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Neighbourhood Edge Product Cordial and Total Neighbourhood Edge Product Cordial Labeling of Graphs

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Abstract

In this paper, we introduce the concepts neighbourhood edge product cordial labeling and total neighbourhood edge product cordial labeling of graph and present the neighbourhood edge product cordial labeling of Paths, kG, $G_1 \cup G_2$,$K_{1,n,n} \cup W_n \cup C_m$, $K_{1,n,n} \cup W_n \cup C_m$ under some conditions. Finally, we investigate the total neighbourhood edge product cordial labeling of Paths under some conditions and $(P_n+2K_1) \cup K_{1,n,n}$.

Keywords: *Edge product cordial graph, Total Edge product cordial graph, Neighbourhood edge product cordial graph, Total neighbourhood edge product cordial graph.*

1. Introduction

By a graph G, we mean a finite, connected, undirected graph without loops and multiple edges, suppose graph G is disconnected means each component of G must contain at least one edge, for terms not defined here, we refer to Harary [3]. For standard terminology and notations related to graph labeling, we refer to Gallian [2]. The concept of edge product cordial labeling of graphs is introduced by Vaidya et al. [5]. In [1], Cahit introduced the concept of cordial labeling of graph. In [7], Vaidya et al. introduced the concept of edge product cordial labeling of graph. The concept of total edge product cordial labeling is introduced by Vaidya et al. [8]. In [4,5], Muthaiyan et. al., introduced the concepts of neighbourhood cordial labeling, total neighbourhood cordial labeling, neighbourhood product cordial labeling and total neighbourhood product cordial labeling of graph. Muthaiyan et. al., introduced the concepts of neighbourhood E-cordial labeling and total neighbourhood E-cordial labeling of graph in [6]. Motivated by the study of edge product cordial labeling and edge neighbourhood concept in Graph Theory, we introduce the new concepts neighbourhood edge product cordial labeling and total neighbourhood edge product cordial labeling of graph. The brief summaries of definitions which are necessary for the present investigation are provided below.

Definition :1.1

The set of all vertices adjacent to a vertex v is called the neighbourhood of the vertex v and it is denoted by N(v).

Definition :1.2

The set of all edges adjacent to a edge e is called the neighbourhood of the edge e and it is denoted by N(e).

Definition :1.3

A complete bipartite graph $K_{1,n}$ is called a star and it has n+1 vertices and n edges. $K_{1,n,n}$ is the graph obtained by the subdivision of the edges of the star $K_{1,n}$.

Definition :1.4

The join of two graphs G_1 and G_2 is a graph G_1+G_2 with $V(G_1+G_2) = V(G_1) \cup V(G_2)$ and $E(G_1+G_2) = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1) \text{ and } v \in V(G_2)\}$.

Definition :1.5

The graph $C_n + K_1$ is called a wheel with n spokes and is denoted by W_n.

Definition :1.6

The graph $P_n + 2K_1$ is called the double fan.

Definition :1.7

A graph labeling is the assignment of unique identifiers to the edges and vertices of a graph.

Definition :1.8

A mapping $f : V(G) \rightarrow \{0,1\}$ is called binary vertex labeling of G and f(v) is called the label of the vertex v of G under f. If for an edge e = uv, the induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ is given by $f^*(e) = | f(u) - f(v) |$. Then $v_f(i)$ = number of vertices of having label i under f and $e_f(i)$ = number of edges of having label i under f^* for i = 0,1. A binary vertex labeling f of a graph G is called a cordial labeling if $| v_f(0) - v_f(1) | \leq 1$ and $| e_f(0) -$

$e_f(1)$ | ≤ 1. A graph G is cordial if it admits cordial labeling.

Definition :1.9

For graph G, the edge labeling function is defined as $f : E(G) \rightarrow \{0,1\}$ and induced vertex labeling function $f^* : V(G) \rightarrow \{0,1\}$ is given as if $e_1, e_2, \ldots, e_n$ are the edges incident to vertex v, then $f^*(v) = f(e_1)\ f(e_2) \ldots f(e_n)$ and $v_f(i)$ is the number of vertices of G having label i under f^*, $e_f(i)$ is the number of edges of G having label i under f for i = 0,1. f is called edge product cordial labeling of graph G if $| v_f(0) - v_f(1) | \leq 1$ and $| e_f(0) - e_f(1) | \leq 1$. A graph G is called edge product cordial if it admits edge product cordial labeling.

