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Bases of piecewise-liner economic-mathematical models with regard to influence of unaccounted factors in Finite-dimensional vector space

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Abstract

For the last 15 years in periodic literature there has appeared a series of scientific publications that has laid the foundation of a new scientific direction on creation of piecewise-linear economic-mathematical models at uncertainty conditions in finite dimensional vector space.

Representation of economic processes in finite-dimensional vector space, in particular in Euclidean space, at uncertainty conditions in the form of mathematical models in connected with complexity of complete account of such important issues as: spatial inhomogeneity of occurring economic processes, incomplete macro, micro and social-political information; time changeability of multifactor economic indices, their duration and their change rate.

The above-listed one in mathematical plan reduces the solution of the given problem to creation of very complicated economic-mathematical models of nonlinear type.

In this connection, it was established in these works that all possible economic processes considered with regard to uncertainty factor in finite-dimensional vector space should be explicitly determined in spatial-time aspect. Owing only to the stated principle of spatial-time certainty of economic process at uncertainty conditions in finite

dimensional vector space it is possible to reveal systematically the dynamics and structure of the occurring process. In addition, imposing a series of softened additional conditions on the occurring economic process, it is possible to classify it in finite-dimensional vector space and also to suggest a new science-based method of multivariant prediction of economic process and its control in finite-dimensional vector space at uncertainty conditions, in particular, with regard to unaccounted factors influence.

Keywords: Finite-dimensional vector space; Unaccounted factors; Principle of certainty of economic process in fine-dimensional space; Multi alternative forecasting; Principle of spatial-time certainty of economic process at uncertainty conditions in fine-dimensional space; Piecewise-linear economic-mathematical models in view of the factor of uncertainty in finite-dimensional vector space.

I. *Principle of certainty of economic process in fine-dimensional space*

Representation of economic problems in finite-dimensional vector space, in particular, in Euclidean space, at uncertainty conditions in the form of mathematical models is connected with difficulty of complete consideration of such important problems as: spatial inhomogeneity of the occurring economic processes; incomplete macro, micro and social-political information; time changeability of multifactor economic indices, their duration and velocity of their change.

The above-listed ones in mathematical plan the solution of this problem to creation of very complex economic-mathematical models of nonlinear form.

In this connection, all possible economic processes considered with regard to uncertainty factor in a finite-dimensional vector space should be determined in spatial-time aspect. Owing only to the formulated principle of spatial-time certainty of the economic process at uncertainty conditions in finite-dimensional vector space, the dynamics and structure of the occurring process may be revealed systematically. Besides, supposing a number of softened additional conditions on the occurring process, it is possible to classify it in finite-dimensional vector space and to suggest a new science based method of multivariant prediction of economic process and its control in finite-dimensional vector space at uncertainty conditions, in particular, with regard to influence of disregarded factors. In connection with above-stated, below formulate the postulate spatial-time certainty of economic process at uncertainty conditions in finite-dimensional vector space.”

II. *Principle of spatial-time certainty of economic process at uncertainty conditions in fine-dimensional space*

We assume that while investigating economic problems in spatial-time system, in finite-dimensional vector space, the occurring economic process possesses spatial inhomogeneity including a series of unaccounted factors of spatial form (especially, insufficient macro, micro and social-political information and etc.). This means that in different statistical points of finite-dimensional vector space, the nature of vector functions of the economic process will be different.

On the other hand, this process will be unstationary in time and this reflects changeability in time of multifactor economic indices and velocity of their change. However, at the point and small volume around the point $\Delta V_n(x_1, x_2, ... x_m)$ of finite-dimensional vector space we accept the economic process as homogeneous. This assumption allows us in the small volume $\Delta V(x_1, x_2, ... x_m)$ of finite-dimensional vector space to represent the economic process in the vector form as a -linear function. While passing from the points of one spatial vector -linear straight line to another spatial vector straight line, the occurring processes will be different by their homogeneity. Such a distinction will be a result of influence of above-mentioned unaccounted external factors that we call "influence functions of unaccounted parameters."

We will call such a basis the principle of "spatial-time certainty of the economic process at uncertainty conditions in finite-dimensional vector space".

Thus, assume that in finite-dimensional vector space, any economic process under consideration will be homogeneous if in the chosen small volume of the space $\Delta V_n(x_1, x_2, ... x_m)$ for a small time interval Δt the table of statistical data or experimental dependence of the vector function on spatial coordinates and time are obtained at the same external conditions, i.e. in the form:

$$\vec{z}_n = \lambda_n \vec{a}_n + \mu_n \vec{a}_{n+1}$$

This circumstance allows correspondence both between -homogeneous small volumes $\Delta V_n(x_1, x_2, ... x_m)$ and $\Delta V_{n+1}(x_1, x_2, ... x_m)$ of neighboring statistical points and changes of economic process occurring in the finite-dimensional vector space.

Mathematically, this means that in the case of homogeneous process, the -linear vector-function $\vec{z}_n$ at the point and at its small vicinity $\Delta V_n(x_1, x_2, ... x_m)$ is an analytic function. The derivatives in coordinates and also velocity of change of the vector function in small time interval will be constant.

Based on this principle,suggest a way for constructing -linear economic-mathematical models in finite-dimensional vector space with regard to influence of unaccounted factors [1-3,6,9-13].

III. *General method for constructing piecewise-linear economic-mathematical models with regard to influence of unaccounted factors in Finite-dimensional vector space*

Let the statistical table describing some economic process in the form of the points (vectors) set $\{\vec{a}_n\}$ of finite-dimensional vector space R_m be given the numbers a_{n1}, a_{n2}, a_{n3}, … a_{nm} be the coordinates of the point (of the vector) $\vec{a}_n$ or in the name the components of the vector. By means of the vectors $\vec{a}_n$ represent all the set of statistical points (vectors) in the space R_m in the vector form in the form of *n*-piecewise-linear equations of the form:

$$\vec{z}_n = \lambda_n \vec{a}_n + \mu_n \vec{a}_{n+1}, \qquad (\lambda_n,\ \mu_n \geq\ 0,\ \lambda_n + \mu_n = 1) \qquad (1)$$

Here λ_n and μ_n are arbitrary positive numbers referred to the *n*-piecewise-linear straight line, whose sums equals 1.

It the numbers λ_n and μ_n are nonnegative, the set of the vectors $\vec{z}_n$ of the Eq. (1) defines the equation of the *n*-th section in the vector form. In the case λ_n and μ_n of any sign the set of vectors $\vec{z}_n$ of the Eq. (1) will determine the equation of the *n*-th straight line in the vector form.

Write the vector equations of piecewise-linear straight lines Eq. (1) with regard to connections of parameters $\lambda_n + \mu_n = 1$ depending only on one parameter λ_n or μ_n in the form (Fig. 1.)

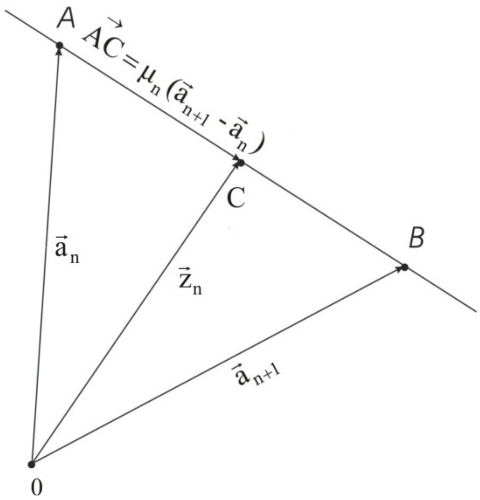

Fig. 1. Dependence of the points (vectors) $\vec{z}_n$ of the straight line AB on two points $\vec{a}_n$ and $\vec{a}_{n+1}$ in finite-dimensional vector space R_m.

$$\vec{z}_n = \vec{a}_n + \mu_n(\vec{a}_{n+1} - \vec{a}_n), \; 0 \le \mu_n \le 1 \tag{2}$$

or

$$\vec{z}_n = \vec{a}_{n+1} + \lambda_n(\vec{a}_n - \vec{a}_{n+1}), \qquad 0 \le \lambda_n \le 1 \tag{3}$$

It becomes clear from equality Eq. (2) that in three-dimensional space, the points $\vec{z}_n$ fill the segment connecting the points $\vec{a}_n$ and $\vec{a}_{n+1}$. This is seen from the fact that the radius-vector $\vec{z}_n$ is the sum of the vector $\vec{a}_n$ and the vector $\mu_n(\vec{a}_{n+1} - \vec{a}_n)$, collinear with the vector $(\vec{a}_{n+1} - \vec{a}_n)$. Thus, the points set Eq. (1) is the segment $[\vec{a}_n, \vec{a}_{n+1}]$ in the space R_3, connecting the points $\vec{a}_n$ and $\vec{a}_{n+1}$. For $\mu_n = 0$, $\vec{z}_n = \vec{a}_n$; for $\lambda = 0$, $\vec{z}_n = \vec{a}_{n+1}$; for any $\lambda_n > 0$ $(\mu_n = 1 - \lambda_n > 0)$ the point $\vec{z}_n$ is an arbitrary point of the segment $[\vec{a}_n, \vec{a}_{n+1}]$.

Note that in 3-dimensional space R_3 the set point:

$$\vec{z}_n = \lambda_n \vec{a}_n + \mu_n \vec{a}_{n+1}, \qquad \lambda_n + \mu_n = 1 \tag{4}$$

where λ_n and μ_n of any sign are piecewise-linear straight lines in the vector form, passing through the points $\vec{a}_n$ and $\vec{a}_{n+1}$.

In the space $R_m (m > 3)$ the points set Eq. (4) will be said a piecewise-linear straight line by definition.

Note that from the principle of spatial-time certainty of economic process at uncertainty conditions in finite-dimensional vector space it follows that in the small volume $\Delta V_n(x_1, x_2, \ldots x_m)$ around the point of finite-dimensional vector space each of the considered piecewise-linear vector functions $\vec{z}_n = \vec{z}_n[\omega_n(\lambda_n, \alpha_{n-1,n})]$ will be homogeneous by itself, i.e., the vector function $\vec{z}_n = \vec{z}_n[\omega_n(\lambda_n, \alpha_{n-1,n})]$ will be an analytic function, and the derivatives of this vector function in coordinates and time will be constant. And homogeneity degree of piecewise-linear vectors will be different [1-3,6,9-13].

