilde{F}$ →x if and only f→x.
Proof: $\tilde{F}$ is a generalized soft filter. D = {(x, (A,E)) ; x $\tilde{\in}$ (A,E) $\in \tilde{F}$ }. $(x_1, (A,E)_1) \leq (x_2, (A,E)_2)$ if $(A,E)_2 \tilde{\subseteq} (A,E)_1$. Consider f: D→X defined as f ((x, (A,E))=x. f is the generalized soft net induced by the generalized soft filter $\tilde{F}$. Now suppose $\tilde{F}$ →x, we want

to prove that $f \rightarrow$x. Let (U,E) be a neighborhood of x. Since $\tilde{F} \rightarrow$x, (U,E)$\in \tilde{F}$ and hence (x, (U,E))$\in$ D. Now (y, (A,E))$\geq$(x, (U,E)) implies (A,E) $\tilde{\subseteq}$ (U,E) and hence f(y, (A,E)) = y $\tilde{\in}$ (A,E) $\tilde{\subseteq}$ (U,E). Hence $\exists$(x,(U,E))$\in$ D such that (y, (A,E))$\geq$(x, (U,E)) implies f(y, (A,E)) $\tilde{\in}$ (U,E). Hence the generalized soft net $f \rightarrow$x.

Conversely, Suppose $f \rightarrow$x, we have to prove that A$\rightarrow$x. Let (U,E) be a neighborhood of *x*. Since $f \rightarrow$x, $\exists$(y, (A,E))$\in$D such that f(z, (B,E)) $\tilde{\in}$ (U,E) for all (z, (B,E))$\geq$(y, (A,E)). Hence z $\tilde{\in}$ (U,E) for all z $\tilde{\in}$ (B,E) which implies (B,E) $\tilde{\subseteq}$ (U,E), (z, (B,E))$\in$D , hence (B,E)$\in \tilde{F}$ which implies (U,E)$\in \tilde{F}$. Hence $\tilde{F} \rightarrow$x.

Conclusions

The soft set theory of Molodtsov [3] offers a general mathematical tool for dealing with uncertain, fuzzy, or vague objects. Molodtsov in [3] has give several possible applications of soft set theory. In this paper, we define the notions of generalized soft filter by using soft sets on an universal set. Also, we investigate their relationships with concepts of filter . We hope that the findings in this paper will help researcher enhance and promote the further study on soft set theory to carry out a general framework for their applications.

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References

[1]. S. Hussain and B. Ahmad, Some properties of soft topological spaces, Comput. Math. Appl. 62, (2011) 4058–4067.

[2]. W. K. Min, A note on soft topological spaces, Comput. Math. Appl. 62, (2011) 3524–3528.

[3]. I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform. 3(2), (2012) 171–185.