Definition :1.10

For a graph G, an edge labeling function $f^* : E(G) \rightarrow \{0, 1\}$ induces a vertex labeling function $f : V(G) \rightarrow \{0,1\}$ defined as $f(v) = \Pi\{f^*(uv)/uv\in E(G)\}$ and $v_f(i)$ is the number of vertices of having label i under f, $e_f(i)$ is the number of edges of having label i under f^* for i = 0,1. The function f^* is called a total edge product cordial labeling of G if $|(v_f(0)+e_f(0)) - (v_f(1)+e_f(1))| \leq 1$. A graph is called total edge product cordial if it admits total edge product cordial labeling in G.

Definition :1.11

An edge product cordial labeling f of a graph G is called a neighbourhood edge product cordial labeling if for every edge $e\in E(G)$, then all the edges adjacent to the edge e have the same label, $|e_f(0) - e_f(1)| \leq 1$ and $|v_f(0) - v_f(1)| \leq 1$. A graph G is neighbourhood edge product cordial if it admits neighbourhood edge product cordial labeling.

Definition :1.12

A total edge product cordial labeling f of a graph G is called a total neighbourhood E-cordial labeling if for every edge $e \in E(G)$, then all the edges adjacent to the edge e have the same label and $|f(0)–f(1)|$, where f(i) denotes sum of the number of vertices and the number of edges labeled with i (i = 0,1). A graph G is total neighbourhood edge product cordial if it admits total neighbourhood edge product cordial labeling.

Definition :1.13

In a graph G, if every edge of G is labeled by 0, then it is called 0 - type edge labeling of graph G.

Definition :1.14

In a graph G, if every edge of G is labeled by 1, then it is called 1 - type edge labeling of graph G.

Definition :1.15

In a graph G, if the edges of G are labeled by 0 or 1, then it is called 01 - type edge labeling of graph G.

Observations : 1.16

(i). If G is a neighbourhood edge product cordial graph, then 0 - type edge labeling or 1- type edge labeling is suitable edge labeling for any component of G.

(ii). If G is a neighbourhood edge product cordial graph, then 01 - type edge labeling is suitable edge labeling for the component, which is either path or cycle of even vertices.

(iii). If G is a neighbourhood edge product cordial graph, then 0 - type edge labeling on any one of the component of G, induces each vertex label on this component as 0.

(iv). If G is a neighbourhood edge product cordial graph, then 1 - type edge labeling on any one of the component of G, induces each vertex label on this component as 1.

(v). If G is a neighbourhood edge product cordial graph G and one of the component is path, then 01 - type edge labeling on path, induces each non pendent vertex label as 0 and pendent vertex label as either 0 or 1.

(vi). If G is a neighbourhood edge product cordial graph G and one of the component is cycle of even vertices, then 01-type edge labeling on cycle of even vertices, induces each vertex label on this component as 0.

2. Main Results

Theorem 2.1

A connected graph G is neighbourhood edge product cordial graph if and only if G is either P_3 or P_4.

Proof.

Let n and m be the number of vertices and edges of a connected graph G. Let G be a neighbourhood edge product cordial graph.

Then G must have 01- type edge labeling. Since G has only one component. This implies G is either path or cycle of even vertices.

Suppose G is cycle of even vertices. Then $v_f(0) = n$ and $| v_f(0) - v_f(1) | = n$, which is contradiction to our assumption.

Therefore G is path and it must have 01- type edge labeling. This implies $| v_f(0) - v_f(1) | \leq 1$ for n = 3, 4 and $| v_f(0) - v_f(1) | > 2$ for all other n. Hence G is P_3 or P_4.

Conversely

Assume that the connected graph G is either P_3 or P_4.

Suppose G is P_3.

Let v_1, v_2, v_3 and e_1, e_2 be the vertices and edges of G respectively.

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Define edge labeling $f : E(G) \rightarrow \{0,1\}$ as follows

$f(e_1) = 0$
$f(e_2) = 1$

The induced vertex labels are

$f^*(v_1) = f^*(v_2) = 0$
$f^*(v_3) = 1$

Then, we have $|e_f(0) - e_f(1)| \leq 1$ and $|v_f(0) - v_f(1)| \leq 1$.

Therefore P_3 is neighbourhood edge product cordial graph.

Suppose G is P_4. Let v_1, v_2, v_3, v_4 and e_1, e_2, e_3 be the vertices and edges of G respectively.

Define edge labeling $f : E(G) \rightarrow \{0,1\}$ as follows

$f(e_1) = f(e_3) = 1$
$f(e_2) = 0$

The induced vertex labels are

$f^*(v_1) = f^*(v_4) = 1$
$f^*(v_2) = f^*(v_3) = 0$

Then, we have $|e_f(0) - e_f(1)| \leq 1$ and $|v_f(0)–v_f(1)| \leq 1$. Therefore P_4 is neighbourhood edge product cordial graph.

Hence G is either P_3 or P_4, and then G is neighbourhood edge product cordial graph.