This means that each piecewise-linear vector-function $\vec{z}_n$ is obtained by observing certain external factors that are inherent to the points of definite small volume $\Delta V_n(x_1, x_2, \ldots x_m)$ at time interval Δt_n.

Thus, economic events occurring in preceding small volumes ΔV_1, ΔV_2, …. $\Delta V_{n\text{-}1}$ will influence on the economic process in the small volume ΔV_n.

Now, using the principle of spatial-time certainty of economic process in finite-dimensional vector space we give a method for constructing the *n*-th piecewise-linear vector-function $\vec{z}_n = \vec{z}_n[\omega_n(\lambda_n, \alpha_{n\text{-}1,n})]$ depending on the first piecewise-linear vector equation and the cosines of the angles between the contiguous piecewise-linear vectors [1-3,6,9-13].

We begin our construction with the second piecewise-linear vector equation. For that we behave as follows. According to Eqs. (1), (2) and (3) we write a vector equation for the first piecewise linear straight line in the form:

$$\vec{z}_1 = \lambda_1 \vec{a}_1 + \mu_1 \vec{a}_2, \qquad \text{for } \lambda_1 + \mu_1 = 1 \tag{5}$$

or

$$\vec{z}_1 = \vec{a}_1 + \mu_1(\vec{a}_2 - \vec{a}_1) \tag{6}$$

or

$$\vec{z}_1 = \vec{a}_2 + \lambda_1(\vec{a}_1 - \vec{a}_2) \tag{7}$$

Here λ_1 and μ_1 are arbitrary numbers corresponding to the points of the first straight line (Fig. 2.). Furthermore, for $\mu_1 = 0$, $\vec{z}_1 = \vec{a}_1$; for $\mu_1 = 1$, $\vec{z}_1 = \vec{a}_2$.

By the fact that the equations of the first and second straight lines do not necessarily intersect at t he point $\vec{a}_2$, i.e., the intersection point of these straight lines are also not coincide with the point $\vec{a}_2$, therefore, denote the intersection point of the first and second piecewise-linear straight lines by $\vec{z}_1^{k_1} = \vec{z}_2^{k_2}$ (Fig. 2.).

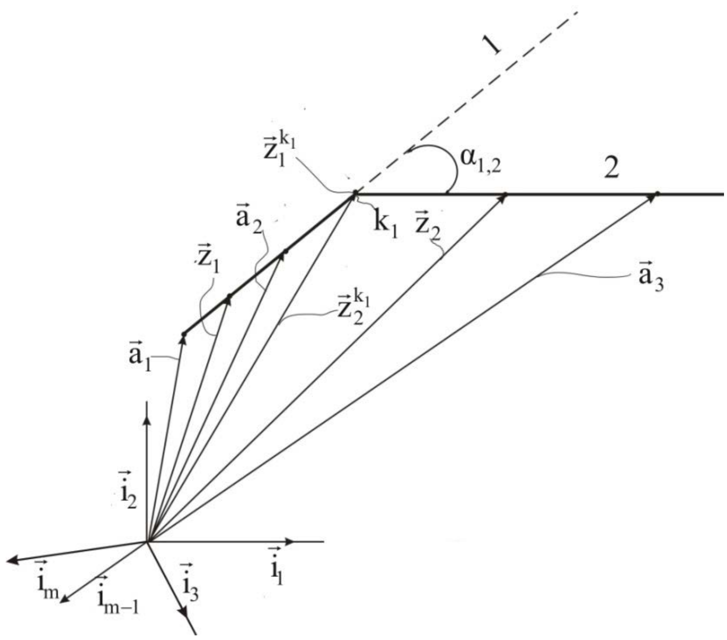

Fig. 2. Two-component piecewise-linear function of economic process in finite-dimensional vector space R_m.

Now, by means of the intersection point $\vec{z}_1^{k_1} = \vec{z}_2^{k_1}$ and arbitrarily given point $\vec{a}_3$, write a vector equation for the second straight line in the form:

$$\vec{z}_2 = \vec{z}_2^{k_1} + \mu_2(\vec{a}_3 - \vec{z}_2^{k_1}) \tag{8}$$

Here μ_2 is an arbitrary parameter corresponding to the points of the second straight line. And for $\mu_2 = 0$, according to Eq. (8), we get the value of the intersection point of the first and second piecewise-linear straight lines, i.e., for $\mu_2 = 0$ We have $\vec{z}_2^{k_1} = \vec{z}_1^{k_1}$.

In this case, a necessary condition for the existence of the contiguity point between the straight lines will be:

$$\vec{z}_1^{k_1} = \vec{z}_2^{k_1} \tag{9}$$

Here the upper index k_1 indicates the conjugation point between the first and second piecewise-linear straight lines.

The vector equation of the second piecewise-linear straight line Eq. (8) with regard to contact condition Eq. (9) will take the form:

$$\vec{z}_2 = \vec{z}_1^{k_1} + \mu_2(\vec{a}_3 - \vec{z}_1^{k_1}) \tag{10}$$

Here $\vec{z}_1^{k_1}$ is the value of the vector (point) of the first piecewise-linear straight line at the k_1-th point. According to Eq. (6), the value $\vec{z}_1^{k_1}$ will be of the form:

$$\vec{z}_1^{k_1} = \vec{a}_1 + \mu_1^{k_1}(\vec{a}_2 - \vec{a}_1) \tag{11}$$

Here the parameter $\mu_1^{k_1}$ is the value of the parameter μ_1 in the k_1-th contact point taken from the side of the first piecewise-linear straight line.

Substituting Eqs. (11) in (10), express the vector equation of the second piecewise-linear function depending on the vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ and the contact value of the parameter $\mu_1^{k_1}$ in the form:

$$\vec{z}_2 = [\vec{a}_1 + \mu_1^{k_1}(\vec{a}_2 - \vec{a}_1)](1 - \mu_2) + \mu_2\vec{a}_3 \tag{12}$$

Taking into account Eq. (11), we give the vector equation of the second piecewise-linear straight line Eq. (12) the following compact form:

$$\vec{z}_2 = (1 - \mu_2)\vec{z}_1^{k_1} + \mu_2\vec{a}_3 \tag{13}$$

Thus, by Eq. (13), the points of the second piecewise-linear straight line are expressed in the vector form by the values of the conjugation point $\vec{z}_1^{k_1}$, of the vector $\vec{a}_3$ given on the second straight line and also on an arbitrary parameter μ_2 corresponding to the points of the second piecewise-linear straight line.

In particular case, if the conjugation point $\vec{z}_1^{k_1}$ coincides with the point $\vec{a}_2$, i.e., $\vec{z}_1^{k_1} = \vec{a}_2$ then in this case the parameter $\mu_1^{k_1} = 1$. By this, the vector equation of the second piecewise-linear straight line will take the following simple form:

$$\vec{z}_2 = \vec{a}_2 + \mu_2(\vec{a}_3 - \vec{a}_2) \tag{14}$$

Now in the brackets of Eq. (12) add and subtract the quantity $\pm\mu_1(\vec{a}_2 - \vec{a}_1)$ and also take into account the expression for $\vec{z}_1$, given by the Eq. (6). As a result we get:

$$\vec{z}_2 = \vec{z}_1\left[1 - (\mu_1 - \mu_1^{k_1})\frac{\vec{a}_2 - \vec{a}_1}{\vec{z}_1} + \mu_2\frac{\vec{a}_3 - \vec{z}_1^{k_1}}{\vec{z}_1}\right]$$

for

$$\mu_1^{k_1} \le \mu_1 \le \mu_1^{k_2},\ 0 \le \mu_2 \le \mu_2^{k_2} \tag{15}$$

In the obtained Eq. (15), the second and third numbers are represented in the form of symtotic operation of division of vectors, more exactly, $\frac{\vec{a}_2 - \vec{a}_1}{\vec{z}_1}$ and $\frac{\vec{a}_3 - \vec{z}_1^{k_1}}{\vec{z}_1}$. However, it is known that in the axiomatic of finite-dimensional vector space, the symbolic operation of division of vectors has not been determined. Taking into account this fact, we consider this question in detail.

Here the main mathematical problem is to find the way allowing to realize the operation of division of a vector by a vector. For resolving this question, in Section 1.3 of Chapter 1 we suggest a method for constructing a conjugated vector in finite-dimensional Euclidean space. By means of the constructed conjugated vector we suggested a necessary condition that a scalar product of any vector of Euclidean space and the developed modified vector conjugated to it will equal 1. The symbolic operation of division of a vector by a vector in Euclidean space is realized by means of this condition. Below given is the applicability of this method in development of piecewise-linear economic-mathematical models in finite dimensional vector space.

Using what has been said, we have to show what will be equal the first vector fraction of Eq. (15). For that, in the vector equation of the first piecewise-linear straight line take an arbitrary piecewise vector, for instance, $(\vec{z}_1^{k_1} - \vec{a}_1)$ that will be collinear to the numerator of the vector fraction $(\vec{a}_2 - \vec{a}_1)$. Now, by means of these two vectors, i.e., of the vector $(\vec{a}_2 - \vec{a}_1)$ and of the vector $(\vec{z}_1^{k_1} - \vec{a}_1)$, and also by means of Theorem 1 of Section 1 we write the form of the conjugated modified vector. It will be in the form:

$$\vec{c} = \eta(\vec{z}_1^{k_1} - \vec{a}_1) = \frac{\vec{z}_1^{k_1} - \vec{a}_1}{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{k_1} - \vec{a}_1|} \tag{16}$$

Here

$$\eta = \frac{1}{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{k_1} - \vec{a}_1|} \tag{17}$$

is an conjugation factor between the vectors $(\vec{a}_2 - \vec{a}_1)$ and $(\vec{z}_1^{k_1} - \vec{a}_1)$.

Now, multiply the numerator and denominator of the first vector fraction of Eq. (15) by modified vector Eq. (16) conjugated to the numerator, and also take into account that the scalar product of the vector $(\vec{a}_2 - \vec{a}_1)$ and its conjugated modified vector $\vec{c} = \eta(\vec{z}_1^{k_1} - \vec{a}_1)$ equals 1, i.e.,

$$(\vec{a}_2 - \vec{a}_1)\cdot\vec{c} = (\vec{a}_2 - \vec{a}_1)\cdot\eta\cdot(\vec{z}_1^{k_1} - \vec{a}_1) = 1 \tag{18}$$

Then the first vector fraction of Eq. (15) will take the form:

$$\frac{\vec{a}_2 - \vec{a}_1}{\vec{z}_1} = \frac{(\vec{a}_2 - \vec{a}_1)\cdot\vec{c}}{\vec{z}_1\cdot\vec{c}} = \frac{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{k_1} - \vec{a}_1|}{\vec{z}_1\,(\vec{z}_1^{k_1} - \vec{a}_1)} \tag{19}$$

This will be the vector notation of the first vector fraction that will equal some scalar. The coordinate representation of (19) will look like:

$$\frac{\vec{a}_2 - \vec{a}_1}{\vec{z}_1} = \frac{\mu_1^{k_1}\sum_{m=1}^{M}(a_{2m} - a_{1m})^2}{\sum_{m=1}^{M}\vec{z}_{1m}(z_{1m}^{k_1} - a_{1m})} \tag{20}$$

Similarly we show the way for getting rid of vector notation of the second vector fraction of Eq. (15). For defining an appropriate conjugated modified vector for the vector $(\vec{a}_3 - \vec{z}_1^{k_1})$, at first on the first piecewise linear straight line we choose an additional vector $(\vec{z}_1 - \vec{z}_1^{k_1})$ that will be adjacent to the vector $(\vec{a}_3 - \vec{z}_1^{k_1})$. The contiguity point of these vectors will be $\vec{z}_1^{k_1}$, and contiguity angle between these vectors will be $\alpha_{1,2}$.

Now by means of these adjacent vectors, i.e., of the vector $(\vec{a}_3 - \vec{z}_1^{k_1})$ and $(\vec{z}_1 - \vec{z}_1^{k_1})$ also of Theorem 2 of Section 1.3 we write the form of the modified vector $\vec{c}$ conjugated to the numerator. It will be of the form:

$$\vec{c} = \eta \cdot (\vec{z}_1 - \vec{z}_1^{k_1}) = \frac{\vec{z}_1 - \vec{z}_1^{k_1}}{|\vec{z}_1 - \vec{z}_1^{k_1}||\vec{a}_3 - \vec{z}_1^{k_1}| cos\alpha_{1,2}} \tag{21}$$

Here,

$$\eta = \frac{1}{|\vec{z}_1 - \vec{z}_1^{k_1}||\vec{a}_3 - \vec{z}_1^{k_1}| cos\alpha_{1,2}}$$

is the conjugation factor between adjacent vectors $(\vec{z}_1 - \vec{z}_1^{k_1})$ and $(\vec{a}_3 - \vec{z}_1^{k_1})$.

Now multiply the numerator and denominator of the second vector fraction of Eq. (15) by modified vector $\vec{c}$ of Eq. (21) conjugated to the numerator, and also take into account that the scalar product of the vector $(\vec{a}_3 - \vec{z}_1^{k_1})$ and its conjugated modified vector $\vec{c} = \eta \cdot (\vec{z}_1 - \vec{z}_1^{k_1})$ equals 1, i.e.,

$$(\vec{a}_3 - \vec{z}_1^{k_1}) \cdot \vec{c} = (\vec{a}_3 - \vec{z}_1^{k_1}) \cdot \eta \cdot (\vec{z}_1 - \vec{z}_1^{k_1}) = 1$$

Then the second vector fraction of Eq. (15) takes the form:

$$\frac{\vec{a}_3 - \vec{z}_1^{k_1}}{\vec{z}_1} = \frac{(\vec{a}_3 - \vec{z}_1^{k_1}) \cdot \vec{c}}{\vec{z}_1 \cdot \vec{c}} = \frac{|\vec{z}_1 - \vec{z}_1^{k_1}||\vec{a}_3 - \vec{z}_1^{k_1}|}{\vec{z}_1 (\vec{z}_1 - \vec{z}_1^{k_1})} cos\alpha_{1,2} \tag{22}$$

Here, the value of $\cos\alpha_{1,2}$ between the first and second piecewise-linear straight lines is determined by means of the equation of scalar product of two adjacent vectors $\overrightarrow{AB} = \vec{z}_2 - \vec{z}_1^{k_1}$ and $\overrightarrow{AC} = \vec{z}_1 - \vec{z}_1^{k_1}$ in the form (Fig. 3):

$$\cos\alpha_{1,2} = \frac{\overrightarrow{AB} \cdot |\overrightarrow{AC}|}{|\overrightarrow{AB}| \cdot |\overrightarrow{AC}|} = \frac{(\vec{z}_1 - \vec{z}_1^{k_1})(\vec{z}_2 - \vec{z}_1^{k_1})}{|\vec{z}_1 - \vec{z}_1^{k_1}| \cdot |\vec{z}_2 - \vec{z}_1^{k_1}|}$$

While calculating the values of $cos\alpha_{1,2}$ one can use any values of arbitrary parameters μ_1 and μ_2.

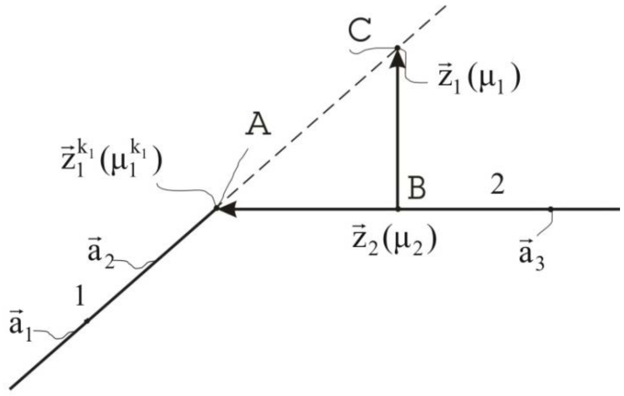

Fig. 3. Dependence of the parameter μ_2 on the parameter μ_1 that corresponds to the points of the first piecewise-linear straight line in finite-dimensional vector space R_m .

Thus, Eq. (22) will be a vector notation of the second vector fraction of Eq. (15) that will equal some scalar. The coordinate representation Eq. (22) will have the following form:

$$\frac{\vec{a}_3 - \vec{z}_1^{k_1}}{\vec{z}_1} =$$

$$= \frac{\sqrt{\sum_{m=1}^{M}(a_{2m} - a_{1m})^2} \cdot \sqrt{\sum_{m=1}^{M}[a_{3m} - a_{1m} - \mu_1^{k_1}(a_{2m} - a_{1m})]^2}}{\sum_{m=1}^{M} z_{1m}(a_{2m} - a_{1m})} cos\alpha_{1,2}$$

Substituting Eqs. (19) and (22) in Eq. (15), we get:

$$\vec{z}_2 = \vec{z}_1\left[1 + (\mu_1^{k_1} - \mu_1)\frac{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{k_1} - \vec{a}_1|}{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)} + \mu_2\frac{|\vec{z}_1 - \vec{z}_1^{k_1}||\vec{a}_3 - \vec{z}_1^{k_1}|}{\vec{z}_1(\vec{z}_1 - \vec{z}_1^{k_1})}\cos\alpha_{1,2}\right] \tag{23}$$

or

$$\vec{z}_2 = \vec{z}_1\left\{1 + (\mu_1^{k_1} - \mu_1)\frac{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{k_1} - \vec{a}_1|}{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}[1 + \right.$$

$$\left. + \frac{\mu_2}{\mu_1^{k_1} - \mu_1}\frac{|\vec{z}_1 - \vec{z}_1^{k_1}||\vec{a}_3 - \vec{z}_1^{k_1}|}{\vec{z}_1(\vec{z}_1 - \vec{z}_1^{k_1})} \cdot \frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{k_1} - \vec{a}_1|}\cos\alpha_{1,2}]\right\} \tag{24}$$

In this case, the initial point of the second piecewise linear straight line corresponding to the value of the parameters $\mu_2 = 0$ and $\mu_1 = \mu_1^{k_1} \neq 0$ will be expressed by the value of the vector-function of the first piecewise-linear straight line $\vec{z}_1$ at the point $\mu_1^{k_1}$ and by the value of the parameter $\mu_1^{k_1}$ corresponding to the intersection point, in the form:

$$\vec{z}_2\big|_{\mu_2=0} = \vec{z}_1\big|_{\mu_1=\mu_1^{k_1}}$$

In the obtained Eq. (24) the fractional expressions represent the numerical expressions dependent only on arbitrary parameters μ_1 and μ_2, changing in the interval $\mu_1^{k_1} \le \mu_1 \le \mu_1^{k_2}$ and $0 \le \mu_2 \le \mu_2^{k_2}$. Therefore, we can re-denote them in the form of numerical coefficients:

$$A = A(\mu_1) = (\mu_1^{k_1} - \mu_1)\frac{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{k_1} - \vec{a}_1|}{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}$$

$$\lambda_2 = \lambda_2(\mu_1, \mu_2) = \frac{\mu_2}{\mu_1^{k_1} - \mu_1}\frac{|\vec{z}_1 - \vec{z}_1^{k_1}||\vec{a}_3 - \vec{z}_1^{k_1}|}{\vec{z}_1(\vec{z}_1 - \vec{z}_1^{k_1})}\cdot\frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{k_1} - \vec{a}_1|}$$

(25)

Here the parameter μ_1 changes in the segment [$\mu_1^{k_1}, \mu_1^{k_2}$], the parameter μ_2 changes in the segment [0, $\mu_2^{k_2}$], the vector $\vec{z}_1$ depends on the parameter $\mu_1 \geq \mu_1^{k_1}$ in the form:

$$\vec{z}_1 = \vec{z}_1(\mu_1).$$

Taking into account denotation Eq. (25), Eq. (24) takes the form:

$$\vec{z}_2 = \vec{z}_1\{1 + A\ (\mu_1)[1 + \lambda_2(\mu_1, \mu_2)cos\alpha_{1,2}]\}$$

For $\mu_1^{k_1} \leq \mu_1 \leq \mu_1^{k_2}$ (26)

Although this equation is applicable for the case $\mu_1 \geq \mu_1^{k_2}$ as well.