Theorem 2.2

Let G be connected graph, then the disconnected graph kG is a neighbourhood edge product cordial graph when (i) G is either P_3 or P_4 and any k and (ii) G is other than P_3 or P_4 and k is even.

Proof.

Let G be connected graph. Let n and m be the number of vertices and edges G. Let $v_1, v_2, ..., v_n$ and $e_1, e_2, ..., e_m$ be the vertices and edges G respectively.

Let $v_1,v_2,...,v_n$, $v_{n+1},v_{n+2},...,v_{2n}$, …, $v_{(k-1)n+1},v_{(k-1)n+2},..., v_{kn}$ and $e_1,e_2,...,e_m$, $e_{m+1},e_{m+2},...,e_{2m}$, …, $e_{(k-1)m+1},e_{(k-1)m+2},..., e_{km}$ be the vertices and edges kG respectively.

Define edge labeling $f : E(G) \rightarrow \{0,1\}$ we consider the following cases.

Case (i) : G is either P_3 or P_4 and k is even.

$f(e_i) = 1$ for $1 \leq i \leq \frac{km}{2}$.

$f(e_i) = 0$ for $\frac{km}{2}+1 \leq i \leq km$.

The induced vertex labels are

$f(v_i) = 1$ for $1 \leq i \leq \frac{kn}{2}$.

$f(v_i) = 0$ for $\frac{kn}{2}+1 \leq i \leq kn$.

Then, $e_f(0) = e_f(1) = \frac{km}{2}$ and $v_f(0) = v_f(1) = \frac{kn}{2}$.

Therefore$|\ e_f(0) - e_f(1)\ | \leq 1$ and $|\ v_f(0) - v_f(1)\ | \leq 1$.

Therefore, kG is a neighbourhood edge product cordial graph for k is even.

Case (ii) : G is either P_3 or P_4 and k is odd.

Subcase (i) : G is P_3 and k is odd.

$f(e_i) = 1$ for $1 \leq i \leq k–1$.
$f(e_i) = 0$ for $(k–1)+1 \leq i \leq 2(k–1)$.
$f(e_{2k–1}) = 1$
$f(e_{2k}) = 0$

The induced vertex labels are

$f(v_i) = 1$ for $1 \leq i \leq \frac{3(k-1)}{2}$.

$f(v_i) = 0$ for $\frac{3(k-1)}{2}+1 \leq i \leq 3(k–1)$.

$f(v_{3k–2}) = 1$
$f(v_{3k–1}) = f(v_{3k}) = 0$

Then, we have $e_f(0) = e_f(1) = k$, $v_f(0) = \frac{3k+1}{2}$ and $v_f(1) = \frac{3k-1}{2}$.

Therefore$|\ e_f(0) - e_f(1)\ | \leq 1$ and $|\ v_f(0) - v_f(1)\ | \leq 1$.

Therefore, kG is a neighbourhood edge product cordial graph for k is odd.

Subcase (i) : G is P_4 and k is odd.

$f(e_i) = 1$ for $1 \leq i \leq \frac{3(k-1)}{2}$.

$f(e_i) = 0$ for $\frac{3(k-1)}{2}+1 \leq i \leq 3(k–1)$.

$f(e_{3k–2}) = f(e_{3k}) = 1$
$f(e_{3k–1}) = 0$

The induced vertex labels are

$f(v_i) = 1$ for $1 \leq i \leq 2(k–1)$.
$f(v_i) = 0$ for $2(k–1)+1 \leq i \leq 4(k–1)$.
$f(v_{4k–3}) = f(v_{4k}) = 1$
$f(v_{4k–2}) = f(v_{4k–1}) = 0$

Then, we have $e_f(0) = \frac{3k-1}{2}$, $e_f(1) = \frac{3k+1}{2}$ and $v_f(0) = v_f(1) = 2k$.

Therefore$|\ e_f(0) - e_f(1)\ | \leq 1$ and $|\ v_f(0) - v_f(1)\ | \leq 1$.

Therefore, kG is a neighbourhood edge product cordial graph for k is odd.

Case (iii) : G is other than P_3 or P_4 and k is even.

$f(e_i) = 1$ for $1 \leq i \leq \frac{km}{2}$.

$f(e_i) = 0$ for $\frac{km}{2}+1 \leq i \leq km$.

The induced vertex labels are

$f(v_i) = 1$ for $1 \leq i \leq \frac{kn}{2}$.

$f(v_i) = 0$ for $\frac{kn}{2}+1 \leq i \leq kn$.

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Then, $e_f(0) = e_f(1) = \frac{km}{2}$ and $v_f(0) = v_f(1) = \frac{kn}{2}$.

Therefore $|e_f(0) - e_f(1)| \le 1$ and $|v_f(0) - v_f(1)| \le 1$.

Therefore, kG is a neighbourhood edge product cordial graph for k is even.

Case (iv) : G is other than P_3 or P_4 and k is odd.

Subcase (i) : Suppose G is any cycle of even order.

Then any suitable neighbourhood edge product cordial labeling will produce $\frac{(k+1)n}{2}$ vertices with label 0 and $\frac{(k-1)n}{2}$ vertices with label 1.

Therefore $|v_f(1) - v_f(0)| = n$. Thus the vertex condition for neighbourhood edge product cordial graph is violated.

Hence, kG is not a neighbourhood edge product cordial graph, when k is odd.

Subcase (ii) : Suppose G is any path of length n, $n \ne 3,4$ and $n \ge 2$.

For n = 2.

Any suitable neighbourhood edge product cordial labeling will produce either $\frac{(k-1)n}{2} + 2$ vertices with label 0 and $\frac{(k-1)n}{2}$ vertices with label 1 or $\frac{(k-1)n}{2}$ vertices with label 0 and $\frac{(k-1)n}{2} + 2$vertices with label 1.

Therefore $|v_f(1) - v_f(0)| = 2$. Thus the vertex condition for neighbourhood edge product cordial graph is violated.

For $n \ge 5$.

Any suitable neighbourhood edge product cordial labeling will produce atleast $\frac{(k-1)n}{2}+(n-2)$ vertices with label 0 and atmost $\frac{(k-1)n}{2}+2$ vertices with label 1.

Therefore $|v_f(1) - v_f(0)| \ge 2$. Thus the vertex condition for neighbourhood edge product cordial labeling of G is violated. Hence, kG is not a neighbourhood edge product cordial graph for k is odd.

Subcase (iii) : Suppose G is any graph other than cycle of even length and any path of length n.

Then G is not suitable for 01- type edge labeling.

Therefore the edge condition for neighbourhood edge product cordial labeling of G is violated. Hence kG is not a neighbourhood edge product cordial graph for k is odd.

Example 2.1

The graph $4K_{1,5}$ and its neighbourhood edge product cordial labeling is shown in figure 2.1.

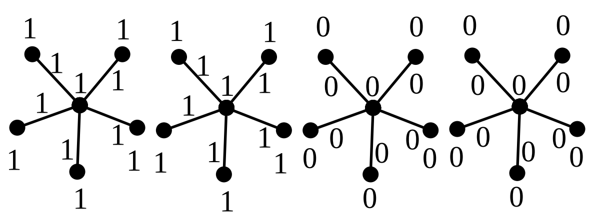

Figure 2.1

Example 2.2

The disconnected graph $5P_4$ and its neighbourhood edge product cordial labeling is shown in Figure 2.2.

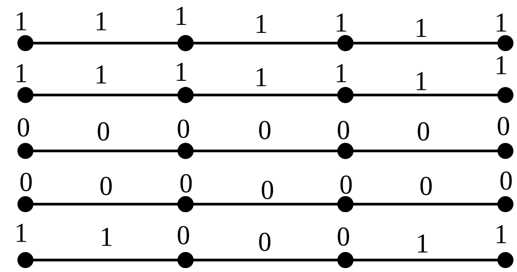

Figure 2.2

Theorem 2.3

Let n_1 and n_2 be the number of vertices and m_1 and m_2 be the number of edges of connected graphs G_1 and G_2 respectively, $|n_1 - n_2| \le 1$ and $|m_1 - m_2| \le 1$, then $G_1 \cup G_2$ is neighbourhood edge product cordial graph.

Proof.

Let G_1 and G_2 be two connected graphs with n_1 and n_2 number of vertices and m_1 and m_2 number of edges respectively. Let $e_1, e_2, ..., e_{m_1}$ and $c_1, c_2, ..., c_{m_2}$ be the edges of G_1 and G_2 respectively. Let $u_1, u_2, ..., u_{n_1}$ and $v_1, v_2, ..., v_{n_2}$ be the vertices of G_1 and G_2 respectively.

Define edge labeling $f : E(G) \to \{0,1\}$ as follows

$f(e_i) = 1$ for $1 \le i \le m_1$.

$f(c_i) = 0$ for $1 \le i \le m_2$.

The induced vertex labels are

$f^*(u_i) = 1$ for $1 \le i \le n_1$.

$f^*(v_i) = 0$ for $1 \le i \le n_2$.

Then, we have $e_f(1) = m_1$, $e_f(0) = m_2$, $v_f(1) = n_1$ and $v_f(0) = n_2$. Here $|n_1 - n_2| \le 1$ and $|m_1 - m_2| \le 1$implies that $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Therefore $G_1 \cup G_2$ is a neighbourhood edge product cordial graph.

Example 2.3

The graph $K_{1,5} \cup C_5$ and its neighbourhood edge product cordial labeling is shown in figure 2.3.