Omitting the brackets in the denoted constants, we can write Eq. (26) in a more convenient form:

$$\vec{z}_2 = \vec{z}_1[1 + A\ (1 + \lambda_2 cos\alpha_{1,2})], \text{ for } \mu_1^{k_1} \leq \mu_1 \leq \mu_1^{k_2} \quad (27)$$

Introducing a new denotation:

$$\omega_2(\lambda_2, \alpha_{1,2}) = \lambda_2 cos\alpha_{1,2} \quad (28)$$

Eq. (27) accepts finally the form:

$$\vec{z}_2 = \vec{z}_1\{1 + A[1 + \omega_2(\lambda_2, \alpha_{1,2})]\}, \text{ for } \mu_1^{k_1} \leq \mu_1 \leq \mu_1^{k_2} \quad (29)$$

And at the intersection point of piecewise-linear straight lines, i.e., $\mu_2 = 0$ we have:

$$\lambda_2 = 0,\ \omega_2(\lambda_2, \alpha_{1,2})\big|_{\mu_2 = 0} = 0$$

Thus, by Eq. (29) we mathematically establish connection of arbitrary points of the second vector equation of a piecewise-linear straight line, on one hand, depending on the equation of the first piecewise-linear straight line $\vec{z}_1$; on the other hand, from the spatial form of the parameter λ_2 and influence function $\omega_2(\lambda_2, \alpha_{1,2})$. And the parameters μ_1 and μ_2 change on the interval $\mu_1^{k_1} \leq \mu_1 \leq \mu_1^{k_2}$ and $0 \leq \mu_2 \leq \mu_2^{k_2}$.

However, it should be noted the fact that the parameters μ_1, μ_2 arbitrarily participate in Eq. (29). We can establish a special mathematical interrelation between these arbitrary parameters using, the condition of equality to zero of the scalar product of two orthogonal vectors. This allows to reduce Eq. (29) to dependence only on one arbitrary parameter. Below we have shown this procedure.

At the arbitrary point *B* of the second piecewise-linear straight line erect a perpendicular to it and continue to the intersection with the first piecewise-linear straight line. By the same token we generate the vector $\overrightarrow{BC}$ that will be perpendicular to the vector $\overrightarrow{BA}$, i.e. $\overrightarrow{BC} \perp \overrightarrow{BA}$ (Fig. 3).

In this case, we write the orthogonality condition $\overrightarrow{BC} \perp \overrightarrow{BA}$ in the form of equality to zero of their scalar product:

$$(\overrightarrow{BC} \cdot \overrightarrow{BA}) = 0 \tag{30}$$

Taking into account Eqs. (6), (10) and (11), write the expressions for the vectors $\overrightarrow{BC}$, $\overrightarrow{BA}$ and get:

$$\begin{aligned}
&1)\ \overrightarrow{BA} = \vec{z}_1^{k_1} - \vec{z}_2(\mu_2) = \vec{z}_1^{k_1} - [\vec{z}_1^{k_1} + \mu_2(\vec{a}_3 - \vec{z}_1^{k_1})] = -\mu_2(\vec{a}_3 - \vec{z}_1^{k_1})\\
&2)\ \overrightarrow{BC} = \vec{z}_1(\mu_1) - \vec{z}_2(\mu_2) = \vec{a}_1 + \mu_1(\vec{a}_2 - \vec{a}_1) - [\vec{z}_1^{k_1} + \mu_2(\vec{a}_3 - \vec{z}_1^{k_1})] =\\
&= \vec{a}_1 + \mu_1(\vec{a}_2 - \vec{a}_1) - \{\vec{a}_1 + \mu_1^{k_1}(\vec{a}_2 - \vec{a}_1) + \mu_2[\vec{a}_3 - \vec{a}_1 - \mu_1^{k_1}(\vec{a}_2 - \vec{a}_1)]\} =\\
&\ = (\vec{a}_2 - \vec{a}_1)(\mu_1 - \mu_1^{k_1}) - \mu_2[\vec{a}_3 - \vec{a}_1 - \mu_1^{k_1}(\vec{a}_2 - \vec{a}_1)]
\end{aligned} \tag{31}$$

Substituting Eq. (31) in scalar product Eq. (30), we get:

$$-\mu_2(\vec{a}_3 - \vec{z}_1^{k_1})\{(\vec{a}_2 - \vec{a}_1)(\mu_1 - \mu_1^{k_1}) - \mu_2[\vec{a}_3 - \vec{a}_1 - \mu_1^{k_1}(\vec{a}_2 - \vec{a}_1)]\} = 0 \tag{32}$$

From Eq. (32) it is seen that one of the values of μ_2 will be $\mu_2 = 0$. In the case $\mu_2 = 0$, the value of the parameter μ_1 will equal:

$$\mu_1 = \mu_1^{k_1} \text{ for } \mu_2 = 0 \tag{33}$$

Condition Eq. (33) will correspond to the conjugation point between the first and second piecewise-linear straight lines.

For an arbitrary value of the parameter $\mu_1 \geq \mu_1^{k_1}$ from Eq. (32) set up a contact condition between the parameters μ_2 and μ_1 in the form:

$$\mu_2 = \frac{(\vec{a}_3 - \vec{z}_1^{k_1})(\vec{a}_2 - \vec{a}_1)}{(\vec{a}_3 - \vec{z}_1^{k_1})^2}(\mu_1 - \mu_1^{k_1}), \text{ for } \mu_1^{k_1} \leq \mu_1 \leq \mu_1^{k_2} \tag{34}$$

Now, using Eq. (34), we can determine the upper value of the parameter μ_1, i.e., $\mu_1^{k_2}$. For that in Eq. (34) accept $\mu_2 = \mu_2^{k_2}$. Then from Eq. (33) we determine its appropriate value $\mu_1^{k_2}$ in the form:

$$\mu_1^{k_2} = \mu_1^{k_1} + \mu_2^{k_2}\frac{(\vec{a}_3 - z_1^{k_1})^2}{(\vec{a}_3 - z_1^{k_1})(\vec{a}_2 - \vec{a}_1)}$$

Taking into account this dependence, the range of the parameter μ_1 by defining the function of the second piecewise-linear straight line will be:

$$\mu_1^{k_1} \leq \mu_1 \leq \mu_1^{k_1} + \mu_2^{k_2}\frac{(\vec{a}_3 - z_1^{k_1})^2}{(\vec{a}_3 - z_1^{k_1})(\vec{a}_2 - \vec{a}_1)} \tag{34.a}$$

Taking into account the rule of scalar product of vectors, we write Eq. (34) in the coordinate form:

$$\mu_2 = \frac{\sum_{m=1}^{M}(a_{2m} - a_{1m})[a_{3m} - a_{1m} - \mu_1^{k_1}(a_{2m} - a_{1m})]}{\sum_{m=1}^{M}[a_{3m} - a_{1m} - \mu_1^{k_1}(a_{2m} - a_{1m})]^2}(\mu_1 - \mu_1^{k_1}) \quad \text{for} \quad \mu_1^{k_1} \leq \mu_1 \leq \mu_1^{k_2} \tag{35}$$

Thus, by Eqs. (34) and (35) we set up mathematical relation between the arbitrary parameters μ_2 and μ_1 corresponding to the points of the first and second piecewise-linear straight lines both in the vector and in coordinate form, respectively [4,5,7-13].

In the particular case, for $\mu_1^{k_1} = 1$, i.e., for $\vec{z}_1^{\mu_1^{k_1}} = \vec{a}_2$, Eqs. (34) and (35) will take a rather simpler form:

$$\mu_2 = \frac{(\vec{a}_2 - \vec{a}_1)(\vec{a}_3 - \vec{a}_2)}{(\vec{a}_3 - \vec{a}_2)^2}(\mu_1 - 1), \text{ for } \mu_1 \geq 1 \tag{36}$$

And in the coordinate form:

$$\mu_2 = \frac{\sum_{m=1}^{M}(a_{3m} - a_{2m})(a_{2m} - a_{1m})}{\sum_{m=1}^{M}(a_{3m} - a_{2m})^2}(\mu_1 - 1), \text{ for } \mu_1 \geq 1 \tag{37}$$

We write a vector equation for the second piecewise-linear straight line Eq. (29) in the vector form. Recall that the vectors $\vec{a}_1$, $\vec{a}_2$, $\vec{a}_3$, $\vec{z}_1$, $\vec{z}_2$, $\vec{z}_1^{k_1}$ were written in *m*-dimensional Euclidean space. In the coordinate form they will look like as:

$$\vec{a}_1 = \sum_{m=1}^{M} a_{1m}\vec{i}_m = a_{11}\vec{i}_1 + a_{12}\vec{i}_2 + \dots\dots\dots + a_{1M}\vec{i}_M$$

$$\vec{a}_2 = \sum_{m=1}^{M} a_{2m}\vec{i}_m = a_{21}\vec{i}_1 + a_{22}\vec{i}_2 + \dots\dots\dots + a_{2M}\vec{i}_M$$

$$\vec{a}_3 = \sum_{m=1}^{M} a_{3m}\vec{i}_m = a_{31}\vec{i}_1 + a_{32}\vec{i}_2 + \dots\dots\dots + a_{3M}\vec{i}_M$$

$$\vec{z}_1 = \sum_{m=1}^{M} z_{1m}\vec{i}_m = z_{11}\vec{i}_1 + z_{12}\vec{i}_2 + \dots\dots\dots + z_{1M}\vec{i}_M$$

$$\vec{z}_2 = \sum_{m=1}^{M} z_{2m}\vec{i}_m = z_{21}\vec{i}_1 + z_{22}\vec{i}_2 + \dots\dots\dots + z_{2M}\vec{i}_M$$

$$\vec{z}_1^{k_1} = \sum_{m=1}^{M} z_{1m}^{k_1}\,\vec{i}_m = z_{11}^{k_1}\vec{i}_1 + z_{12}^{k_1}\vec{i}_2 + \dots\dots\dots + z_{1mM}^{k_1}\vec{i}_M \tag{38}$$

Here $\vec{i}_m$ are unit vectors of *m*-dimensional space.

Taking into account Eq. (38), in the following coordinate form we write the following expressions of scalar products in of vectors and vector module:

1) $\left|\vec{a}_2 - \vec{a}_1\right| = \sqrt{\sum_{m=1}^{M}(a_{2m} - a_{1m})^2}$

2) $\left|\vec{z}_1^{k_1} - \vec{a}_1\right| = \sqrt{\sum_{m=1}^{M}(z_{1m}^{k_1} - a_{1m})^2} =$

$$= \sqrt{\sum_{m=1}^{M}\left\{a_{1m} + \mu_1^{k_1}(a_{2m} - a_{1m}) - a_{1m}\right\}^2} = \mu_1^{k_1}\sqrt{\sum_{m=1}^{M}(a_{2m} - a_{1m})^2}$$

3) $\left|\vec{z}_1 - \vec{z}_1^{k_1}\right| = \sqrt{\sum_{m=1}^{M}(z_{1m} - z_{1m}^{k_1})^2} = \sqrt{(\mu_1 - \mu_1^{k_1})^2\sum_{m=1}^{M}(a_{2m} - a_{1m})^2} =$

$$= (\mu_1 - \mu_1^{k_1})\sqrt{\sum_{m=1}^{M}(a_{2m} - a_{1m})^2}$$

4) $\left|\vec{a}_3 - \vec{z}_1^{k_1}\right| = \sqrt{\sum_{m=1}^{M}(a_{3m} - z_{1m}^{k_1})^2} =$

$$= \sqrt{\sum_{m=1}^{M}\{a_{3m} - [a_{1m} + \mu_1^{k_1}(a_{2m} - a_{1m})]\}^2}$$

5) $\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1) = \sum_{m=1}^{M} z_{1m}\mu_1^{k_1}(a_{2m} - a_{1m}) = \mu_1^{k_1}\sum_{m=1}^{M} z_{1m}(a_{2m} - a_{1m})$

6) $\vec{z}_1(\vec{z}_1 - \vec{z}_1^{k_1}) = \sum_{m=1}^{M} z_{1m}(z_{1m} - \vec{z}_1^{k_1}) =$

$$= \sum_{m=1}^{M} = (\mu_1 - \mu_1^{k_1})\sum_{m=1}^{M} z_{1m}(a_{2m} - a_{1m}) \qquad (39)$$

Substituting Eq. (39) in Eq. (29) and taking into account Eq. (35), we write a vector equation for the points of the second piecewise-linear straight-line in the coordinate form:

$$\vec{z}_2 = \vec{z}_1[1 + A\,[1 + \omega_2(\lambda_2, \alpha_{12})]]\text{, for } \mu_1 \geq \mu_1^{k_1} \qquad (40)$$

Here

$$A = (\mu_1^{k_1} - \mu_1)\frac{\sum_{m=1}^{M}(a_{2m} - a_{1m})^2}{\sum_{m=1}^{M} z_{1m}(a_{2m} - a_{1m})},$$

$$\lambda_2 = \frac{\mu_2}{\mu_1^{k_1} - \mu_1}\cdot\frac{\sqrt{\sum_{m=1}^{M}\{a_{3m} - [a_{1m} + \mu_1^{k}(a_{2m} - a_{1m})]\}^2}}{\sqrt{\sum_{m=1}^{M}(a_{2m} - a_{1m})^2}},$$

$$\omega_2(\lambda_2, \alpha_{1,2}) = \lambda_2 cos\alpha_{1,2},$$

$$\mu_2 = \frac{\sum_{m=1}^{M}(a_{2m} - a_{1m})[a_{3m} - a_{1m} - \mu_1^{k_1}(a_{2m} - a_{1m})]}{\sum_{m=1}^{M}[a_{3m} - a_{1m} - \mu_1^{k_1}(a_{2m} - a_{1m})]^2}(\mu_1 - \mu_1^{k_1}),$$

for $\mu_1 \geq \mu_1^{k_1}$ (41)

We should underline the following points. The parameter λ_2 depends on an arbitrary value of the parameter μ_2 that in its turn by Eq. (36) depends on the parameter μ_1. Therefore, defining by Eq. (40) the points of the second piecewise-linear straight line, it is necessary to give the value of the parameter $\mu_1 \geq \mu_1^{k_1}$ and by means of Eq. (36) to find its appropriate parameter. After that, this value of the parameter μ_2 should be substituted in Eq. (40) and the points of the second piecewise-linear straight line depending on the parameter μ_2 corresponding to the appropriate second piecewise-linear straight line be defined.

Taking into account that in finite-dimensional vector space it holds the expression of the vectors $\vec{z}_2 = \sum_{m=1}^{M} z_{2m}\vec{i}_m$ and $\vec{z}_1 = \sum_{m=1}^{M} z_{nm}\vec{i}_m$, then according to Eq. (40) the coordinates of the vector $\vec{z}_2$, i.e., z_{2m} will be expressed by the coordinates of the first piecewise-linear vector z_{1m}, spatial parameter λ_2 and the influence function $\omega_2(\lambda_2, \alpha_{1,2})$ in the form:

$$z_{2m} = [1 + A\ (1 + \omega_2(\lambda_2, \alpha_{1,2}))]z_{1m}, \text{ for } m = 1,2,3,..., M \quad (42)$$

Here the coefficients λ_2 and $\omega_2(\lambda_2, \alpha_{1,2})$ have the form of Eq. (41).

From Eq. (42) it is seen that by giving coordinates of the first piecewise-linear straight line z_{1m}, and also the angle α_{12} between the piece-wize linear straight lines, by means of Eq. (42) the coordinates of the points of the second piecewise straight line are determined automatically.

Now, by means of the intersection point $\vec{z}_3^{k_2}$ and arbitrarily given point (vector) $\vec{a}_4$ taken on the third piecewise-linear vector line, we write a vector function for the third piecewise-linear straight line in the form (Fig. 4):

$$\vec{z}_3 = \vec{z}_3^{k_2} + \mu_3(\vec{a}_4 - \vec{z}_3^{k_2}) \quad (43)$$

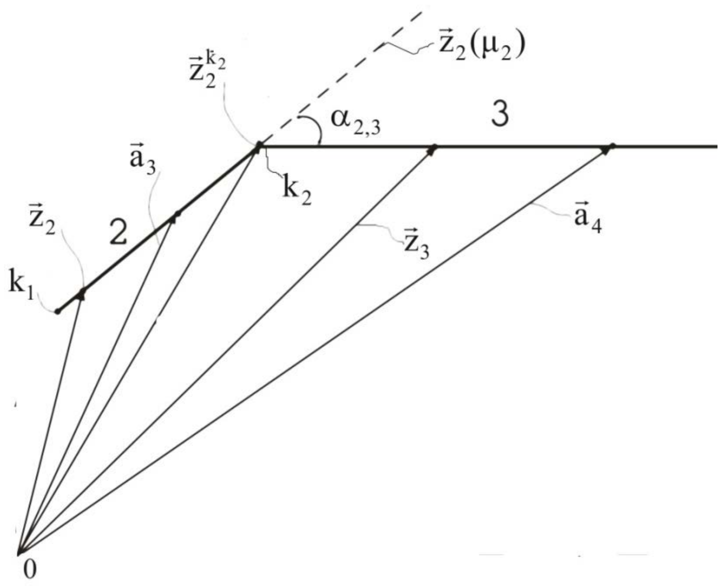

Fig. 4. The scheme of construction of the third piecewise-linear straight line in finite-dimensional vector space R_m.

Here μ_3 is an arbitrary parameter corresponding to the points of the third straight line. And for $\mu_3 = 0$, according to Eq. (43) we get the value of the intersection point of the second and third piecewise-linear straight lines, i.e., for $\mu_3 = 0$ we have:

$$\vec{z}_3 = \vec{z}_3^{k_2}$$

In this case, the condition of existence of the adjacent point between the second and third piecewise-lines straight linear will be:

$$\vec{z}_3^{k_2} = \vec{z}_2^{k_2} \quad (44)$$

Taking into account contact condition Eq. (44), the equation for the third piecewise-linear straight line in vector form Eq. (43) will accept the form:

$$\vec{z}_3 = \vec{z}_2^{k_2} + \mu_3(\vec{a}_4 - \vec{z}_2^{k_2}) \tag{45}$$

Here $\vec{z}_2^{k_2}$ is the value of the point of the second piecewise-linear straight line at the k_2-th conjugation point.

In Eq. (45) the point (vector) $\vec{z}_2^{k_2}$ is calculated by means of the earlier derived Eq. (15). Therefore, in Eq. (15), accepting instead of μ_2 the value of $\mu_2^{k_2}$, the value of the vector $\vec{z}_2^{k_2}$ at the k_2-th point will be of the form:

$$\vec{z}_2^{k_2} = \vec{z}_1\left[1 - (\mu_1 - \mu_1^{k_1})\frac{\vec{a}_2 - \vec{a}_1}{\vec{z}_1} + \mu_2^{k_2}\frac{\vec{a}_3 - \vec{z}_1^{k_1}}{\vec{z}_1}\right] \text{ for } \mu_1^{k_1} \le \mu_1 \le \mu_1^{k_2}$$

Here the parameter $\mu_1^{k_2}$ is connected with the parameter $\mu_2^{k_2}$ in the form:

$$\mu_1^{k_2} = \mu_1^{k_1} + \mu_2^{k_2}\frac{(\vec{a}_3 - \vec{a}_2)^2}{(\vec{a}_3 - \vec{z}_1^{k_1})(\vec{a}_2 - \vec{a}_1)} \tag{46}$$

The equation of the second straight line passing through the two given conjugation points k_1 and k_2 to which there will correspond the values of the parameters $\mu_1^{k_1}$ and $\mu_2^{k_2}$ will be set up by Eq. (46). And in this case, $\vec{z}_2^{k_2}(\mu_1)$ will depend on the parameter μ_1 [4,5,7-13].

Further, substitute Eq. (46) in Eq. (45) and get:

$$\vec{z}_3 = \vec{z}_1\left[1 - (\mu_1 - \mu_1^{k_1})\frac{\vec{a}_2 - \vec{a}_1}{\vec{z}_1} + \mu_2^{k_2}\frac{\vec{a}_3 - \vec{z}_1^{k_1}}{\vec{z}_1}\right](1 - \mu_3) + \mu_3\vec{a}_4 =$$
$$= \vec{z}_2^{k_2}(1 - \mu_3) + \mu_3\vec{a}_4$$

or

$$\vec{z}_3 = \vec{z}_1\left[1 + (\mu_1^{k_1} - \mu_1)\frac{\vec{a}_2 - \vec{a}_1}{\vec{z}_1} + \mu_2^{k_2}\frac{\vec{a}_3 - \vec{z}_1^{k_1}}{\vec{z}_1}\right] -$$
$$- \mu_3\vec{z}_1\frac{\vec{z}_1^{k_1} + \mu_2^{k_2}(\vec{a}_3 - \vec{z}_1^{k_1}) - \vec{a}_4}{\vec{z}_1} \tag{47}$$

Now, according to Eq. (13) the value of the point $\vec{z}_2^{k_2}$ for $\mu_2 = \mu_2^{k_2}$ will be of the form:

$$\vec{z}_2^{k_2} = (1 - \mu_2^{k_2})\vec{z}_1^{k_1} + \mu_2^{k_2}\vec{a}_3$$

Substituting this value in (47), we get:

$$\vec{z}_3 = \vec{z}_1\left[1 + (\mu_1^{k_1} - \mu_1)\frac{\vec{a}_2 - \vec{a}_1}{\vec{z}_1} + \mu_2^{k_2}\frac{\vec{a}_3 - \vec{z}_1^{k_1}}{\vec{z}_1} + \mu_3\frac{\vec{a}_4 - \vec{z}_2^{k_2}}{\vec{z}_1}\right] \text{ for } \mu_1 \ge \mu_1^{k_2} \tag{48}$$

In Eq. (48) the numbers are represented in the form of symbolic operation of division of vectors. Above we showed the way for getting rid of the vector notation of the first two fraction expressions. The results of the first two fraction expressions were represented in the form of Eqs. (19) and (22).