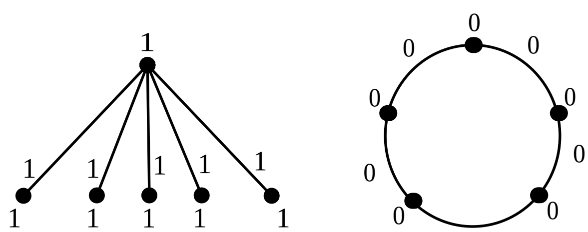

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Figure 2.3

Theorem 2.4

The disconnected graph $K_{1,n,n} \cup W_n \cup C_m$ is neighbourhood edge product cordial graph, when (i). n is odd and m = n–1, n+1 and (ii). n is even and m = n, m ≥ 3

Proof.

Let G be the disconnected graph $K_{1,n,n} \cup W_n \cup C_m$.

Let v, $v_1, v_2, \ldots, v_n, v_{n+1}, v_{n+2}, \ldots, v_{2n}$, w, $w_1, w_2, \ldots, w_n$, and $u_1, u_2, \ldots, u_m$ be the vertices of $K_{1,n,n}$, W_n and C_m respectively.

Let $e_1, e_2, \ldots, e_{2n}, e_{2n+1}, e_{2n+2}, \ldots, e_{4n}$, and $e_{4n+1}, e_{4n+2}, \ldots, e_{4n+m}$ be the vertices of $K_{1,n,n}$, W_n and C_m respectively.

Then |V(G)| = 3n+m+2 and |E(G)| = 4n+m.

Case (i) : n is odd and m = n–1

Define edge labeling f : E(G) → {0,1} as follows.

$f(e_i) = 1$ for $1 \le i \le 2n$,
$f(e_{2n+i}) = 0$ for $1 \le i \le 2n$.
$f(e_{4n+2i-1}) = 1$ for $1 \le i \le \frac{m}{2}$.
$f(e_{4n+2i}) = 0$ for $1 \le i \le \frac{m}{2}$.

The induced vertex labels are

$f^*(v) = 1$
$f^*(v_i) = 1$ for $1 \le i \le 2n$,
$f^*(w) = 0$
$f^*(w_i) = 0$ for $1 \le i \le n$,
$f^*(u_i) = 0$ for $1 \le i \le m$

In view of the above labeling pattern we have, $e_f(1) = e_f(0) = 2n + \frac{m}{2}$, $v_f(1) = 2n+1$ and $v_f(0) = n+m+1 = 2n$.

Therefore, $| e_f(0) - e_f(1) | \le 1$ and $| v_f(0) - v_f(1) | \le 1$.

Hence G is a neighbourhood edge product cordial graph.

Case (ii) : n is odd and m = n+1

Define edge labeling f : E(G) → {0,1} as follows.

$f(e_i) = 1$ for $1 \le i \le 2n$,
$f(e_{2n+i}) = 0$ for $1 \le i \le 2n$.
$f(e_{4n+2i-1}) = 1$ for $1 \le i \le \frac{m}{2}$.
$f(e_{4n+2i}) = 0$ for $1 \le i \le \frac{m}{2}$.

The induced vertex labels are

$f^*(v) = 1$
$f^*(v_i) = 1$ for $1 \le i \le 2n$,
$f^*(w) = 0$
$f^*(w_i) = 0$ for $1 \le i \le n$,
$f^*(u_i) = 0$ for $1 \le i \le m$

In view of the above labeling pattern we have, $e_f(1) = e_f(0) = 2n + \frac{m}{2}$, $v_f(1) = 2n$ and $v_f(0) = n+m+1 = 2n+2$.

Therefore, $| e_f(0) - e_f(1) | \le 1$ and $| v_f(0) - v_f(1) | \le 1$.

Hence G is a neighbourhood edge product cordial graph.

Case (iii) : n is even and m = n

Define edge labeling f : E(G) → {0,1} as follows.

$f(e_i) = 1$ for $1 \le i \le 2n$,
$f(e_{2n+i}) = 0$ for $1 \le i \le 2n$.
$f(e_{4n+2i-1}) = 1$ for $1 \le i \le \frac{m}{2}$.
$f(e_{4n+2i}) = 0$ for $1 \le i \le \frac{m}{2}$.

The induced vertex labels are

$f^*(v) = 1$
$f^*(v_i) = 1$ for $1 \le i \le 2n$,
$f^*(w) = 0$
$f^*(w_i) = 0$ for $1 \le i \le n$,
$f^*(u_i) = 0$ for $1 \le i \le m$

In view of the above labeling pattern we have, $e_f(1) = e_f(0) = 2n + \frac{m}{2}$, $v_f(1) = 2n+1$ and $v_f(0) = n+m+1 = 2n+1$.

Therefore, $| e_f(0) - e_f(1) | \le 1$ and $| v_f(0) - v_f(1) | \le 1$.

Hence G is neighbourhood edge product cordial graph.

From the above cases, G is a neighbourhood edge product cordial graph, when (i). n is odd and m = n–1, n+1 and (ii). n is even and m = n, m ≥ 3

Example 2.4

The graph $K_{1,5,5} \cup W_{,5} \cup C_4$ and its neighbourhood edge product cordial labeling is shown in figure 2.4.

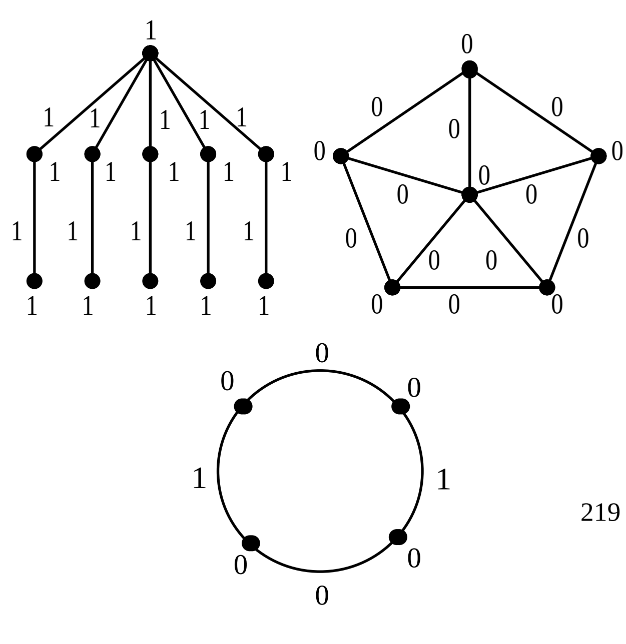

Figure 2.4

Theorem 2.5

The disconnected graph $K_{1,n,n} \cup W_n \cup P_m$ is neighbourhood Edge product cordial graph, when (i). n is odd and m = n–1,n and (ii). n is even and m = n, n+1, $n \geq 3$

Proof.

Let G be the disconnected graph $K_{1,n,n} \cup W_n \cup P_m$.

Let $v, v_1, v_2, ..., v_n, v_{n+1}, v_{n+2}, ..., v_{2n}, w, w_1, w_2, ..., w_n$, and $u_1, u_2, ..., u_m$ be the vertices of $K_{1,n,n}$, W_n and P_m respectively.

Let $e_1, e_2, ..., e_{2n}, e_{2n+1}, e_{2n+2}, ..., e_{4n}$, and $e_{4n+1}, e_{4n+2}, ..., e_{4n+m-1}$ be the vertices of $K_{1,n,n}$, W_n and P_m respectively.

Then |V(G)| = 3n+m+2 and |E(G)| = 4n+m–1.

Case (i) : n is odd and m = n–1

Define edge labeling $f : E(G) \rightarrow \{0,1\}$ as follows.

$f(e_i) = 1$ for $1 \leq i \leq 2n$,
$f(e_{2n+i}) = 0$ for $1 \leq i \leq 2n$.
$f(e_{4n+2i-1}) = 0$ for $1 \leq i \leq \frac{m}{2}$.
$f(e_{4n+2i}) = 1$ for $1 \leq i \leq \frac{m-2}{2}$.

The induced vertex labels are

$f^*(v) = 1$
$f^*(v_i) = 1$ for $1 \leq i \leq 2n$,
$f^*(w) = 0$
$f^*(w_i) = 0$ for $1 \leq i \leq n$,
$f^*(u_i) = 0$ for $1 \leq i \leq m$

In view of the above labeling pattern we have, $e_f(1) = 2n+\frac{m}{2}$, $e_f(0) = 2n+\frac{m}{2}-1$, $v_f(1) = 2n+1$ and $v_f(0) = n+m+1 = 2n$.

Therefore, $| e_f(0) – e_f(1) | \leq 1$ and $| v_f(0) – v_f(1) | \leq 1$.

Hence G is neighbourhood edge product cordial graph.

Case (ii) : n is odd and m = n

Define edge labeling $f : E(G) \rightarrow \{0,1\}$ as follows.

$f(e_i) = 1$ for $1 \leq i \leq 2n$,
$f(e_{2n+i}) = 0$ for $1 \leq i \leq 2n$.
$f(e_{4n+2i-1}) = 0$ for $1 \leq i \leq \frac{m-1}{2}$.
$f(e_{4n+2i}) = 1$ for $1 \leq i \leq \frac{m-1}{2}$.