Now get rid of the vector notation of the third fraction $\frac{\vec{a}_4 - \vec{z}_2^{k_2}}{\vec{z}_1}$. For that construct a modified vector $\vec{c}$ conjugate to the vector $(\vec{a}_4 - \vec{z}_2^{k_2})$. For that, on the vector equation of the second piecewise-linear straight line choose an additional piecewise-linear vector $(\vec{z}_2 - \vec{z}_2^{k_2})$ for $\mu_2 > \mu_2^{k_2}$ that will be adjacent to the vector $(\vec{a}_4 - \vec{z}_2^{k_2})$. The contiguity point of these vectors will be $\vec{z}_2^{k_2}$, and the adjacent angle between these vectors will be $\alpha_{2,3}$. And this angle $\alpha_{2,3}$ will lie in the plane of these vectors. Now, by means of these adjacent vectors, i.e., of the vectors $(\vec{a}_4 - \vec{z}_2^{k_2})$, and $(\vec{z}_2 - \vec{z}_2^{k_2})$, also of Theorem 2 of Section 3.1 of Chapter 1 we write the form of the conjugate modified vector $\vec{c}$. It will have the form:

$$\vec{c} = \eta \cdot (\vec{z}_2 - \vec{z}_2^{k_2}) = \frac{\vec{z}_2 - \vec{z}_2^{k_2}}{\left|\vec{z}_2 - \vec{z}_2^{k_2}\right| \left|\vec{a}_4 - \vec{z}_2^{k_2}\right| cos\alpha_{2,3}} \tag{49}$$

Here

$$\eta = \frac{1}{\left|\vec{z}_2 - \vec{z}_2^{k_2}\right| \left|\vec{a}_4 - \vec{z}_2^{k_2}\right| cos\alpha_{2,3}}$$

of is the coefficient between the adjacent vectors $(\vec{z}_2 - \vec{z}_2^{k_2})$ and $(\vec{a}_4 - \vec{z}_2^{k_2})$.

Here the value of $cos\alpha_{2,3}$ between the second and third piecewise-linear straight lines is determined by means of the scalar product of two adjacent vectors $\overrightarrow{AB} = \vec{z}_3(\mu_3) - \vec{z}_2^{k_2}(\mu_2^{k_2})$ and $\overrightarrow{AC} = \vec{z}_2(\mu_2) - \vec{z}_2^{k_2}(\mu_2)$ in the form (Fig. 5):

$$cos\alpha_{2,3} = \frac{\overrightarrow{AB} \cdot \left|\overrightarrow{AC}\right|}{\left|\overrightarrow{AB}\right| \cdot \left|\overrightarrow{AC}\right|} = \frac{[\vec{z}_3(\mu_3) - \vec{z}_2^{k_2}][\vec{z}_2(\mu_2) - \vec{z}_2^{k_2}]}{\left|\vec{z}_3(\mu_3) - \vec{z}_2^{k_2}\right| \cdot \left|\vec{z}_2(\mu_2) - \vec{z}_2^{k_2}\right|}$$

Now multiply the numerator and denominator of the third vector fraction of Eq. (48) by the modified vector $\vec{c}$ Eq. (49) conjugate to the vector $(\vec{a}_4 - \vec{z}_2^{k_2})$, and also take into account that the scalar product of the vector $(\vec{a}_4 - \vec{z}_2^{k_2})$ and the modified vector $\vec{c} = \eta \cdot (\vec{z}_2 - \vec{z}_2^{k_2})$ conjugated to it, equals 1, i.e.,

$$(\vec{a}_4 - \vec{z}_2^{k_2}) \cdot \vec{c} = (\vec{a}_4 - \vec{z}_2^{k_2}) \cdot \eta \cdot (\vec{z}_2 - \vec{z}_2^{k_2}) = 1$$

Then the third vector fraction of Eq. (48) takes the form:

$$\frac{\vec{a}_4 - \vec{z}_2^{k_2}}{\vec{z}_1} = \frac{(\vec{a}_4 - \vec{z}_2^{k_2}) \cdot \vec{c}}{\vec{z}_1 \cdot \vec{c}} = \frac{\left|\vec{z}_2 - \vec{z}_2^{k_2}\right| \left|\vec{a}_4 - \vec{z}_2^{k_2}\right|}{\vec{z}_1 (\vec{z}_2 - \vec{z}_2^{k_2})} cos\alpha_{2,3} \tag{50}$$

This will be the vector notation of the third vector fraction of Eq. (48) that will be equal to some scalar. The coordinate notation of Eq. (50) will be of the form:

$$\frac{\vec{a}_4 - \vec{z}_2^{k_2}}{\vec{z}_1} = \frac{\sqrt{\sum_{m=1}^{M} (z_{2m} - z_{2m}^{k_2})^2} \cdot \sqrt{\sum_{m=1}^{M} (a_{4m} - z_{2m}^{k_2})^2}}{\sum_{m=1}^{M} z_{1m} (z_{2m} - z_{2m}^{k_2})} \cdot cos\alpha_{2,3}$$

Substituting Eqs. (50), (19) and (22) in Eq. (48), write the vector equation for the points of the third piecewise-linear straight line in the form:

$$\vec{z}_3 = \vec{z}_1 \left\{ 1 + (\mu_1^{k_1} - \mu_1) \frac{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{k_1} - \vec{a}_1|}{z_1(\vec{z}_1^{k_1} - \vec{a}_1)} + \mu_2^{k_2} \frac{|\vec{z}_1 - \vec{z}_1^{k_1}||\vec{a}_3 - \vec{z}_1^{k_1}|}{z_1(\vec{z}_1 - \vec{z}_1^{k_1})} cos\alpha_{1,2} + \right.$$

$$\left. + \mu_3 \cdot \frac{|\vec{z}_2 - \vec{z}_2^{k_2}||\vec{a}_4 - \vec{z}_2^{k_2}|}{z_1(\vec{z}_2 - \vec{z}_2^{k_2})} cos\alpha_{2,3} \right\}, \text{ for } \quad \mu_1 \geq \mu_1^{k_2}, \ \mu_3 \geq 0 \qquad (51)$$

or

$$\vec{z}_3 = \vec{z}_1 \left\{ 1 + (\mu_1^{k_1} - \mu_1) \frac{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{k_1} - \vec{a}_1|}{z_1(\vec{z}_1^{k_1} - \vec{a}_1)} \left[1 + \right. \right.$$

$$+ \frac{\mu_2^{k_2}}{\mu_1^{k_1} - \mu_1} \cdot \frac{|\vec{z}_1 - \vec{z}_1^{k_1}||\vec{a}_3 - \vec{z}_1^{k_1}|}{z_1(\vec{z}_1 - \vec{z}_1^{k_1})} \cdot \frac{z_1(\vec{z}_1^{k_1} - \vec{a}_1)}{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{k_1} - \vec{a}_1|} cos\alpha_{1,2} +$$

$$\left. \left. + \frac{\mu_3}{\mu_1^{k_1} - \mu_1} \cdot \frac{|\vec{z}_2 - \vec{z}_2^{k_2}||\vec{a}_4 - \vec{z}_2^{k_2}|}{z_1(\vec{z}_2 - \vec{z}_2^{k_2})} \cdot \frac{z_1(\vec{z}_1^{k_1} - \vec{a}_1)}{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{k_1} - \vec{a}_1|} cos\alpha_{2,3} \right] \right\}$$

for

$$\mu_1 \geq \mu_1^{k_2}, \mu_3 \geq 0 \qquad (52)$$

Note that all the points of the third piecewise-linear straight line in *m*-dimensional vector space will be determined by Eq. (52). And the case $\mu_3 = 0$ will correspond to the value of the initial point of the third piecewise-linear straight line that will be defined by the vector function of the first piecewise-linear straight line $\vec{z}_1$, the values of the parameters $\mu_1^{k_1}$ and $\mu_2^{k_2}$ of intersection points of piecewise-linear straight lines, and also on $cos\alpha_{1,2}$ generated between the first and second piecewise-linear straight lines. It has the form [4,5,7-13]:

$$\vec{z}_3\Big|_{\mu_3=0} = \vec{z}_1 \left\{ 1 + (\mu_1^{k_1} - \mu_1) \frac{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{k_1} - \vec{a}_1|}{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)} \left[1 + \right. \right.$$

$$\left. \left. + \frac{\mu_2^{k_2}}{\mu_1^{k_1} - \mu_1} \cdot \frac{|\vec{z}_1 - \vec{z}_1^{k_1}||\vec{a}_3 - \vec{z}_1^{k_1}|}{z_1(\vec{z}_1 - \vec{z}_1^{k_1})} \cdot \frac{z_1(\vec{z}_1^{k_1} - \vec{a}_1)}{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{k_1} - \vec{a}_1|} \right] cos\alpha_{1,2} \right\}$$

Using Eq. (29), calculate the following expressions:

1) $\left|\vec{z}_2 - \vec{z}_2^{k_2}\right| = \left|A\vec{z}_1[\omega_2(\lambda_2, \alpha_{1,2}) - \omega_2(\lambda_2^{k_2}, \alpha_{1,2})]\right|$

2) $\left|\vec{a}_4 - \vec{z}_2^{k_2}\right| = \left|\vec{a}_4 - \vec{z}_1[1 + A(1 + \omega_2(\lambda_2^{k_2}, \alpha_{1,2}))]\right|$

3) $(\vec{z}_2 - \vec{z}_2^{k_2}) = \vec{z}_1[1 + A(1 + \omega_2(\lambda_2, \alpha_{1,2}))] -$

$- \vec{z}_1[1 + A(1 + \omega_2(\lambda_2^{k_2}, \alpha_{1,2}))] =$

$$= A\vec{z}_1[\omega_2(\lambda_2, \alpha_{1,2}) - \omega_2(\lambda_2^{k_2}, \alpha_{1,2})] \qquad (53)$$