The induced vertex labels are

$f^*(v) = 1$
$f^*(v_i) = 1$ for $1 \leq i \leq 2n$,
$f^*(w) = 0$
$f^*(w_i) = 0$ for $1 \leq i \leq n$,
$f^*(u_i) = 0$ for $1 \leq i \leq m–1$
$f^*(u_m) = 0$

In view of the above labeling pattern we have,

$e_f(1) = e_f(0) = 2n+\frac{m-1}{2}$, $v_f(1) = 2n+1$ and $v_f(0) = n+m+1 = 2n+1$

Therefore, $| e_f(0) – e_f(1) | \leq 1$ and $| v_f(0) – v_f(1) | \leq 1$.

Hence G is neighbourhood edge product cordial graph.

Case (iii) : n is even and m = n

Define edge labeling $f : E(G) \rightarrow \{0,1\}$ as follows.

$f(e_i) = 1$ for $1 \leq i \leq 2n$,
$f(e_{2n+i}) = 0$ for $1 \leq i \leq 2n$.
$f(e_{4n+2i-1}) = 0$ for $1 \leq i \leq \frac{m}{2}$.
$f(e_{4n+2i}) = 1$ for $1 \leq i \leq \frac{m-2}{2}$.

The induced vertex labels are

$f^*(v) = 1$
$f^*(v_i) = 1$ for $1 \leq i \leq 2n$,
$f^*(w) = 0$
$f^*(w_i) = 0$ for $1 \leq i \leq n$,
$f^*(u_i) = 0$ for $1 \leq i \leq m$

In view of the above labeling pattern we have, $e_f(1) = 2n+\frac{m}{2}$, $e_f(0) = 2n+\frac{m-2}{2}$, $v_f(1) = m+n = 2n+2$ and $v_f(0) = 2n+2$. Therefore, $|e_f(0)–e_f(1)| \leq 1$ and $| v_f(0) – v_f(1) | \leq 1$.

Hence G is neighbourhood Edge product cordial graph.

Case (iv) : n is even and m = n+1

Define edge labeling $f : E(G) \rightarrow \{0,1\}$ as follows.

$f(e_i) = 1$ for $1 \leq i \leq 2n$,
$f(e_{2n+i}) = 0$ for $1 \leq i \leq 2n$.
$f(e_{4n+2i-1}) = 0$ for $1 \leq i \leq \frac{m-1}{2}$.
$f(e_{4n+2i}) = 1$ for $1 \leq i \leq \frac{m-1}{2}$.

The induced vertex labels are

$f^*(v) = 1$
$f^*(v_i) = 1$ for $1 \leq i \leq 2n$,
$f^*(w) = 0$
$f^*(w_i) = 0$ for $1 \leq i \leq n$,
$f^*(u_i) = 0$ for $1 \leq i \leq m–1$
$f^*(u_m) = 1$

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In view of the above labeling pattern we have, $e_f(1) = e_f(0) = 2n+\frac{m-1}{2}$, $v_f(1) = 2n+2$ and $v_f(0) = n+m = 2n+1$.

Therefore, $| e_f(0) – e_f(1) | \leq 1$ and $| v_f(0) – v_f(1) | \leq 1$.

Hence G is neighbourhood edge product cordial graph.

From the above cases, G is a neighbourhood edge product cordial graph, when (i). n is odd and m = n–1, n and (ii). n is even and m = n,n+1, $n \geq 3$.

Example 2.5

The graph $K_{1,5,5} \cup W_{,5} \cup P_6$ and its neighbourhood Edge product cordial labeling is shown in figure 2.5.

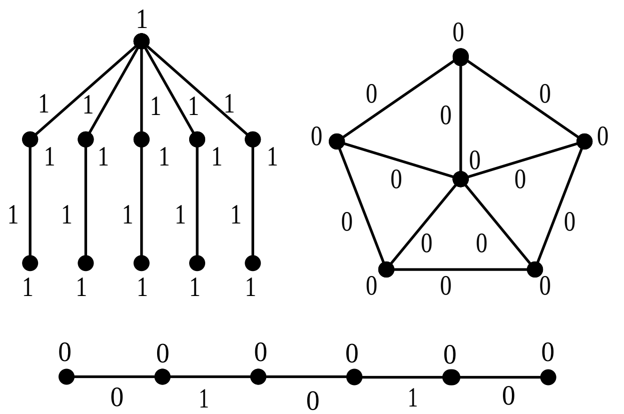

Figure 2.5

Theorem 2.6

A connected graph G is a total neighbourhood edge product cordial graph if and only if G is P_3 or P_4 or P_6.

Proof.

Let n and m be the number of vertices and edges of a connected graph G. Let G be a total neighbourhood edge product cordial graph.

Then G must have 01- type edge labeling. Since G has only one component. This implies G is either path or cycle of even vertices.