Here the form of the expression of the unaccounted factors parameter $\lambda_2^{k_2}$ is the expression of the form λ_2 given by Eq. (25), where the parameter μ_2 was replaced by the parameter $\mu_2^{k_2}$, i.e.:

$$\lambda_2^{k_2} = \frac{\mu_2^{k_2}}{\mu_1^{k_1} - \mu_1} \cdot \frac{\left|\vec{z}_1 - \vec{z}_1^{k_1}\right| \left|\vec{a}_3 - \vec{z}_1^{k_1}\right|}{\vec{z}_1(\vec{z}_1 - \vec{z}_1^{k_1})} \cdot \frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{k_1} - \vec{a}_1\right|}$$

$$\omega_2(\lambda_2^{k_2}, \alpha_{1,2}) = \lambda_2^{k_2} cos\alpha_{1,2}$$

$$A = (\mu_1^{k_1} - \mu_1) \frac{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{k_1} - \vec{a}_1\right|}{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)} \tag{54}$$

Taking into account Eq. (54), vector function for the third piecewise-linear straight line Eq. (52) will be of the form:

$$\vec{z}_3 = \vec{z}_1\{1 + A\,[(1 + \omega_2(\lambda_2^{k_2}, \alpha_{1,2}) + \omega_3(\lambda_3, \alpha_{2,3})]\}$$

or

$$\mu_1 \geq \mu_1^{k_2} \tag{55}$$

Here

$$\omega_3(\lambda_3, \alpha_{2,3}) = \lambda_3 cos\alpha_{2,3} =$$

$$= \frac{\mu_3}{\mu_1^{k_1} - \mu_1} \cdot \frac{\left|\vec{z}_2 - \vec{z}_2^{k_2}\right| \left|\vec{a}_4 - \vec{z}_2^{k_2}\right|}{\vec{z}_1(\vec{z}_2 - \vec{z}_2^{k_2})} \cdot \frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{k_1} - \vec{a}_1\right|} cos\alpha_{2,3}$$

for

$$\mu_1 \geq \mu_1^{k_2} \tag{56}$$

And at the intersection point of the second and third piecewise-linear straight lines, i.e., for $\mu_3 = 0$ we have:

$$\lambda_3 = 0, \quad \omega_3(\lambda_3, \alpha_{2,3}) = 0$$

In order to write the unaccounted parameters spatial function $\omega_3(\lambda_3, \alpha_{2,3})$ by the values of the first vector $\vec{z}_1$ and $\vec{z}_1^{k_1}$ in Eq. (56), it is necessary to express the vector-functions $\vec{z}_2$ and $\vec{z}_2^{k_2}$ by means of Eqs. (25) and (29) by $\vec{z}_1$ and $\vec{z}_1^{k_1}$. Performing this operation, represent the function $\omega_3(\lambda_3, \alpha_{2,3})$ in the form:

$$\omega_3(\lambda_3, \alpha_{2,3}) = \lambda_3 cos\alpha_{2,3} = \frac{\mu_3}{\mu_1^{k_1} - \mu_1} \cdot$$

$$\cdot \frac{\left|A\vec{z}_1\right| \left|\vec{z}_1[\omega_2(\lambda_2, \alpha_{1,2}) - \omega_2(\lambda_2^{k_2}, \alpha_{1,2})]\right| \left|\vec{a}_4 - \vec{z}_1[1 + A(1 + \omega_2(\lambda_2^{k_2}, \alpha_{1,2})]\right|}{A\vec{z}_1^2[\omega_2(\lambda_2, \alpha_{1,2}) - \omega_2(\lambda_2^{k_2}, \alpha_{1,2})]} \cdot$$

$$\cdot \frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{k_1} - \vec{a}_1\right|} cos\alpha_{2,3} \tag{57}$$

Thus, by Eq. (55) we mathematically set up the relation of arbitrary points of the third vector equation of piecewise-linear straight line depending on the vector equation of the first piecewise-linear straight line, of spatial parameters $\lambda_2^{k_2}$ and λ_3 and also on the unaccounted parameter spatial influence function $\omega_3(\lambda_3, \alpha_{2,3})$ Eq. (56).

$$(\overrightarrow{BC} \cdot \overrightarrow{BA}) = 0 \tag{58}$$

Note that in Eq. (55) the parameters μ_1 and μ_3 participate arbitrarily. Below we have established mathematical relation between these parameters. At the arbitrary point ***B*** of the third piecewise-linear straight line, erect a perpendicular to it and continue it to the intersection with the second piecewise-linear straight line. Thus, we form the vector ***BC*** with its origin at the point ***B*** on the third straight line and the finite point ***C*** on the second straight line. By the same token, we geometrically form the vector $\overrightarrow{BC}$ that will be perpendicular to the vector $\overrightarrow{BA}$, i.e., $\overrightarrow{BC} \perp \overrightarrow{BA}$ (Fig. 5).

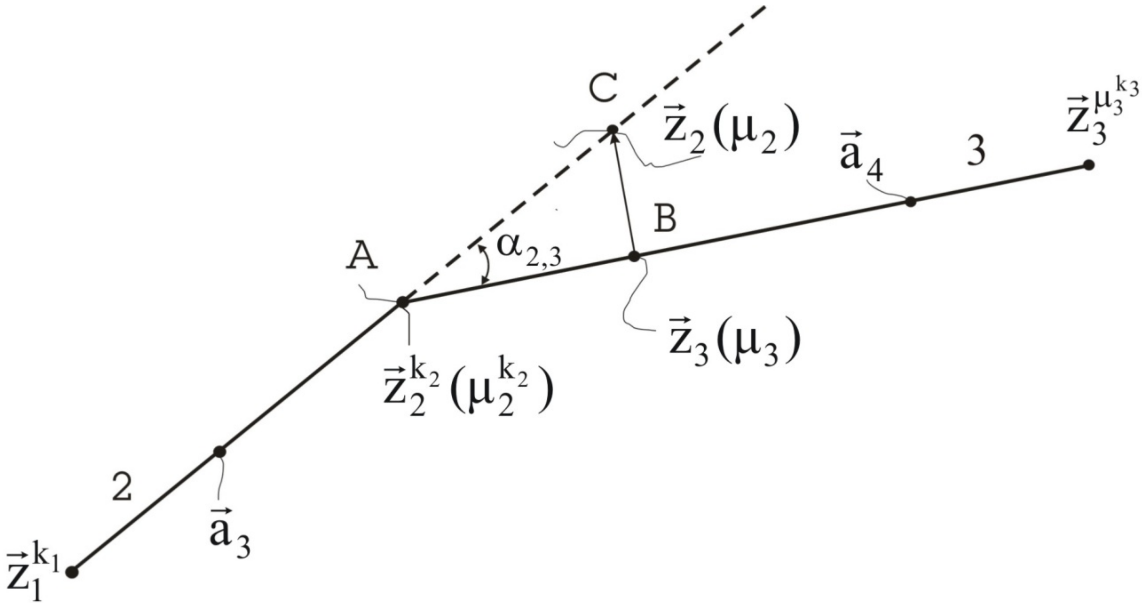

Fig. 5. Dependence of the parameter μ_3 on the parameter μ_2 that corresponds to the points of the second piecewise-linear straight line in finite-dimension vector space R_m.

In this case, we write the orthogonality condition $\overrightarrow{BC} \perp \overrightarrow{BA}$ in the form of equality to zero of their scalar product (Fig. 5).

According to Fig. 5, we have

1. $\vec{z}_3 = z_2^{k_2} + \mu_3(\vec{a}_4 - \vec{z}_2^{k_2})$ for $z_3^{k_2} = \vec{z}_2^{k_2}$ (59)

2. $\vec{z}_2 = z_1^{k_1} + \mu_2(\vec{a}_3 - \vec{z}_1^{k_1})$ (60)

3. for $\mu_2 = \mu_2^{k_2}$ (from Eq. (60)), we get:

$$\vec{z}_2^{k_2} = z_1^{k_1} + \mu_2^{k_2}(\vec{a}_3 - \vec{z}_1^{k_1}) \quad (61)$$

Taking into account Eqs. (59)–(61), write the expressions of the vectors $\overrightarrow{BC}$, $\overrightarrow{BA}$ and get:

$$\overrightarrow{BA} = \vec{z}_2^{k_2} - \vec{z}_3(\mu_3) = \vec{z}_2^{k_2} - [\vec{z}_2^{k_2} + \mu_3(\vec{a}_4 - \vec{z}_2^{k_2})] = \\ = \mu_3(\vec{a}_4 - \vec{z}_2^{k_2}) \quad (62)$$

$$\overrightarrow{BC} = \vec{z}_2(\mu_2) - \vec{z}_3(\mu_3) = (1 - \mu_2)\vec{z}_1^{k_1} + \mu_2\vec{a}_3 - [\vec{z}_2^{k_2} + \\ \mu_3(\vec{a}_4 - \vec{z}_2^{k_2})] = \vec{z}_2(\mu_2) - \vec{z}_2^{k_2} - \mu_3(\vec{a}_4 - \vec{z}_2^{k_2})$$

or, allowing for Eqs. (59) and (61), get:

$$\overrightarrow{BC} = (\mu_2 - \mu_2^{k_2})(\vec{a}_3 - \vec{z}_1^{k_1}) - \mu_3[\vec{a}_4 - z_1^{k_1} - \mu_2^{k_2}(\vec{a}_3 - \vec{z}_1^{k_1})] \quad (63)$$

Substituting Eqs. (62) and (63) in vector product Eq. (58), we get:

$$\mu_3(\vec{a}_4-\vec{z}_2^{k_2})\{(\mu_2-\mu_2^{k_2})(\vec{a}_3-\vec{z}_1^{k_1})- \\ -\mu_3[\vec{a}_4-\mu_2^{k_2}(\vec{a}_3-\vec{z}_1^{k_1})-\vec{z}_1^{k_1}]\}=0 \tag{64}$$

From Eq. (64) it is seen that one of the values $\mu_3=0$. This means that the parameter μ_3 begins from the end of the second piecewise-linear straight line.