Suppose G is cycle of even vertices. Then $f(0) = n+\frac{m}{2}$ and $f(1) = \frac{m}{2}$. This implies $| f(0) – f(1) | = n$, which is contradiction to our assumption.

Therefore G is path and it must have 01- type edge labeling. This implies $| f(0) – f(1) | \leq 1$ for n = 3, 4, 6 and $|f(0) – f(1) | > 2$ for all other n.

Hence G is G is P_3 or P_4 or P_6.

Conversely

Assume that the connected graph G is P_3 or P_4 or P_6.

Suppose G is P_3.

By theorem 2.1, f(0) = f(1) + 1. This implies P_3 is total neighbourhood edge product cordial graph.

Suppose G is P_4.

By theorem 2.1, f(1) = f(0) + 1. This implies P_4 is total neighbourhood edge product cordial graph.

Suppose G is P_6.

Let $v_1, v_2, …, v_6$, and $e_1, e_2, …, e_6$ be the vertices and edges of G respectively.

Define edge labeling $f : E(G) \to \{0,1\}$ as follows

$f(e_i) = 1$ for i is odd
$f(e_i) = 0$ for i is even

The induced vertex labels are

$f^*(v_i) = 1$ for i = 1,6
$f^*(v_i) = 0$ for i = 2,3,4,5

Then, we have f(0) = 6 and f(1) = 5 and $|f(0)–f(1)| \leq 1$. Therefore P_6 is total neighbourhood edge product cordial graph.

Hence G is P_3 or P_4 or P_6, then G is a total neighbourhood edge product cordial graph.

Theorem 2.7

The disconnected graph $(P_n+2K_1)\cup K_{1,n,n}$ is total neighbourhood edge product cordial graph, $n \geq 3$.

Proof.

Let G be the disconnected graph $(P_n+2K_1) \cup K_{1,n,n}$.

Let u, w, $w_1, w_2, ..., w_n$ and v, $v_1, v_2, ..., v_{2n}$ be the vertices of P_n+2K_1 and $K_{1,n,n}$ respectively.

Let $e_1, e_2, ..., e_{3n–1}$ and $e_{3n}, e_{3n+1}, ..., e_{5n–1}$ be the edges of (P_n+2K_1) and $K_{1,n,n}$ respectively.

Then $|V(G)| = 3n+3$ and $|E(G)| = 5n–1$.

Define edge labeling $f : E(G) \to \{0,1\}$ as follows.

$f(e_i) = 1$ for $1 \leq i \leq 3n–1$,
$f(e_i) = 0$ for $3n \leq i \leq 5n–1$.

The induced vertex labels are

$f^*(u) = 1$
$f^*(w) = 1$
$f^*(w_i) = 1$ for $1 \leq i \leq n$,
$f^*(v) = 0$
$f^*(v_i) = 0$ for $1 \leq i \leq 2n$,

In view of the above labeling pattern we have, $e_f(1) = 3n–1$, $e_f(0) = 2n$, $v_f(1) = n+2$ and $v_f(0) = n+1$.

Thus f(0) = f(1) = 4n+1.

Therefore, $|f(0) – f(1) | \leq 1$.

Hence G is a total neighbourhood edge product cordial graph, $n \geq 2$.

Example 2.6

The graph $(P_5+2K_1)\cup K_{1,5,5}$ and its total neighbourhood Edge product cordial labeling is shown in figure 2.6.

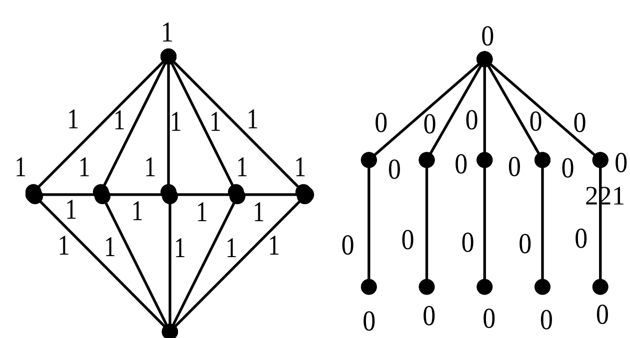

Figure 2.6

Conclusions

In this paper, the concepts neighbourhood edge product cordial labeling and total neighbourhood edge product cordial labeling of graph are introduced. The neighbourhood edge product cordial labeling of Paths, kG, $G_1 \cup G_2$, $K_{1,n,n} \cup W_n \cup C_m$, $K_{1,n,n} \cup W_n \cup C_m$ under some conditions are presented. Finally, the total neighbourhood edge product cordial labeling of Paths under some conditions and $(P_n+2K_1) \cup K_{1,n,n}$ are investigated.

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