For an arbitrary value of the parameter μ_3, we get the following relation between the parameters:

$$\mu_3=(\mu_2-\mu_2^{k_2})\frac{(\vec{a}_3-\vec{z}_1^{k_1})(\vec{a}_4-\vec{z}_2^{k_2})}{(\vec{a}_4-\vec{z}_2^{k_2})^2}, \quad \text{for} \qquad \mu_2\geq\ \mu_2^{k_2},\ \mu_1\geq\ \mu_1^{k_2} \tag{65}$$

Thus, by Eq. (65) we established mathematical relation of an arbitrary parameter μ_3 and parameter μ_2 that corresponds to the point of the second piecewise-linear straight line beginning from $\mu_2=\ \mu_2^{k_2}$. On the other hand, by Eq. (34) the parameter μ_2 is connected with the parameter μ_1. If we substitute the value of μ_2 definable by Eq. (34), in Eq. (65), we get a catenary dependence of the parameter μ_3 on μ_1.

Now we can give Eq. (65) the following coordinate form:

$$\mu_3=(\mu_2-\mu_2^{k_2})\frac{\sum_{m=1}^{M}(a_{3m}-z_{1m}^{k_1})(a_{4m}-z_{2m}^{k_2})}{\sum_{m=1}^{M}(a_{4m}-z_{2m}^{k_2})^2}, \quad \text{for} \qquad \mu_{2\geq}\ \mu_2^{k_2} \tag{66}$$

By similar calculations we can follow the method for constructing a vector equation for the fourth piecewise-linear straight line. According to the above-indicated method, set up the dependence of the points of the fourth piecewise-linear straight line on difference of economic process rate holding between the third and fourth piecewise-linear straight lines, in the following form:

$$\vec{z}_4=\vec{z}_1\{1+A\,[(1+\omega_2(\lambda_2^{k_2},\alpha_{1,2})+ \\ +\omega_3(\lambda_3^{k_3},\alpha_{2,3})+\omega_4(\lambda_4,\alpha_{3,4})]\} \tag{67}$$

Here,

$$\omega_2(\lambda_2^{k_2},\alpha_{1,2})=\lambda_2^{k_2}cos\alpha_{1,2}= \\ =\frac{\mu_2^{k_2}}{\mu_1^{k_1}-\mu_1}\cdot\frac{|\vec{z}_1-\vec{z}_1^{k_1}||\vec{a}_3-\vec{z}_1^{k_1}|}{\vec{z}_1(\vec{z}_1-\vec{z}_1^{k_1})}\cdot\frac{\vec{z}_1(\vec{z}_1^{k_1}-\vec{a}_1)}{|\vec{a}_2-\vec{a}_1||\vec{z}_1^{k_1}-\vec{a}_1|}cos\alpha_{1,2}$$

$$\omega_3(\lambda_3^{k_3},\alpha_{2,3})=\lambda_3^{k_3}cos\alpha_{2,3}= \\ =\frac{\mu_3^{k_3}}{\mu_1^{k_1}-\mu_1}\cdot\frac{|\vec{z}_2-\vec{z}_2^{k_2}||\vec{a}_4-\vec{z}_2^{k_2}|}{\vec{z}_1(\vec{z}_2-\vec{z}_2^{k_2})}\cdot\frac{\vec{z}_1(\vec{z}_1^{k_1}-\vec{a}_1)}{|\vec{a}_2-\vec{a}_1||\vec{z}_1^{k_1}-\vec{a}_1|}cos\alpha_{2,3}$$

$$\omega_4(\lambda_4,\alpha_{3,4})=\lambda_4cos\alpha_{3,4}= \\ =\frac{\mu_4}{\mu_1^{k_1}-\mu_1}\cdot\frac{|\vec{z}_3-\vec{z}_3^{k_3}||\vec{a}_5-\vec{z}_3^{k_3}|}{\vec{z}_1(\vec{z}_3-\vec{z}_3^{k_3})}\cdot\frac{\vec{z}_1(\vec{z}_1^{k_1}-\vec{a}_1)}{|\vec{a}_2-\vec{a}_1||\vec{z}_1^{k_1}-\vec{a}_1|}cos\alpha_{3,4} \tag{68}$$

$$\mu_4=(\mu_3-\mu_3^{k_3})\frac{(\vec{a}_4-\vec{z}_2^{k_2})(\vec{a}_5-\vec{z}_3^{k_3})}{(\vec{a}_5-\vec{z}_3^{k_3})^2}, \text{for} \qquad \mu_{3\geq}\ \mu_3^{k_3} \tag{69}$$

By the recurrent method it is easy to get in finite-dimensional Euclidean space the dependence of any n-th piecewise-linear vector equation $\vec{z}_n$ on the first piecewise-linear function $\vec{z}_1$ and all spatial type influence function of unaccounted parameters $\omega_n(\lambda_n^{k_n}, \alpha_{n\text{-}1,n})$ influencing on the preceding interval of economic event, in the form (Fig. 6) [4,5,7-13]:

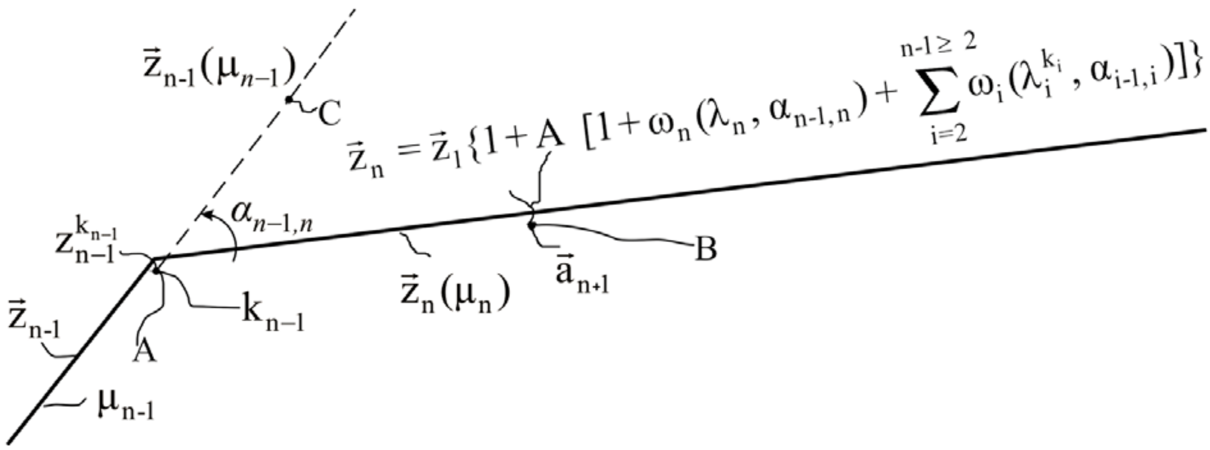

Fig. 6. The scheme of construction of the n-th piecewise-linear vector equation in finite-dimensional vector space R_m.

$$\vec{z}_n = \vec{z}_1\left\{1 + A\left[1 + \omega_n(\lambda_n, \alpha_{n\text{-}1,n}) + \sum_{i=2}^{n\text{-}1\geq 2} \omega_i(\lambda_i^{k_i}, \alpha_{i\text{-}1,i})\right]\right\} \tag{70}$$

Here

$$\omega_i(\lambda_i^{k_i}, \alpha_{i\text{-}1,i}) = \lambda_i^{k_i} cos\alpha_{i\text{-}1,i} = = \frac{\mu_i^{k_i}}{\mu_1^{k_1} - \mu_1} \cdot \frac{\left|\vec{z}_{i\text{-}1} - \vec{z}_{i\text{-}1}^{k_{i\text{-}1}}\right|\left|\vec{a}_{i+1} - \vec{z}_{i\text{-}1}^{k_{i\text{-}1}}\right|}{\vec{z}_1(\vec{z}_{i\text{-}1} - \vec{z}_{i\text{-}1}^{k_{i\text{-}1}})} \cdot \frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right|\left|\vec{z}_1^{k_1} - \vec{a}_1\right|} cos\alpha_{i\text{-}1,i} \tag{71}$$

$$\mu_i = (\mu_{i\text{-}1} - \mu_{i\text{-}1}^{k_{i\text{-}1}})\frac{(\vec{a}_i - \vec{z}_{i\text{-}2}^{k_{i\text{-}2}})(\vec{a}_{i+1} - \vec{z}_{i\text{-}1}^{k_{i\text{-}1}})}{(\vec{a}_{i+1} - \vec{z}_{i\text{-}1}^{k_{i\text{-}1}})^2}, \text{ for } \mu_{i\text{-}1\geq}\ \mu_{i\text{-}1}^{k_{i\text{-}1}} \tag{72}$$

$$\omega_n(\lambda_n, \alpha_{n\text{-}1,n}) = \lambda_n cos\alpha_{n\text{-}1,n} = = \frac{\mu_n}{\mu_1^{k_1} - \mu_1} \cdot \frac{\left|\vec{z}_{n\text{-}1} - \vec{z}_{n\text{-}1}^{k_{n\text{-}1}}\right|\left|\vec{a}_{n+1} - \vec{z}_{n\text{-}1}^{k_{n\text{-}1}}\right|}{\vec{z}_1(\vec{z}_{n\text{-}1} - \vec{z}_{n\text{-}1}^{k_{n\text{-}1}})} \frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right|\left|\vec{z}_1^{k_1} - \vec{a}_1\right|} cos\alpha_{n\text{-}1,n}$$

(73)

$$A = (\mu_1^{k_1} - \mu_1)\frac{\left|\vec{a}_2 - \vec{a}_1\right|\left|\vec{z}_1^{k_1} - \vec{a}_1\right|}{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)} \tag{74}$$

$$\mu_n = (\mu_{n\text{-}1} - \mu_{n\text{-}1}^{k_{n\text{-}1}})\frac{(\vec{a}_n - \vec{z}_{n\text{-}2}^{k_{n\text{-}2}})(\vec{a}_{n+1} - \vec{z}_{n\text{-}1}^{k_{n\text{-}1}})}{(\vec{a}_{n+1} - \vec{z}_{n\text{-}1}^{k_{n\text{-}1}})^2},\ \mu_{n\text{-}1\geq}\ \mu_{n\text{-}1}^{k_{n\text{-}1}} \tag{75}$$

The value of $\cos\alpha_{n\text{-}1,n}$ between the (n–1)-th and the n-th piecewise linear vector equations is determined according to Fig. 6. in the form:

$$cos\alpha_{n-1,n} = \frac{(\vec{z}_{n-1} - \vec{z}_{n-1}^{\,k_{n-1}})(\vec{a}_{n+1} - \vec{z}_{n-1}^{\,k_{n-1}})}{\left|\vec{z}_{n-1} - \vec{z}_{n-1}^{\,k_{n-1}}\right|\left|\vec{a}_{n+1} - \vec{z}_{n-1}^{\,k_{n-1}}\right|} \tag{76}$$

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